Italian Association of Aeronautics and Astronautics
XXVI International Congress
31st August – 3rd September 2021
On-line event hosted by the Tuscany AIDAA Section in Pisa

"Paper for the Special Session on DEVISU Project"

PERIODICITY AND QUASI-PERIODICITY EFFECTS ON VIBRATION BAND GAPS: NUMERICAL INVESTIGATIONS ON ONE-DIMENSIONAL STRUCTURES

A. Pecoraro [1*], D. Magliacano [1], G. Petrone [1], M. Filippi [2], F. Franco [1], S. De Rosa [1]

[1] PASTA-Lab (Laboratory for Promoting experiences in Aeronautical STructures and Acoustics),
Department of Industrial Engineering - Aerospace Section, Università degli Studi di Napoli "Federico II",
Via Claudio 21, 80125 Naples, Italy

[2] Department of Mechanical and Aerospace Engineering
Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, ITALY

* ale.pecoraro@studenti.unina.it

ABSTRACT

Periodic structures have found a big interest in engineering application because they introduce frequency band effects due to the impedance mismatch generated by periodic discontinuities in the geometry, material, or boundary conditions. Adding periodicity to structures leads to wave mode interaction, which generates pass- and stop-bands. The frequencies at which stop-bands occur are related to the periodic nature of the structure. The presence of defects or irregularity in the structure leads to a partial loss of regular periodicity (quasi periodic structure) that can have a noticeable impact on the vibrational and/or acoustic behavior of the elastic structure.
The aim of this study is to analyze the dynamic behaviors of periodic and quasi-periodic structures, and to investigate where band gaps appear in the frequency range. First, the behavior of an infinite periodic structure through the dispersion diagrams is studied compared with the results obtained through the Carrera Unified Formulation (CUF). Then, numerical studies on different beams are done to compare band gaps position at different periodic and quasi-periodic sequences.

1 INTRODUCTION

The advantage of dealing with periodic configurations is that it is possible to study the base cell by applying the periodic boundary conditions, thus reducing computational costs. A one-dimensional base cell is considered, and the dispersion curves are obtained [1]. A first study is then carried out with the COMSOL software. The results are compared with those obtained with the Carrera Unified Formulation (CUF) method [2, 3]. In the framework of the unified formulation, classical theories assumptions are systematically neglected by enriching the kinematic field with an arbitrary number of terms that expand the mechanical variables over the cross-section domain, with certain functions of the xz-plane. In its compact form it can be expressed as:

$$\mathbf{u}(x, y, z) = F_\tau(x, y)\, \boldsymbol{u_\tau}(y), \quad \tau = 1, 2, \ldots, M$$

where $\mathbf{u}(x, y, z)$ is the three-dimensional displacement field, $\boldsymbol{u_\tau}(y)$ is the vector of generalized displacements, M is the number of terms of the kinematic field and $F_\tau(x, y)$ are the so-called *expansion functions*. The choice of the theory of structure becomes in this way arbitrary and can be introduced in the structural beam model as an input when defining F_τ [4]. In this case Taylor Expansions are chosen. The reference wave guide is shown in Figure 1.

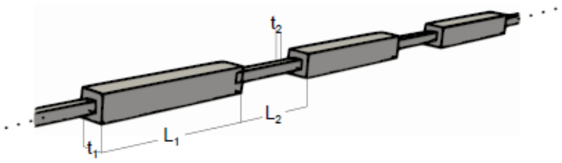

Figure 1: Sketch of a multi-section wave guide.

The geometric characteristics are: L_1=0.17 m, L_2=0.1 m, t_1=0.025 m and t_2=0.01 m. Material characteristics are: Young modulus E=210 GPa, Poisson ratio ν=0.30, density ρ=7850 kg/m³. For the first analysis, the unit cell is discretized in three Lagrangian elements considering a first case with 3 nodes for each portion of segment, and a second case with 4 nodes. The second analysis is made with the software COMSOL and the geometry is discretized with 373 triangular and tetrahedral elements. Periodic boundary conditions of Floquet-Bloch are applied [5,6].

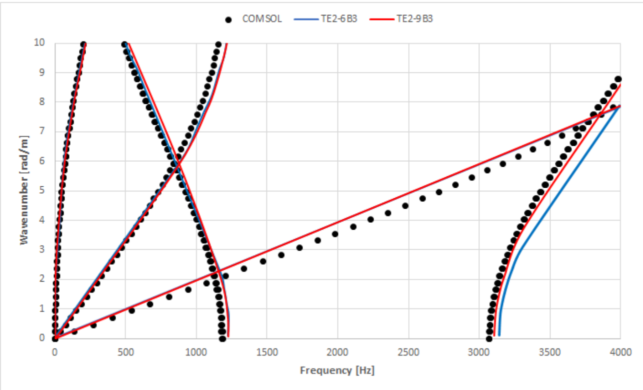

Figure 2: Dispersion curves (COMSOL and TE2).

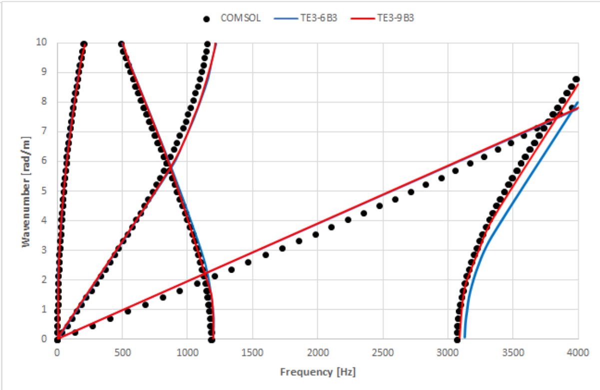

Figure 3: Dispersion curves (COMSOL and TE3).

Figure 2 and Figure 23 show the results of the dispersion curves. About CUF method, the solutions are indicated with the acronym TEN, where N is the polynomial order. Instead, the notations 6B3 and 9B3 indicate respectively the case with 3 and 4 nodes for each portion of segment. The numerical results show that two stopbands are generated. These are 210-480 Hz and 1200-3090 Hz. Both theories predict approximately the same position and length of the stopbands and as the degree of the polynomial increases, the solutions tend to overlap with those obtained with COMSOL analysis.

2 PERIODICITY EFFECTS ON VIBRATION BAND GAPS

2.1 Geometric impedance

For finite structures, vibrations levels are calculated by the frequency response function (FRF) [7,8]. A first study is carried out on 1D beams of fixed length L=1.2 m and made of aluminum (E=73 GPa, ν=0.33, ρ=2700 kg/m³). The base structure is divided in 12 cells of equal length, each cell is modelled on finite element (FE) in 3 beams element. To obtain the normalized response, the entire structure is loaded with a flexural load on extreme point of 1 N. To consider the physics of the problem, a 1% damping is also considered. No constraints have been placed on the structure which therefore has free-free boundary conditions. To compare the answer between the various analyses a constant mass of 5 kg is set for each beam and the height of the individual cells is obtained. The solution is evaluated considering the mean energy value in dB that pass thought the structures. The reference case has cells of equal height. The first periodic structure analyzed is that with two periodic cells: A, B. The unit cells, that are repeated all over the structures are: AB and BA. Two different cases of their height's ratio $\frac{h_a}{h_b}$ are considered to analyze the effects of the impedance on bandgaps: case 1 with $\frac{h_a}{h_b} = 2.6$ and case 2 with $\frac{h_a}{h_b} = 1.6$.

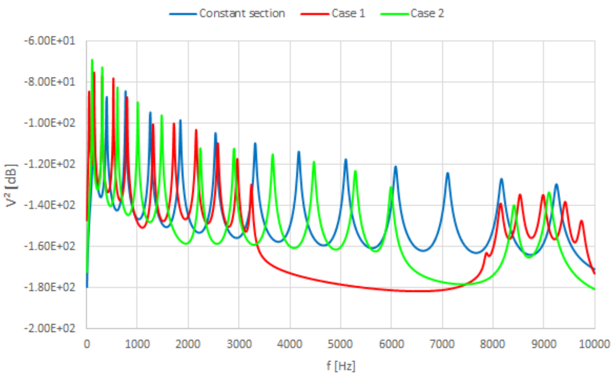

Figure 4: Mean value of the squares of velocities (AB cells).

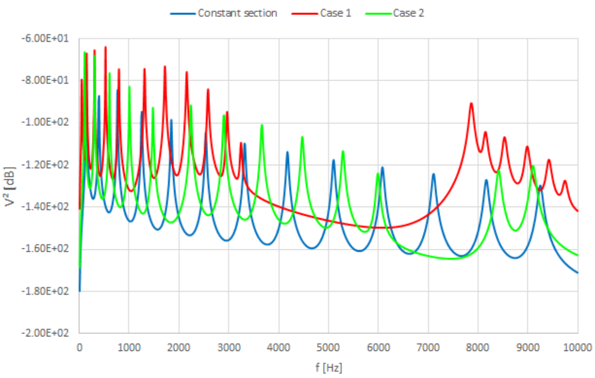

Figure 5: Mean value of the squares of velocities (BA cells).

As expected, the higher is the impedance mismatch, the higher is the frequency range in which the bandgap appears. In AB sequence (Figure 4), the bandgap is in the same frequency range of the BA one (Figure 5), the only difference being the lower energy level, because the load is applied on B cell. This allows more energy to enter in the structure.

2.2 Material impedance

Impedance mismatch generated by material is now analyzed: aluminum (cell A) and steel (cell B, E=210 GPa, ν=0.26, ρ=7800 kg/m³) are chosen.

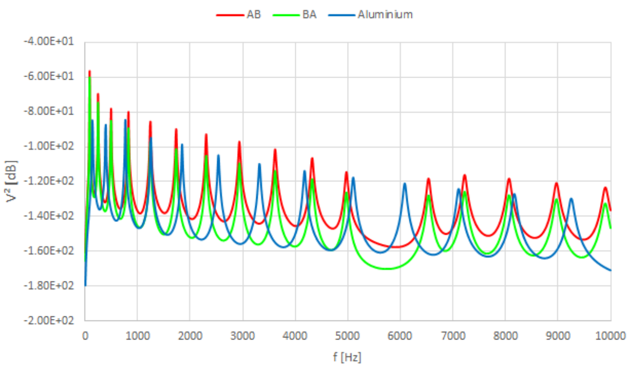

Figure 6: Mean value of the squares of velocities (material impedance effect).

No extended bandgaps are generated, and only one between 5000-6000 Hz is visible (Figure 6). As expected, loading the aluminum cell, more energy enters through the system.

3 QUASI-PERIODICITY EFFECTS ON VIBRATION BAND GAPS

The presence of imperfections in the structure, such as defects or irregularities, leads to the loss of the periodicity and thus can have a noticeable impact on the vibrational and/or acoustic behavior of the elastic structure [9,10].

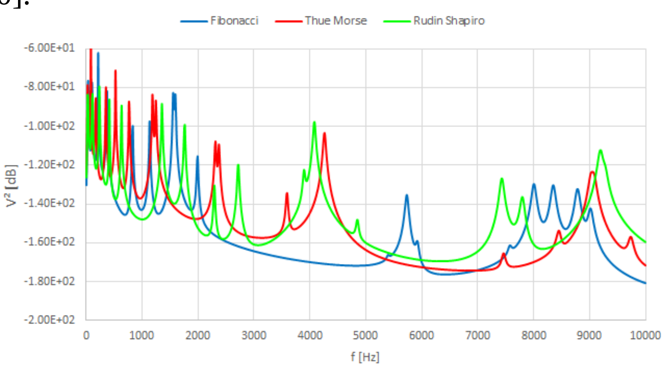

Figure 7: Comparison of quasi periodic sequences.

In this case, the property of quasi-periodicity is defined [11]. The quasi-periodic structures considered in this work are of the type generally known as *substitutional sequences* and are: Fibonacci sequence, Thue Morse sequence, Rudin Shapiro sequence [12,13].
Results in Figure 7 show that, compared to periodic sequences, quasi-periodic structures have an efficient impact on reducing the response in lower frequency but do not show large band gap widths.

4 CONCLUSIONS

The contents presented herein deal with the FE models of beams focused on the spectral analysis and the damped forced responses: a FE model is developed to predict the modal and frequency response of different configurations of beams. The average energy is evaluated, and results show that the frequency range in which band gaps appear depends on periodic sequences, and the higher the impedance mismatch, the more extended it is. The quasi-periodic structures with geometrical impedance mismatch have an efficient impact on reducing the response in lower frequency regimes.

ACKNOWLEDGMENTS

The authors acknowledge the support of the Italian Ministry of Education, University and Research (MIUR) through the project DEVISU, funded under the scheme PRIN-2107 – grant agreement No. 22017ZX9X4K006.

BIBLIOGRAPHY

[1] Parthkumar G Domadiya et al. “Numerical and experimental investigation of stop-bands in finite and infinite periodic one-dimensional structures”. In: *Vibration and Control* (2015).
[2] E. Carrera. “Theories and finite elements for multilayered anisotropic, composite plates and shells”. In: *Archives of Computational Methods in Engineering* (2012).
[3] E. Carrera. “Theories and finite elements for multilayered plates and shells: a unified compact formulation with numerical assessment and benchmarking.” In: *Archives of Computational Methods in Engineering* (2003).
[4] E. Carrera et al. “Extension of MITC to higher-order beam models and shear locking analysis for compact, thin-walled and composite structures”. In: *International journal for numerical methods in engineering* (2017).
[5] Pablo Gómez García and José-Paulino Fernández-Álvarez. “Floquet-Bloch Theory and Its Application to the Dispersion Curves of Nonperiodic Layered Systems”. In: *Hindawi Publishing Corporation* Volume 2015 Article ID 475364 (2015).
[6] Dario Magliacano. “Vibroacoustics of porous media with periodic inclusions”. PhD thesis. Universite de Franche-Comte (Besancon FR), Università degli studi di Napoli "Federico II" (Napoli IT), 2020.
[7] D. J. Mead. “Wave propagation in continuous periodic structures: research contributions from Southampton”. In: *Journal of Sound and Vibration* 190(3):495-524 (1996).
[8] S. Timorian et al. “Spectral analysis and structural response of periodic and quasi-periodic beams”. In: *Journal Mechanical Engineering Science* 233(23-24):7498-7512 (2019).
[9] M. Gei. “Wave propagation in quasi periodic structures: stop/pass band distribution and prestress effects”. In: *International Journal of Solids and Structures* 47:3067-3075 (2010).
[10] V.R. Velasco and J.E. Zárate. “Elastic waves in quasiperiodic structures”. In: *Progress in Surface Science* 67: 383-402 (2001).
[11] Vladislav S. Sorokin. “Longitudinal wave propagation in a one-dimensional quasi periodic waveguide”. In: *Proceedings of the Royal Society of London. Series A* 475: 20190392 (2019).
[12] Z. Hou, F. Wu, and Y. Liu. “Acoustic wave propagating in one-dimensional Fibonacci binary composite systems”. In: *Physica B: Condensed Matter* 344: 391-397 (2004).
[13] E.L. Albuquerque and M.G. Cottam. “Theory of elementary excitations in quasi periodic structures”. In: *Physics Reports* 376: 225-337 (2002).