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An optimal distributed PID-like control for the output containment and leader-following of heterogeneous high-order multi-agent systems

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ABSTRACT

This paper addresses the leader-tracking and the containment control problems for heterogeneous high-order Multi-Agent Systems (MASs) sharing information through a directed communication topology. To solve both the control problems, a fully-distributed Proportional-Integral-Derivative (PID) control strategy is proposed, whose stability is analytically proven by exploiting the regulator equations and the Static Output Feedback (SOF) procedure adapted to the MASs framework. The application of SOF allows recasting the PID control design problem into a state-feedback control design one and, hence, finding the proper values of the proportional, integral and derivative actions via classical state-feedback approaches, such as the Linear-Quadratic-Regulator (LQR) strategy. Numerical simulations confirm the effectiveness of the proposed approach in guaranteeing that each follower tracks the leader behavior in the case of leader-tracking and that each follower converges to the convex hull spanned by the multiple leaders in the case of containment control.

1. Introduction

In the last years, distributed consensus-based controllers for Multi-Agent Systems (MASs) have attracted compelling attention due to their wide range of applications (e.g. sensor networks, robotics, electric power systems, autonomous vehicles and so on) and their intrinsic ability to overcome some of the well-known limits of the centralized control approaches, for example in the presence of a limited spatial distribution of sensors or short-range communications (see [1] and references therein for an overview). Early works in this research field mainly address the consensus problem without considering the presence of a leader node, so that all nodes are driven to converge toward a common, non prescribed, evolution [2]. More recently, in order to achieve a commanded trajectory, the leader-following approach has been introduced, where a leading node is exploited so to impose the desired behavior to the agents [3].

However, in several engineering applications (e.g, when dealing with earth monitoring or the obstacles avoidance for mobile robots), there might be more than one leader [4]. In these cases, a containment control problem arises, whose objective is to design distributed control protocols driving the followers to enter into the convex hull spanned by multiple leaders [5].

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The presence of a single leader or of multiple leaders has been often addressed in the wide technical literature related to MAS. However, the control design is tailored to the specific problem to be solved, i.e. leader-tracking or containment, usually under the restrictive assumption of homogeneous dynamics ranging from single or double integrator to high-order linear or non-linear systems (see for example [6] and the references therein). For homogeneous MASs, the control approaches leverage different techniques, from optimal control strategies [7] to adaptive [8,9] and robust approaches [10].

In practice, agents dynamics can not be modeled by the same nominal behavior and, hence, more recently, the realistic case of heterogeneous MASs, where each agent reacts according to its own dynamics, has received great attention. Naturally, the heterogeneous framework is more challenging and complex with respect to the analytical derivation, since the overall closed-loop system can not be described in a global form by leveraging the Kronecker product and a set of regulator equations have to be solved [11]. In this context, different state feedback control techniques have been proposed. For example, diffusive state feedback and H_∞ controllers have been proposed in [11] and in [12], respectively, to solve the leader-tracking problem in presence of external disturbances acting on agents dynamics. With the same aim at coping with uncertainties on the leading node dynamics, the leader-tracking problem has been addressed by designing a diffusive state feedback control associated with a state observer in [13] and an adaptive full-state feedback strategy in [14]. State feedback protocols have been also proposed to solve containment, as in [15] for MASs composed of singular systems, or in [16] where the presence of communication delays has been also accounted for, or in [17] for heterogeneous MASs in the presence of asynchronous updates. Moreover, a state feedback finite-time containment control protocol has been proposed in [18] for homogeneous second-order nonlinear MASs. To cope with model uncertainties, external disturbances, and thruster faults, [19,20] suggest adaptive state feedback finite-time containment control strategies for heterogeneous second-order nonlinear MASs with application to Ocean Bottom Flying Node. Finally, state feedback control methods are also suggested to solve the containment control problem for general linear discrete-time MASs under the asynchronous setting in [21], where the network topology is not subjected to any structural restriction and the roles of the leaders and the followers are entirely determined by the network topology.

Anyhow, full state information is not always available in practical applications that, instead, require to drive the control action based on a reduced amount of output measurements [11]. Most of the existing consensus and synchronization output feedback protocols are purely diffusive and usually exploit distributed proportional actions based on current information. These kinds of approaches are proposed for the output leader-tracking of linear heterogeneous multi-agent systems (see e.g. [22,23]), of nonlinear heterogeneous MASs (see e.g. [24]) and for containment control of heterogeneous linear high-order MASs (see e.g. [25,26,15]). Fewer attempts are made in the MAS field for improving steady-state and transient closed-loop performances via additional distributed integral and derivative actions (see e.g. [27] and references therein). Despite its well-known benefits, PID control and its variations are lightly addressed in the distributed MAS control and they are mainly dealt with homogeneous MASs (see e.g. [28] and references therein).

For instance, distributed PI and PD protocols are proposed in [29] to solve the synchronization problem for linear homogeneous second order MASs in the presence of external disturbances. Herein a H_2 norm metric is also provided to evaluate the coherence, i.e. the variance of nodal state fluctuations. To address the same problem, but for the case of linear high-order agents, a robust PID consensus control strategy has been proposed under the restrictive assumption of an undirected communication graph in [30]. A PD protocol is proposed in [27] to solve, instead, the problem of the average consensus under a fast arbitrarily switching topology for the case of first-order nonlinear homogeneous MASs with Lipschitz dynamics. To solve the leader-tracking for uncertain high-order homogeneous MASs, robust PID protocols have been investigated [31,32] with the aim at coping with the presence of both parameter uncertainties and delays in the communication among agents.

Nevertheless, practical applications in the distributed framework require nonidentical models due to the unavoidable mismatches and differences among the agents. In this context, only few distributed protocols aim at extending the PID advantages to the high-order heterogeneous MASs framework.

Along this line, the mathematical derivation usually needs restrictive assumptions on the input matrix B_i of the agents and this strongly limits the applicability of the approach for solving practical problems, where matrices may have a generic structure. More specifically, to solve the leader-tracking control problem, in [33] it is required that each agent must have a full row rank input matrix B_i and, as observed by the authors themselves, this condition can be very stringent. Instead, in [34] each agent must be characterized by an input matrix B_i that has to be properly selected such that each of the input acts only on one specific state variable. Conversely, distributed PID controller for the output containment has been rarely addressed in the technical literature. For instance, [35] suggests a distributed generalized PID control strategy to drive the homogeneous high-order MASs into the convex hull spanned by the leaders. However, the herein proposed approach is not fully-distributed since it requires, for its proper design, the knowledge about the communication graph topology.

With the aim at designing a fully-distributed PID controller that can be applied to generic high-order heterogeneous MASs, sharing information via directed communication topologies, in our work we remove any hypothesis on the input matrix structure of agents dynamics. We provide a flexible mathematical framework allowing to solve both output leader-tracking and output containment control problems, so that the proposed PID structure may accomplish different requirements depending on the practical applications, i.e.: *i*) tracking a generic time-varying reference behavior if there exists only one leader; *ii*) reaching the convex hull spanned by multiple leaders.

The closed-loop stability analysis leverages both the regulator equations conditions for dealing with the agents heterogeneity [11] and the application of the Static Output Feedback (SOF) transformation [36] to the MASs framework. This allows: *i*) recasting the PID control design into a state feedback one and, hence, to find, for each agent of the MAS, optimal gains values by exploiting the classic Linear-Quadratic-Regulator (LQR) approach and its typical performance index; *ii*) handling the derivative action more easily w.r.t. other complex approaches presented in the technical literature, such as the descriptor one [31]. Note that, due to SOF transformation, our approach allows embedding any state feedback control strategy, hence providing flexibility to different control gains tuning procedure. Numerical simulations confirm the theoretical derivation and disclose the ability of the approach in solving both the output leader tracking and the output containment control problems. Most notably, to better appreciate the potential applications of our approach, we apply our method for solving two practical problems in the engineering literature, namely cooperative driving of autonomous connected ground vehicles and three-dimensional containment control of Unmanned Aerial Vehicles. Moreover, a comparison analysis w.r.t. alternative control approaches, proposed in the very recent literature on both output leader-tracking and output containment for heterogeneous high-order MAS, is presented to disclose the advantages of the proposed control algorithm in terms of closed-loop performances and computational burden. Summarizing, the main contributions of the work are:

- We propose a fully-distributed optimal PID strategy able to solve both output leader-tracking and output containment control problems for generic heterogeneous high-order linear MASs, where agents share information via a directed communication topology. It is worth noting that, although PID controllers are often proposed to solve the leader-tracking control problem, they are never proposed to solve the containment one in the case of heterogeneous agents.
- We extend the current results on PID-like protocols for solving the containment problem, while also overcoming at the same time some of the limitations on the leader-tracking in the case of agents heterogeneity. Differently from the alternative PID-like approaches proposed in [33,34], the control design does not require the fulfillment of additional constraints on the control input matrix of each dynamical agent. Indeed, in our case, the input matrix has a more general structure and, hence, our fully-distributed PID can be also applied to Multi-Input Multi-Output (MIMO) MASs.
- We leverage the SOF transformation in the MASs framework to mathematical handle the derivative action without requiring the more complex procedure based on descriptor transformation.
- We extend the Static Output Feedback transformation to the MAS framework so as to recast the fully-distributed PID output feedback control problem into a state feedback control one. In so doing, we propose a PID gains tuning algorithm which allows exploiting any state-feedback control method, hence providing a greater flexibility w.r.t. additional control requirements depending on the specific application. Note that, among these different control techniques, we consider the LQR one since it computes an optimal control action sequence that allows reaching the control objective while minimizing the control effort and enjoying some important robustness properties.
- We propose a control architecture that is easier to implement and is able to reduce the computational demand w.r.t. alternative methods in the current literature. Differently from distributed model predictive or adaptive approaches, requiring high computational burden due to the need for on-line solving some optimization problems or for computing the adaptation law in real-time [22,23], we propose a more computationally effective approach that is based on the classical fixed-gain framework, where the optimization of the control action is off-line performed during the control design phase via LQR. Moreover, differently from the output feedback approaches in [15,25], our control architecture does not require any additional local observers to estimate the full state of the agents within the MAS, hence reducing the complexity of the overall control architecture.

Finally, the paper is organized as follows. Mathematical preliminaries are provided in Section II. Section III is dedicated to the problem statement, while the analytical derivation proving the ability of the fully-distributed PID in achieving both output leader-tracking and output containment is detailed in Section IV. In Section V, numerical simulations are presented to validate the effectiveness of the theoretical results. Conclusions and future works are drawn in Section VI.

2. Mathematical preliminaries and assumptions

A generic MAS composed of N followers and M leaders can be modeled as a directed graph $\mathcal{G}_{N+M} = \{\mathcal{V}_{N+M}, \mathcal{E}_{N+M}\}$, where $\mathcal{V}_{N+M}$ is the set of vertices, while $\mathcal{E}_{N+M} \subseteq \{\mathcal{V}_{N+M} \times \mathcal{V}_{N+M}\}$ is the set of edges that define the communication network topology. Node $v_i \in \mathcal{V}_{N+M}$ can be connected with node $v_j \in \mathcal{V}_{N+M}$ through a direct communication link $(v_i, v_j) \in \mathcal{E}_{N+M}$ or via a directed path. A directed path from node $v_{i1} \in \mathcal{V}_{N+M}$ to node $v_{il} \in \mathcal{V}_{N+M}$ is a sequence of ordered direct edges of the form $(v_{ik}, v_{ik+1}) \in \mathcal{E}_{N+M}$ with $k = 1, \dots, l-1$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+M)\times(N+M)}$, with $a_{ij} = 1$ if there is a link from node j to node i; 0 otherwise. In this context, we assume that the self-loops are not present and that any leader does not receive information from other agents. Now, indicating as $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}_{N+M}\}$ the set of all nodes with edges incoming to i, we can define the in-degree matrix $\mathcal{D} = \text{diag}(d_i)$, being $d_i = \sum_{j=1}^{N+M} a_{ij}$ the weighted in-degree of node i. The associated Laplacian matrix, defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, can be partitioned as follows:

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix}, \tag{1}$$

where $M = 1$ in the case of a single leader, while $M > 1$ in the multi-leader case; $\mathcal{L}_2 \in \mathbb{R}^{N\times M}$ reports how followers are connected to the leader/s; $\mathcal{L}_1 \in \mathbb{R}^{N\times N}$ describes the connections among the followers.

The following Assumptions and Lemmas hold.

Assumption 1. [15] $\mathscr{G}_{N+M}$ is such that for each follower, there exists, at least, one leader that has a directed path with the follower.

Lemma 1. *[37] Under the Assumption 1, all eigenvalues of $\mathscr{L}_1$ have positive real part, each entry of $-\mathscr{L}_1^{-1}\mathscr{L}_2$ is non-negative and the sum of each of his row is equal to 1.*

We recall here some definitions and technical results exploited in the following theoretical derivation.

Definition 1. Denote as $\text{dist}(x,\mathscr{O})$ the distance from $x \in R^n$ to the set $\mathscr{O} \subset R^n$ in the sense of Euclidean norm, i.e. [26]:

$$\text{dist}(x,\mathscr{O}) = \inf_{y\in\mathscr{O}}\|x-y\|.$$

Definition 2. Let $\mathscr{O}$ be a set in a real vector space $D \subseteq R^m$. The set $\mathscr{O}$ is convex if, for any x and y in $\mathscr{O}$, the point $(1-\tau)x+\tau y$ is in $\mathscr{O}$ for any $\tau \in [0,1]$. The convex hull spanned by a set of points $X = \{x_1,\ldots,x_m\}$ in D is the minimal convex set containing all points in X. We use $\text{Co}(X)$ to denote the convex hull spanned by X.

Definition 3. [Moore–Penrose pseudoinverse [38]] Consider a generic matrix $A \in \mathbb{R}^{n\times m}$ with $n \geqslant m$ and rank m. Hence, we can define the left pseudoinverse as $A^\dagger = \left(A^TA\right)^{-1}A^T$ such that $A^\dagger A = I_m$. In a similar way, if $A \in \mathbb{R}^{n\times m}$ with $n \leqslant m$ and rank n, there exists the right pseudoinverse as $A^\dagger = A^T\left(AA^T\right)^{-1}$ such that $AA^\dagger = I_n$. Obviously, if A is a square matrix $\in \mathbb{R}^{n\times m}$, with $n = m$ and full rank, the pseudoinverse is equal to the standard inverse.

Finally, we have the following claim.

Consider the linear time-invariant system

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{2}$$

with the following PID controller

$$u = F_1 y + F_2\int_0^t y\,d\tau + F_3\frac{dy}{dt}, \tag{3}$$

where $x \in R^n$ is the state variable, $u \in R^l$ is the control input, $y \in R^m$ is the output; A, B, C are matrices with appropriate dimensions; $F_1, F_2, F_3 \in R^{1\times m}$ are matrices to be designed.

Condition 1. The matrix $(I - F_3CB)$ is invertible.

Lemma 2. *[Well-posedness of multi-variable PID [36]] Given system (2) under the action of PID controller in (3), the closed-loop system is well-posed [39] if and only if Condition 1 holds.*

3. Problem statement

Consider a MAS composed of N heterogeneous agents, plus M leaders that impose the reference behaviour for the whole MAS sharing information via a directed communication topology. The dynamics of each follower i $(i = 1,\ldots,N)$ are described by the following linear high-order dynamical system:

$$\begin{aligned}\dot{x}_i(t) &= A_ix_i(t) + B_iu_i(t),\\ y_i(t) &= C_ix_i(t),\end{aligned} \tag{4}$$

being $x_i(t) \in \mathbb{R}^n, y_i(t) \in \mathbb{R}^q, u_i(t) \in \mathbb{R}^m$, the state, the output and the control input vectors, respectively, while $A_i \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^{n\times m}$ and $C_i \in \mathbb{R}^{q\times n}$ with $q \leqslant n/2$.

The reference dynamics provided by the M leaders are, instead, described via the following multi-output neutrally stable exosystem:

$$\begin{aligned}\dot{\xi}_k(t) &= S\xi_k(t)\\ l_k(t) &= R\xi_k(t),\end{aligned} \tag{5}$$

being $k = N+1,\ldots,N+M$ and $\xi_k(t) \in \mathbb{R}^n, l_k \in \mathbb{R}^q$ the state and output vectors, respectively; $R \in \mathbb{R}^{q\times n}$ is the output matrix and $S \in \mathbb{R}^{n\times n}$ is the dynamic matrix such that for all $\lambda \in \rho(S)$, being $\rho(S)$ the spectrum of S (i.e., $\rho(S) = \{\lambda : det(\lambda I_n - S) = 0\}$), it holds [23]:

$$\operatorname{rank}\begin{bmatrix} A_i - \lambda I_n & B_i \\ C_i & 0 \end{bmatrix} = n + q, \; i = 1, \ldots, N. \tag{6}$$

Assumption 2. The leader/s matrices S and R are assumed to be directly available for each follower i $(i = 1, \ldots, N)$ [25].

Now, if there is just one leader $(M = 1)$ a leader-tracking control problem arises, while we have, instead, a containment control problem in the case of multiple leaders $(M > 1)$. More specifically, the following problems can be stated for the high-order heterogeneous MASs.

Problem 1. [*Leader-Tracking*] Let $M = 1$ in (5). The multi-agent system (4)–(5) (being $i = 1, \ldots, N; k = N + 1$) is said to achieve output leader-tracking if appropriate control law $u_i(t)$ in (4) is such that the followers outputs track the leader dynamics as time approaches infinity, i.e. $\forall i$ we have:

$$\lim_{t\to\infty} \|y_i(t) - l_{N+1}(t)\| = \lim_{t\to\infty} \|e_i(t)\| = 0. \tag{7}$$

Problem 2. [*Output Containment*] Let $M > 1$ in (5). The multi-agent system (4)–(5) (being $i = 1, \ldots, N; k = N + 1, \ldots, N + M$) is said to achieve output containment if appropriate control law $u_i(t)$ in (4) is such that the followers outputs converge to the convex hull spanned by the outputs of the dynamic leaders as time approaches infinity, i.e. $\forall i$ we have:

$$\lim_{t\to\infty} \operatorname{dist}(y_i(t), \operatorname{Co}(\{l_{N+1}, \ldots, l_{N+M}\})) = 0. \tag{8}$$

Remark 1. *Assumption* 1 guarantees that the leader tracking/containment control problem is well-posed. Without this assumption, there would be at least one isolated follower that can not obtain any information from any leader.

To solve both Problem 1 and Problem 2, for each agent i, here we propose a fully-distributed output feedback PID protocol that updates its action based on the i-th agent output information and the information shared among neighbors as:

$$u_i(t) = K_{Pi}(y_i(t) - C_i\Pi_i\zeta_i(t)) + K_{Ii}\int_0^t (y_i(s) - C_i\Pi_i\zeta_i(s))ds + K_{Di}\left(\dot{y}_i(t) - C_i\Pi_i\dot{\zeta}_i(t)\right) + \Gamma_i\zeta_i(t) \tag{9}$$

where $K_{Pi}, K_{Ii}, K_{Di} \in \mathbb{R}^{m\times q}$ are the proportional, integral and derivative control gains vectors of positive values such that *Condition 1* holds; $\zeta_i(t) \in \mathbb{R}^n$ is $\forall i$ the synchronized references signal, whose dynamics depend on the shared information about the synchronized references generator of the neighboring agents within its communication range and the information about the references behaviour provided by the leader/s within its communication range [40,41,11], i.e.:

$$\dot{\zeta}_i(t) = S\zeta_i(t) + \sigma\left[\sum_{j=1}^{N} a_{ij}\left(\zeta_j(t) - \zeta_i(t)\right) + \sum_{k=N+1}^{N+M} a_{ik}(\xi_k(t) - \zeta_i(t))\right], \tag{10}$$

where $\sigma \in \mathbb{R}_+; a_{ij}$ and a_{ik} models the communication network topology according to Section 2; matrices $\Pi_i \in \mathbb{R}^{n\times n}$ and $\Gamma_i \in \mathbb{R}^{m\times n}$ $(i = 1, \ldots, N)$ are such that the following regulator equations are fulfilled [42]:

$$A_i\Pi_i + B_i\Gamma_i = \Pi_i S, \quad C_i\Pi_i = R. \tag{11}$$

The solvability of (11) is guaranteed by choosing the spectrum of S as in (6) (see [40]).

Note that, the distributed control protocol $u_i(t) \in \mathbb{R}^{m\times 1}$ $(i = 1, \ldots, N)$ in (9) is, hence, based both on the networked information shared among neighboring agents and the fulfillment of regulator Eq. (11), that provides for each agent i the matrices $\Pi_i, \Gamma_i \in \mathbb{R}^{m\times n}$. Note that the solution of (11) requires for each agent i that Assumption 2 holds.

Finally, we remark that the PID control strategy (9), is fully-distributed since it needs no global information about the communication topology (e.g., the smallest positive eigenvalue of the Laplacian matrix) [43,22,15].

4. Main result

In this section, we analytically prove the ability of the proposed fully-distributed PID control strategy (9) in achieving both leader-tracking and containment control for high-order linear heterogeneous MASs. The theoretical results are derived by leveraging both the regulator equations in (11) and the application of SOF transformation to the MASs framework. In so doing, the fully-distributed PID output feedback control problem is recast into a state optimal feedback one whose solution provides the proportional, integral and derivative control gains.

Theorem 1. *Consider a MAS composed of $M \geqslant 1$ leaders and N followers with heterogeneous dynamics as in (5)–(4) under the action of the distributed PID protocol as in (9). Let the associated extended graph $\mathcal{G}_{N+M}$ be such that there exists at least one leader that has a directed path with the follower and define the following matrices $(i = 1, \ldots, N)$:*

$$\overline{A}_i = \begin{bmatrix} A_i & 0_{n\times q} \\ C_i & 0_{q\times q} \end{bmatrix} \in \mathbb{R}^{(n+q)\times(n+q)}, \quad \overline{B}_i = \begin{bmatrix} B_i \\ 0_{q\times m} \end{bmatrix} \in \mathbb{R}^{(n+q)\times m}. \tag{12}$$

Given a sufficiently large value $\sigma \in \mathbb{R}_+$, if all couples $\left(\overline{A}_i, \overline{B}_i\right)$ are controllable, then

(a) *the leader-tracking control problem stated in Problem 1 is solved in the case of $M = 1$;*
(b) *each agent i is driven to the convex hull spanned by the leaders if $M > 1$, i.e. the containment control problem as in Problem 2 is solved.*

Proof 1. Consider the agents and leader/s dynamics as in (4) and (5), respectively.

Define the tracking error of the i-th agent w.r.t. the synchronized reference signal in (10) as

$$\varepsilon_i(t) = x_i(t) - \Pi_i\zeta_i(t), \tag{13}$$

being matrix Π_i as in (11). Then, rewrite the control input $u_i(t)$ in (9) as function of $\varepsilon_i(t)$, i.e.:

$$u_i(t) = u_{fb,i}(t) + \Gamma_i\zeta_i(t), \tag{14}$$

being

$$u_{fb,i}(t) = K_{Pi}C_i\varepsilon_i(t) + K_{Ii}C_i\int_0^t \varepsilon_i(s)ds + K_{Di}C_i\dot{\varepsilon}_i(t). \tag{15}$$

The error dynamics for each agent i can be derived from (4) and (10), after some algebraic manipulation, as

$$\begin{aligned} \dot{\varepsilon}_i(t) &= \dot{x}_i(t) - \Pi_i\dot{\zeta}_i(t) = A_i\varepsilon_i(t) + A_i\Pi_i\zeta_i(t) - \Pi_i\dot{\zeta}_i(t) + B_iu_i(t) \\ &= A_i\varepsilon_i(t) + A_i\Pi_i\zeta_i(t) + B_iu_i(t) - \Pi_iS\zeta_i(t) - \sigma\Pi_i\left[\sum_{j=1}^{N} a_{ij}\left(\zeta_j(t) - \zeta_i(t)\right) + \sum_{k=N+1}^{N+M} a_{ik}(\xi_k(t) - \zeta_i(t))\right]. \end{aligned} \tag{16}$$

By substituting the control (14) into (16), the closed-loop dynamics for the i-th agent are:

$$\begin{aligned} \dot{\varepsilon}_i(t) &= A_i\varepsilon_i(t) + A_i\Pi_i\zeta_i(t) + B_iK_{Pi}C_i\varepsilon_i(t) + B_iK_{Ii}C_i\int_0^t \varepsilon_i(s)ds + B_iK_{Di}C_i\dot{\varepsilon}_i(t) + B_i\Gamma_i\zeta_i(t) - \Pi_iS\zeta_i(t) \\ &\quad - \sigma\Pi_i\left[\sum_{j=1}^{N} a_{ij}\left(\zeta_j(t) - \zeta_i(t)\right) + \sum_{k=N+1}^{N+M} a_{ik}(\xi_k(t) - \zeta_i(t))\right]. \end{aligned} \tag{17}$$

From the fulfilling of the regulators Eq. (11), the closed-loop can be rewritten as

$$\dot{\varepsilon}_i(t) = A_i\varepsilon_i(t) + B_iu_{fb,i}(t) - \sigma\Pi_i\left[\sum_{j=1}^{N} a_{ij}\left(\zeta_j(t) - \zeta_i(t)\right) + \sum_{k=N+1}^{N+M} a_{ik}(\xi_k(t) - \zeta_i(t))\right]. \tag{18}$$

Now, considering (18), under the fulfilment of both Condition 1 and Lemma 2, after some algebraic manipulations, (15) can be recast as

$$\begin{aligned} u_{fb,i}(t) &= (I_m - K_{Di}C_iB_i)^{-1} \\ &\quad\times\left(K_{Pi}C_i\varepsilon_i(t) + K_{Ii}C_i\int_0^t \varepsilon_i(s)ds + K_{Di}C_iA_i\varepsilon_i(t) - K_{Di}C_i\sigma\Pi_i\sum_{j=1}^{N} a_{ij}\left(\zeta_j(t) - \zeta_i(t)\right) - K_{Di}C_i\sigma\Pi_i\sum_{k=N+1}^{N+M} a_{ik}(\xi_k(t) - \zeta_i(t))\right). \end{aligned} \tag{19}$$

Define the following augmented state variables:

$$z_{1i}(t) = \varepsilon_i(t) \in \mathbb{R}^n, \tag{20a}$$

$$z_{2i}(t) = \int_0^t y_{\varepsilon_i}(s)ds = C_i\int_0^t \varepsilon_i(s)ds \in \mathbb{R}^q. \tag{20b}$$

By leveraging the Static Output Feedback (SOF) transformation [36] and considering (19), the closed-loop system in (18) can be further recast as

$$\dot{z}_{1i}(t) = \dot{\varepsilon}_i(t) = A_iz_{1i}(t) + B_i\tilde{u}_i(t) - \sigma\Pi_i\omega_i(t), \tag{21a}$$

$$\dot{z}_{2i}(t) = C_i\varepsilon_i(t) = C_iz_{1i}(t), \tag{21b}$$

where

$$\omega_i(t) = \sum_{j=1}^{N} a_{ij}\big(\zeta_j(t) - \zeta_i(t)\big) + \sum_{k=N+1}^{N+M} a_{ik}(\xi_k(t) - \zeta_i(t)), \tag{22}$$

while

$$\tilde{u}_i(t) = \widetilde{K}_{Pi} C_i z_{1i}(t) + \widetilde{K}_{Ii} z_{2i}(t) + \widetilde{K}_{Di} C_i A_i z_{1i}(t) - \widetilde{K}_{Di} C_i \sigma \Pi_i \sum_{j=1}^{N} a_{ij}\big(\zeta_j(t) - \zeta_i(t)\big) - \widetilde{K}_{Di} C_i \sigma \Pi_i \sum_{k=N+1}^{N+M} a_{ik}(\xi_k(t) - \zeta_i(t)), \tag{23}$$

with

$$\widetilde{K}_{Pi} = (I_m - K_{Di} C_i B_i)^{-1} K_{Pi} \in \mathbb{R}^{m\times q}, \tag{24a}$$
$$\widetilde{K}_{Ii} = (I_m - K_{Di} C_i B_i)^{-1} K_{Ii} \in \mathbb{R}^{m\times q}, \tag{24b}$$
$$\widetilde{K}_{Di} = (I_m - K_{Di} C_i B_i)^{-1} K_{Di} \in \mathbb{R}^{m\times q}. \tag{24c}$$

Introducing the state vector $z_i(t) = \left[z_{1i}^\top(t)\ \ z_{2i}^\top(t)\right]^\top$, after some algebraic manipulations, the closed-loop system for each agent i can be rewritten in a more compact form as

$$\dot{z}_i(t) = \left(\overline{A}_i + \overline{B}_i \widetilde{K}_i \overline{C}_i\right) z_i(t) - \left(\overline{B}_i \gamma_i + \overline{E}_i\right)\omega_i(t), \tag{25}$$

being

$$\overline{A}_i = \begin{bmatrix} A_i & 0_{n\times q} \\ C_i & 0_{q\times q}\end{bmatrix};\ \overline{B}_i = \begin{bmatrix} B_i \\ 0_{q\times m}\end{bmatrix};\ \overline{E}_i = \begin{bmatrix} \sigma\Pi_i \\ 0_{q\times n}\end{bmatrix}; \tag{26a}$$
$$\overline{C}_i = \begin{bmatrix} C_i & 0_{q\times q} \\ 0_{q\times n} & I_{q\times q} \\ C_i A_i & 0_{q\times q}\end{bmatrix} \in \mathbb{R}^{3q\times(n+q)}; \tag{26b}$$
$$\widetilde{K}_i = \begin{bmatrix} \widetilde{K}_{Pi} & \widetilde{K}_{Ii} & \widetilde{K}_{Di}\end{bmatrix} \in \mathbb{R}^{m\times 3q}; \tag{26c}$$
$$\gamma_i = \sigma \widetilde{K}_{Di} C_i \Pi_i \in \mathbb{R}^{m\times n}. \tag{26d}$$

Consider now the whole MAS network and define the following state vectors:

$$\underline{z}(t) = \left[z_1^T(t), z_2^T(t), \ldots, z_N^T(t)\right]^T \in \mathbb{R}^{N(n+q)}, \tag{27a}$$
$$\underline{\zeta}(t) = \left[\zeta_1^T(t), \zeta_2^T(t), \ldots, \zeta_N^T(t)\right]^T \in \mathbb{R}^{nN}, \tag{27b}$$
$$\underline{\xi}(t) = \left[\xi_{N+1}^T(t), \xi_{N+2}^T(t), \ldots, \xi_{N+M}^T(t)\right]^T \in \mathbb{R}^{nM}. \tag{27c}$$

Given (5) and (10), the overall leading and synchronized reference signal dynamics can be expressed in compact form as

$$\dot{\underline{\xi}}(t) = (I_M \otimes S)\underline{\xi}(t), \tag{28a}$$
$$\dot{\underline{\zeta}}(t) = (I_N \otimes S)\underline{\zeta}(t) - \sigma\left[(\mathscr{L}_1 \otimes I_n)\underline{\zeta}(t) + (\mathscr{L}_2 \otimes I_n)\underline{\xi}(t)\right], \tag{28b}$$

while the closed-loop in (25) can be recast as the following dynamical system:

$$\dot{\underline{z}}(t) = \mathscr{A}\underline{z}(t) + \mathscr{B}\left[(\mathscr{L}_1 \otimes I_n)\underline{\zeta}(t) + (\mathscr{L}_2 \otimes I_n)\underline{\xi}(t)\right], \tag{29}$$

being

$$\mathscr{A} = \mathrm{diag}\left\{\overline{A}_i + \overline{B}_i \widetilde{K}_i \overline{C}_i\right\} \in \mathbb{R}^{N(n+q)\times N(n+q)}, \tag{30a}$$
$$\mathscr{B} = \mathrm{diag}\left\{\overline{B}_i \gamma_i + \overline{E}_i\right\} \in \mathbb{R}^{N(n+q)\times Nn}. \tag{30b}$$

Define now the following augmented vector:

$$\underline{\eta}(t) = \underline{\zeta}(t) + \left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\underline{\xi}(t) \in \mathbb{R}^{\mathbb{nN}}. \tag{31}$$

From (28) we obtain:

$$\begin{aligned}\dot{\underline{\eta}}(t) &= \dot{\underline{\zeta}}(t) + \left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\dot{\underline{\xi}}(t) = [(I_N \otimes S) - \sigma(\mathscr{L}_1 \otimes I_n)]\underline{\zeta}(t) - \sigma(\mathscr{L}_2 \otimes I_n)\underline{\xi}(t) + (I_N \otimes S)\left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\underline{\xi}(t) \\ &= [(I_N \otimes S) - \sigma(\mathscr{L}_1 \otimes I_n)]\left[\underline{\eta}(t) - \left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\underline{\xi}(t)\right] - \sigma(\mathscr{L}_2 \otimes I_n)\underline{\xi}(t) + (I_N \otimes S)\left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\underline{\xi}(t). \\ &= [(I_N \otimes S) - \sigma(\mathscr{L}_1 \otimes I_n)]\underline{\eta}(t).\end{aligned} \tag{32}$$

By leveraging (31), the system in (29) can be rewritten as

$$\dot{\underline{z}}(t) = \mathscr{A}\underline{z}(t) + \mathscr{B}(\mathscr{L}_1 \otimes I_n)\left[\underline{\zeta}(t) + \left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\underline{\xi}(t)\right] = \mathscr{A}\underline{z}(t) + \mathscr{B}(\mathscr{L}_1 \otimes I_n)\underline{\eta}(t). \quad (33)$$

Finally, by defining the following augmented state vector

$$\underline{\tilde{x}}(t) = \left[\underline{z}(t)^T, \underline{\eta}(t)^T\right]^T \in \mathbb{R}^{N(2n+q)},$$

the global synchronization error dynamics can be obtained from (32) to (33) as

$$\dot{\underline{\tilde{x}}}(t) = \begin{bmatrix} \mathscr{A} & \mathscr{B}(\mathscr{L}_1 \otimes I_n) \\ 0_{nN \times N(n+q)} & (I_N \otimes S) - \sigma(\mathscr{L}_1 \otimes I_n) \end{bmatrix}. \quad (34)$$

Consider the upper triangular structure of the closed-loop matrix in (34). Under Assumption 1 we have that all the eigenvalues of the matrix $\mathscr{L}_1$ have positive real part according to Lemma 1. Hence, by choosing $\sigma \in \mathbb{R}_+$ such that the matrix $(I_N \otimes S) - \sigma(\mathscr{L}_1 \otimes I_n)$ is Hurwitz [40], it follows $\eta_i(t) \to 0$, $\forall i$.

Now, for proving the asymptotic stability of the closed-loop it suffices that the matrix $\mathscr{A}$, defined as in (30a), is also Hurwitz. This can be always guaranteed since, under the hypothesis that each pair $\left(\overline{A}_i, \overline{B}_i\right)$ is controllable (i.e. the closed-loop system (25) is completely controllable), there always exist suitable matrices

$$\overline{F}_i = \widetilde{K}_i\overline{C}_i \in \mathbb{R}^{m \times (n+q)} \quad (35)$$

$\forall i$ ensuring this. In this way, we have $\varepsilon_i(t) \to 0, \forall i$.

Since the aims of the theorem are twofold, in what follows we consider at first the case when $M = 1$ and, then, we prove the achievement of output containment in the case when $M > 1$.

(a) *Leader-Tracking ($M = 1$).* Consider the output leader-tracking error ($i = 1, \dots, N$), i.e.:

$$\begin{aligned} e_i(t) &= y_i(t) - l_{N+1}(t) = C_i x_i(t) - R\xi_{N+1}(t) = C_i\varepsilon_i(t) + C_i\Pi_i\zeta_i(t) - R\xi_{N+1}(t) = C_i\varepsilon_i + C_i\Pi_i(\zeta_i(t) - \xi_{N+1}(t)) \\ &= C_i\varepsilon_i + C_i\Pi_i\eta_i(t), \end{aligned} \quad (36)$$

where the last equality is obtained according to (11). Given the asymptotic stability of (34), $\forall i$ we have $\varepsilon_i(t) \to 0$ and $\eta_i(t) \to 0$. Therefore, from (36), it follows $e_i(t) \to 0$. In so doing, the leader-tracking (defined as in Problem 1) is achieved.

(b) *Output Containment ($M > 1$).* Consider the containment error with respect to the followers outputs ($i = 1, \dots, N$) in the multiple leaders case as

$$e_i(t) = \sum_{j=1}^{N} a_{ij}\left(y_i(t) - y_j(t)\right) + \sum_{k=N+1}^{N+M} a_{ik}(y_i(t) - l_k(t)), \quad (37)$$

and define the following vectors related the whole closed-loop MAS:

$$\underline{e}(t) = \left[e_1(t)^\top, e_2(t)^\top, \dots, e_N(t)^\top\right] \in \mathbb{R}^{Nq}, \quad (38a)$$
$$\underline{\varepsilon}(t) = \left[\varepsilon_1(t)^\top, \varepsilon_2(t)^\top, \dots, \varepsilon_N(t)^\top\right] \in \mathbb{R}^{Nn}, \quad (38b)$$
$$\underline{y}(t) = \left[y_1(t)^\top, y_2(t)^\top, \dots, y_N(t)^\top\right] \in \mathbb{R}^{Nq}, \quad (38c)$$
$$\underline{l}_k(t) = \left[l_{N+1}(t)^\top, l_{N+2}(t)^\top, \dots, l_{N+M}(t)^\top\right] \in \mathbb{R}^{Mq}. \quad (38d)$$

By exploiting the regulator equations in (11), the containment error vector can be expressed in the following compact form:

$$\begin{aligned} \underline{e}(t) &= \left(\mathscr{L}_1 \otimes I_q\right)\left[\underline{y}(t) + \left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_q\right)\underline{l}_k(t)\right] = \left(\mathscr{L}_1 \otimes I_q\right)\left[\underline{Cx}(t) + (I_N \otimes R)\left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\underline{\xi}(t)\right] \\ &= \left(\mathscr{L}_1 \otimes I_q\right)\left[\underline{C\varepsilon}(t) + \underline{C\Pi\zeta}(t) + (I_N \otimes R)\left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_n\right)\underline{\xi}(t)\right] = \left(\mathscr{L}_1 \otimes I_q\right)\underline{C\varepsilon}(t) + \left(\mathscr{L}_1 \otimes I_q\right)\underline{C\Pi\eta}(t), \end{aligned} \quad (39)$$

being $\underline{C} = \mathrm{diag}\{C_i\}$ and $\underline{\Pi} = \mathrm{diag}\{\Pi_i\}$.

Given the asymptotic stability of (34), we have $\underline{\varepsilon}(t) \to 0, \underline{\eta}(t) \to 0$ and, hence, $\underline{e}(t) \to 0$ which implies that $\lim_{t\to\infty}\underline{y}(t) = -\left(\mathscr{L}_1^{-1}\mathscr{L}_2 \otimes I_q\right)\underline{l}_k(t)$. Therefore, according to [41], the output containment control problem stated in Problem 2 is solved. This completes the proof.

Remark 2. Note that, the parameter σ only influences the convergence of the distributed observer $\zeta_i(t)$ in (10), $\forall i$, but does not involve the control design (see e.g. [15] and references therein). Indeed, in the distributed MAS framework the separation principle still holds [44]. This implies that the distributed observer, the controller, as well as the related control gains, can be designed separately, as also disclosed by the triangular structure of the closed-loop dynamic matrix in (34).

To guarantee the convergence of the distributed observer $\zeta_i(t), \forall i$, according to [15], we select σ large sufficiently so to make the dynamics of the distributed observer faster than the controller ones and to ensure the convergence of control protocol (9) without any a priori knowledge about the graph topology, such as the minimal eigenvalue of the Laplacian Matrix. Therefore, the proposed PID protocol in (9) solves the output leader-tracking and the output containment problems in a fully-distributed manner.

4.1. Optimal gain tuning procedure

Once proved the achievement of the control objective according to Theorem 1, it is possible to optimally tune the Proportional, Integral and Derivative gains according to the design procedure summarized in Algorithm 1.

Indeed, under the hypotheses of Theorem 1, there always exist matrices $\overline{F}_i$ ($i = 1, \ldots, N$) such that the closed-loop system in (34) is asymptotically stable. These matrices $\overline{F}_i$ are selected in order to minimize the control effort needed for reaching the control goals according to the typical LQR performance index [39] and, then, from (35), since $\widetilde{K}_i \in \mathbb{R}^{m\times 3q}$ and $\overline{C}_i \in \mathbb{R}^{3q\times n+q}$, it is possible to derive the gain vector $\widetilde{K}_i$ as

$$\widetilde{K}_i = \overline{F}_i \overline{C}_i^{\dagger}, \tag{40}$$

where $\overline{C}_i^{\dagger}$ is the right Moore–Penrose pseudoinverse of the full-row rank matrix $\overline{C}_i$ (see Definition 3). Note that, if $\overline{C}_i$ is a square matrix, then $\overline{C}_i^{\dagger}$ coincides with $\overline{C}_i^{-1}$.

Finally, from (24a), (24b) and (24c), it is possible to compute the gains of each i-th proportional, integrative and derivative action ($i = 1, \ldots, N$) within the distributed control protocol (9) as

$$K_{Di} = \widetilde{K}_{Di}\left(I_q + C_i B_i \widetilde{K}_{Di}\right)^{-1}, \tag{41a}$$

$$K_{Pi} = (I_m - K_{Di} C_i B_i)\widetilde{K}_{Pi}, \tag{41b}$$

$$K_{Ii} = (I_m - K_{Di} C_i B_i)\widetilde{K}_{Ii}. \tag{41c}$$

Note that, the distributed observer design method as in Remark 2 allows preserving the optimal features of the LQR control approach.

Remark 3. In order to compute the control gains according to (40), it is necessary to assume for each agent dynamics in (4) that $q \leqslant n/2$. Indeed, since $\overline{F}_i$ in (35) $\in \mathbb{R}^{m\times(n+q)}$, the gain matrices $\widetilde{K}_i \in \mathbb{R}^{m\times 3q}$ can be algebraically derived as in (40) only in two possible cases: *i*) by exploiting the right Moore-Pseudoinverse of the matrix $\overline{C}_i$ if $q < n/2$; *ii*) the standard inverse of $\overline{C}_i$ if $q = n/2$. Conversely, if $q > n/2$, (40) does not admit solution.

Algorithm 1 Control Design Procedure

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for $i = 1, \ldots, N$ **do**
  evaluate (32) and (33)
  choose $\sigma \in \mathbb{R}_+$ sufficiently large such that (32) is asymptotically stable
  define $\overline{F}_i = \widetilde{K}_i \overline{C}_i$
  Design $\overline{F}_i$ via state feedback control methods, e.g. the LQR approach
  evaluate $\widetilde{K}_i = \left[\widetilde{K}_{Pi}\ \widetilde{K}_{Ii}\ \widetilde{K}_{Di}\right] = \overline{F}_i \overline{C}_i^{\dagger}$
  evaluate K_{Di} from (41a)
  evaluate K_{Pi} from (41b)
  evaluate K_{Ii} from (41c)
end for

---

5. Numerical validation

To show the effectiveness of the proposed PID control strategy in solving both the output leader-tracking and the output containment control problems, we consider an exemplary heterogeneous MAS composed of six followers whose dynamics are given as in (4) with matrices ($i = 1, \ldots, 6$):

$$A_i = \left(0 a_{12,i} - a_{21,i} - a_{22,i}\right), \quad B_i = \left(0 b_{21,i}\right), C_i = \left(c_{11,i} c_{12,i}\right), \tag{42}$$

being

$$a_{12,i} = \{1.6, 1.0, 2.5, 0.5, 3.5.1.5\},\ a_{21,i} = \{0.5, 0.2, 1.2, 0.3, 2.5, 1.5\},\ a_{22,i} = \{3.0, 1.0, 0.5, 2.5, 5.5, 3.5\},\ b_{21,i} = \{1.0, 2.0, 1.5, 4.5, 1.5, 4.5\},\ c_{11,i} = 1, c_{12,i} = 0.$$

Given the followers dynamics, in what follows we analyze the ability of the proposed control approach in solving both leader-tracking and output containment control problems.

5.1. Leader-tracking

Consider one single leader ($M = 1$) that imposes the desired behaviour to the MAS of 6 followers ($i = 1, \dots, N$) in (42) via the communication structure depicted in Fig. 1. The dynamics of the single leader ($k = 7$) are described as in (5) with matrices:

$$S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

For each follower i, the regulator Eqs. (11) allows computing the matrices Π_i and Γ_i in (9) as

$$\Pi_i = \begin{pmatrix} 1 & 0 \\ 0 & 1/a_{12,i} \end{pmatrix}, \quad \Gamma_i = \begin{pmatrix} a_{12,i}/b_{21,i} & a_{22,i}/(b_{21,i}a_{12,i}) \end{pmatrix}.$$

Now, to find the control gains K_{Pi}, K_{Ii} and K_{Di} in (9), we apply the tuning procedure described in section (4.1) and summarized in Algorithm 1. Specifically, after verifying that each couple $\left(\overline{A}_i, \overline{B}_i\right)$ ($\forall i$) in (12) is controllable, we tune the matrix $\overline{F}_i$ in (35) by leveraging the LQR control design approach with a proper selection of weighting matrices $\mathcal{Q}_i$ and $\mathcal{R}_i$. Control gains can be then derived from (41). All weighting matrices, agents initial conditions, as well as the results of the tuning procedure are summarized in Table 1.

Numerical results, depicted in Fig. 2 and Fig. 3, confirm the theoretical derivation and disclose how each follower under the action of the distributed PID in (9), starting from different initial conditions (see Table 1), tracks the desired behaviour imposed by the leader.

5.2. Containment control

Consider three leaders ($M = 3$) driving the MAS of 6 followers in (42) via the communication structure shown in Fig. 4. The dynamics of the k-th leader ($k = 7, 8, 9$) are as in (5) with matrices:

$$S = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

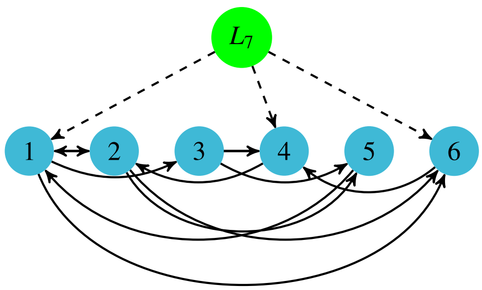

Fig. 1. Leader-tracking. Communication graph topology.

Table 1
Leader-tracking. Control parameters (see Fig. 1).

Agent	$\mathcal{Q}_i$	$\mathcal{R}_i$	K_{Pi}	K_{Ii}	K_{Di}	$y_i(0)$	$l_k(0)$
1	$3000I_3$	1	−82.67	−54.77	−33.88	35	–
2	$1000I_3$	1	−55.16	−31.62	−31.98	20	–
3	$100I_3$	0.5	−18.79	−14.14	−6.35	15	–
4	$500I_3$	1	−50.16	−22.36	−44.11	−15	–
5	$2000I_3$	1	−55.08	−44.72	−12.56	−20	–
6	$1000I_3$	0.5	−48.19	−31.62	−20.90	−35	–
L_7	–	–	–	–	–	–	5

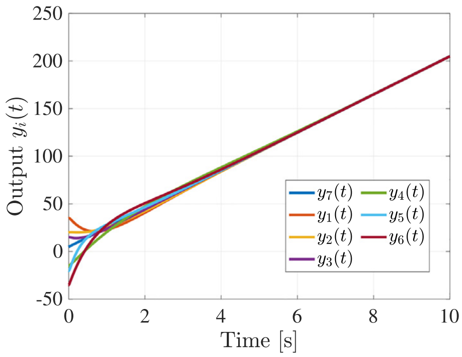

Fig. 2. Leader-tracking. Time history of the outputs $y_i(t)$ $(i = 1, \ldots, 7)$.

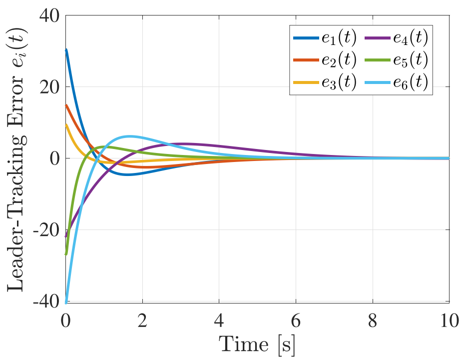

Fig. 3. Leader-tracking. Time history of the output leader-tracking error $e_i(t)$ $(i = 1, \ldots, 6)$ computed as in (36).

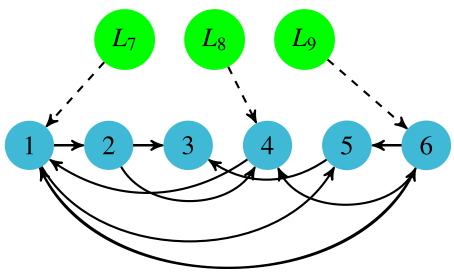

Fig. 4. Output containment. Communication graph topology.

Solving the regulator equations in (11), it is possible to compute the following matrices Π_i and Γ_i $(i = 1, \ldots, 6)$:

$$\Pi_i = \begin{pmatrix} 1 & 0 \\ \frac{1}{a_{12,i}} & -\frac{2}{a_{12,i}} \end{pmatrix}, \quad \Gamma_i = \begin{pmatrix} \frac{a_{22,i}+a_{21,i}a_{12,i}-1}{b_{21,i}a_{12,i}} & -\frac{2a_{22,i}}{a_{12,i}b_{21,i}} \end{pmatrix}.$$

According to the theoretical derivation, from the controllability condition of each couple $(\overline{A}_i, \overline{B}_i)$ in (12), it follows that the output containment can be achieved under the action of the distributed PID in (9) whose gains can be set according to the optimal tuning procedure in Algorithm 1. Weighting matrices $\mathcal{Q}_i$ and $\mathcal{R}_i$, as well as the optimal control gains values, are reported in Table 2. Results, depicted in Fig. 5a and Fig. 5b, confirm the theoretical derivation and show that the output trajectories of the followers, starting from different initial conditions (see Table 2), converge within the envelope formed by the leaders (depicted with the dashed black lines in Fig. 5a) with null output containment error (see Fig. 5b).

5.3. Containment control for heterogeneous multi-input–multi-output multi-agent systems in power-grid networks

To further disclose the effectiveness of the proposed PID protocol (9) in solving the containment control problem for a MIMO MAS, in this section, we consider the case study of the vector control problem for 6 heterogeneous high-order under-

Table 2
Output containment. Control parameters (see Fig. 4).

Agent	$\mathcal{Q}_i$	$\mathcal{R}_i$	K_{Pi}	K_{Ii}	K_{Di}	$y_i(0)$	$l_k(0)$
1	$3000I_3$	1	−132.27	−50.77	−54.21	35	–
2	$1000I_3$	5	−24.88	−14.14	−14.50	20	–
3	$1000I_3$	1	−105.67	−31.62	−33.44	15	–
4	$3000I_3$	10	−6.21	−5.47	−5.19	−15	–
5	$2000I_3$	1	−192.78	−44.72	−43.98	−20	–
6	$1000I_3$	1	−72.29	−31.62	−31.35	−35	–
L_7	–	–	–	–	–	–	10
L_8	–	–	–	–	–	–	15
L_9	–	–	–	–	–	–	−5

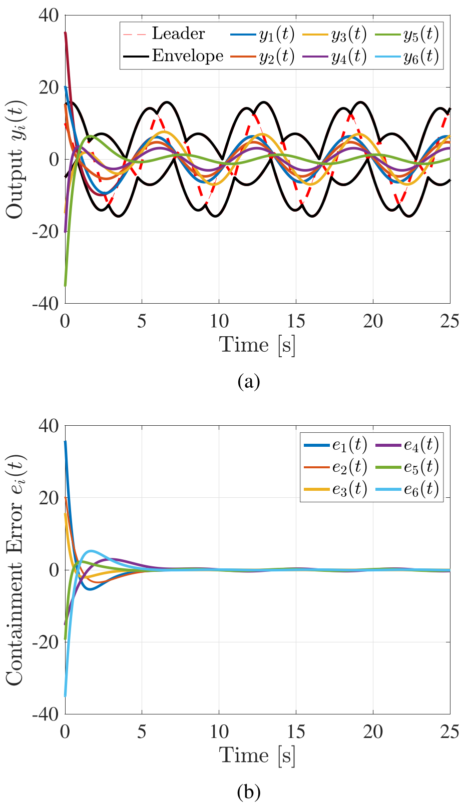

Fig. 5. Output containment for heterogeneous high-order MAS: a) Time history of the output $y_i(t)$ $(i = 1, \ldots, 6)$ and of the output $l_k(t)$ $(k = 7, 8, 9)$; b) Time history of the output containment error $e_i(t)$ $(i = 1, \ldots, 6)$ computed as in (37).

damped harmonic oscillators plus 3 leaders imposing the reference behaviour for the ensemble. Agents share information over the directed communication network modeled by the graph in Fig. 4. In this case study, the containment control problem consists in simultaneously regulating the amplitude and the phase of the 6 oscillators within the network to the ones imposed by the multiple leaders. Note that, this numerical example could be framed within the practical problem of frequency and voltage control in a power-grid network, where agents behaviour can be described via an oscillator dynamics [45].

According to [46], each follower agent is described by the following MIMO system:

$$\begin{pmatrix} \dot{x}_{d,i}(t) \\ \ddot{x}_{d,i}(t) \\ \dot{x}_{q,i}(t) \\ \ddot{x}_{q,i}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ a_{21,i} & a_{22,i} & a_{23,i} & a_{24,i} \\ 0 & 0 & 0 & 1 \\ a_{41,i} & a_{42,i} & a_{43,i} & a_{44,i} \end{pmatrix} \begin{pmatrix} x_{d,i}(t) \\ \dot{x}_{d,i}(t) \\ x_{q,i}(t) \\ \dot{x}_{q,i}(t) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ b_{21,i} & 0 \\ 0 & 0 \\ 0 & b_{42,i} \end{pmatrix} \begin{pmatrix} u_{d,i}(t) \\ u_{q,i}(t) \end{pmatrix},$$

$$\begin{pmatrix} y_{d,i}(t) \\ y_{q,i}(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_{d,i}(t) \\ \dot{x}_{d,i}(t) \\ x_{q,i}(t) \\ \dot{x}_{q,i}(t) \end{pmatrix}, \tag{43}$$

being $x_i(t) = [x_{d,i}(t),\ \dot{x}_{d,i}(t),\ x_{q,i}(t),\ \dot{x}_{q,i}(t)]^\top \in \mathbb{R}^4$ the state variable vector of the i-th oscillator; $u_i(t) = [u_{d,i}(t),\ u_{q,i}(t)]^\top \in \mathbb{R}^2$ is the input vector to be imposed so to guarantee the containment control behaviour; $y_i(t) = [y_{d,i}(t),\ y_{q,i}(t)]^\top \in \mathbb{R}^2$ is the output vector of the i-th oscillator.

Parameters in (43) are selected as follows:

$$\begin{aligned}
a_{21,i} &= \{3.0, 1.0, 1.0, 1.0, 2.0, 1.0\},\\
a_{22,i} &= \{2.0, 5.0, 5.0, 1.0, 7.0, 5.0\},\\
a_{23,i} &= \{-1.0, -2.0, -6.0, -1.0 - 6.0, -1.0\},\\
a_{24,i} &= \{4.0, 1.0, 10.0, 1.0, 4.0, 8.0\},\\
a_{41,i} &= \{2.0, 3.0, 1.0, 15.0, 2.0, 2.0\},\\
a_{42,i} &= \{3.0, 3.0, 3.5, 9.0, 6.0, 1.0\},\\
a_{43,i} &= \{-4.0, -4.0, -1.0, -4.0, -1.0, -8.0\},\\
a_{44,i} &= \{-1.0, -2.0, -10.0, -10.0, -1.0, -1.0\},\\
b_{21,i} &= \{1.0, 1.5, 2.0, 2.0, 10.0, 5.0\},\\
b_{42,i} &= \{1.0, 2.0, 3.0, 3.0, 1.0, 2.0\}.
\end{aligned}$$

The dynamics of the k-th leader ($k = 7, 8, 9$) are defined as in (5) with matrices:

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Solving the regulator equations in (11), we obtain the following matrices $\Pi_i \in \mathbb{R}^{4\times 4}$ and $\Gamma_i \in \mathbb{R}^{2\times 4} (i = 1, \ldots, 6)$:

$$\Pi_i = I_4, \quad \Gamma_i = \begin{pmatrix} \frac{-(1+a_{21,i})}{b_{21,i}} & \frac{-a_{22,i}}{b_{21,i}} & \frac{-a_{23,i}}{b_{21,i}} & \frac{-a_{24,i}}{b_{21,i}} \\ \frac{-a_{41,i}}{b_{42,i}} & \frac{-a_{42,i}}{b_{42,i}} & \frac{-(1+a_{43,i})}{b_{42,i}} & \frac{-a_{44,i}}{b_{42,i}} \end{pmatrix}.$$

Properly setting the weighting matrices $\mathcal{Q}_i$ and $\mathcal{R}_i$, according to the theoretical derivation and the tuning procedure in Algorithm 1, we obtain the following optimal gain matrices:

- Proportional gain matrix

$$K_{Pi} = \begin{pmatrix} K_{Pi,11} & K_{Pi,12} \\ K_{Pi,21} & K_{Pi,22} \end{pmatrix} \tag{44}$$

with

$$\begin{aligned}
K_{Pi,11} &= \{764.5, 761.2, 758.0, 762.1, 746.4, 761.9\},\\
K_{Pi,12} &= \{-10.4, 36.0, -75.9, 2.8, 140.8, -2.8\},\\
K_{Pi,21} &= \{12.1, -38.7, 78.8, 1.4, -150.3, 3.8\},\\
K_{Pi,22} &= \{703.1, 704.1, 703.0, 705.8, 691.8, 703.1\};
\end{aligned}$$

- Integral gain matrix

$$K_{Ii} = \begin{pmatrix} K_{Ii,11} & K_{Ii,12} \\ K_{Ii,21} & K_{Ii,22} \end{pmatrix} \tag{45}$$

with

$$\begin{aligned}
K_{Ii,11} &= \{0.100, 0.099, 0.100, 0.100, 0.098, 0.100\},\\
K_{Ii,12} &= \{-0.001, 0.005, -0.010, 0.001, 0.020, 0.000\},\\
K_{Ii,21} &= \{0.001, -0.005, 0.010, -0.001, -0.020, 0.000\},\\
K_{Ii,22} &= \{0.100, 0.099, 0.100, 0.100, 0.098, 0.100\};
\end{aligned}$$

- Derivative gain matrix

$$K_{Di} = \begin{pmatrix} K_{Di,11} & K_{Di,12} \\ K_{Di,21} & K_{Di,22} \end{pmatrix} \tag{46}$$

with

$$\begin{aligned} K_{Di,11} &= \{41.264, 35.636, 30.030, 28.128, 13.180, 18.493\}, \\ K_{Di,12} &= \{3.597, 2.117, 2.632, 0.571, 5.219, 1.563\}, \\ K_{Di,21} &= \{3.597, 2.823, 3.948, 0.856, 0.522, 0.625\}, \\ K_{Di,22} &= \{36.722, 25.523, 19.009, 18.618, 35.899, 26.069\}. \end{aligned}$$

Results in Fig. 6a–6b show the effectiveness of the theoretical derivation and disclose how the proposed distributed optimal PID approach in (9) drives the followers outputs from random initial conditions to the envelope formed by the leaders output trajectories. Accordingly, the output containment errors converge towards zero as depicted in Figs. 6d.

5.4. Cooperative driving for heterogeneous connected ground vehicles

The output leader-tracking capability of the proposed PID (9) is here validated in the practical engineering application of the cooperative driving for autonomous connected vehicles, referred into the technical literature to platooning [31]. Platooning mainly consists in a union of autonomous vehicles connected through wireless communication networks, via the Vehicle-to-Vehicle (V2V) or Vehicle-to-Infrastructure (V2I) paradigm based on IEEE 802.11p protocol, in order to reach a common velocity, generally given by the first vehicle within the group (i.e. the leader), or by an external infrastructure, while keeping a desired inter-vehicular distance among them. Traveling maintaining this formation brings different advantages, such as the reduction of environmental pollution and traffic congestion, as well as an improvement of safety and road efficiency [47]. In what follows, we consider a platoon of $N = 5$ vehicles plus a leader, connected through a wireless vehicular network whose topology is the one depicted in Fig. 7. According to [31], each vehicle i, $(i = 1, \ldots, 5)$ can be described via the following dynamical system:

$$\begin{aligned} \begin{bmatrix} \dot{p}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{T_i} \end{bmatrix} \begin{bmatrix} p_i(t) \\ v_i(t) \\ a_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_i} \end{bmatrix} u_i(t) \\ y_i(t) &= [1 \quad 0 \quad 0] \begin{bmatrix} p_i(t) \\ v_i(t) \\ a_i(t) \end{bmatrix}, \quad i = 1, \ldots, 5, \end{aligned} \tag{47}$$

being $p_i(t)$ $[m]$, $v_i(t)$ $[m/s]$ and $a_i(t)$ $[m/s^2]$ the i-th absolute vehicle position, velocity and acceleration, respectively; $u_i(t)$ is the control input as in (9); $y_i(t)$ is the i-th vehicle output; T_i $[s]$ is the powertrain time constant and equal to $[T_1\ T_2\ T_3\ T_4\ T_5] = [0.50\ 0.30\ 0.70\ 0.45\ 0.80]$ $[s]$ [31]. The leader dynamics, imposing the reference behaviour for the fleet and moving with a constant velocity $v^\star = 30$ $[m/s]$, are described as

$$\begin{aligned} \dot{p}_6(t) &= v_6(t) \\ \dot{v}_6(t) &= 0 \\ l_6(t) &= p_6(t). \end{aligned} \tag{48}$$

Let $x_i(t) = [p_i(t), v_i(t), a_i(t)]^\top \in \mathbb{R}^{3\times1}$, $\zeta_6(t) = [p_6(t), v_6(t), a_6(t)]^\top \in \mathbb{R}^{3\times1}$ and introduce the vector $d^\star = [d_{i,6}, 0, 0]^\top \in \mathbb{R}^{3\times1}$, where $d_{i,6}$ is the desired inter-vehicular distance between the i-th vehicle and the leader. Defining the tracking error $\varepsilon_i(t)$ as

$$\varepsilon_i(t) = x_i(t) - \Pi_i\zeta_i(t) - d^\star, \tag{49}$$

under the action of the proposed PID control strategy in (9), the i-th closed-loop vehicular system takes the form of (17), i.e.:

$$\dot{\varepsilon}_i(t) = A_i\varepsilon_i(t) + B_iu_i(t) + A_i\Pi_i\zeta_i(t) - \Pi_iS\zeta_i(t) - \sigma\Pi_i\omega_i(t), \tag{50}$$

being $\omega_i(t)$ defined as in (22). Solving the regulator equations in (11) we have $\Pi_i = I_3$ and $\Gamma_i = [0\ 0\ 1]$ $\forall i = 1, \ldots, 5$. After verifying that each couple $(\overline{A}_i, \overline{B}_i)$ $(\forall i)$ in (12) is controllable, we tune the optimal PID control gains according to Algorithm 1. Properly setting $\forall i$ the weighting matrices $\mathcal{Q}_i$ and $\mathcal{R}_i$, we obtain: $[K_{P1}\ K_{P2}\ K_{P3}\ K_{P4}\ K_{P5}] = [-1.2098\ -1.1704\ -0.5728\ -0.4084\ -0.4170]$, $[K_{I1}\ K_{I2}\ K_{I3}\ K_{I4}\ K_{I5}] = [-0.1000\ -0.1000\ -0.0447\ -0.0316\ -0.0316]$ and $[K_{D1}\ K_{D2}\ K_{D3}\ K_{D4}\ K_{D5}] = [-2.3180\ -1.8492\ -1.4323\ -1.0560\ -1.1681]$.

In order to validate the effectiveness of the proposed approach and to disclose its advantages w.r.t. the technical literature in solving this practical engineering problem, we compare its performances with the ones achievable via the purely diffusive control strategy proposed in [22] to solve the leader-tracking for heterogeneous high-order MAS. Numerical results are depicted in Fig. 8. Herein, the red lines refer to performances achievable via the proposed optimal PID control strategy while

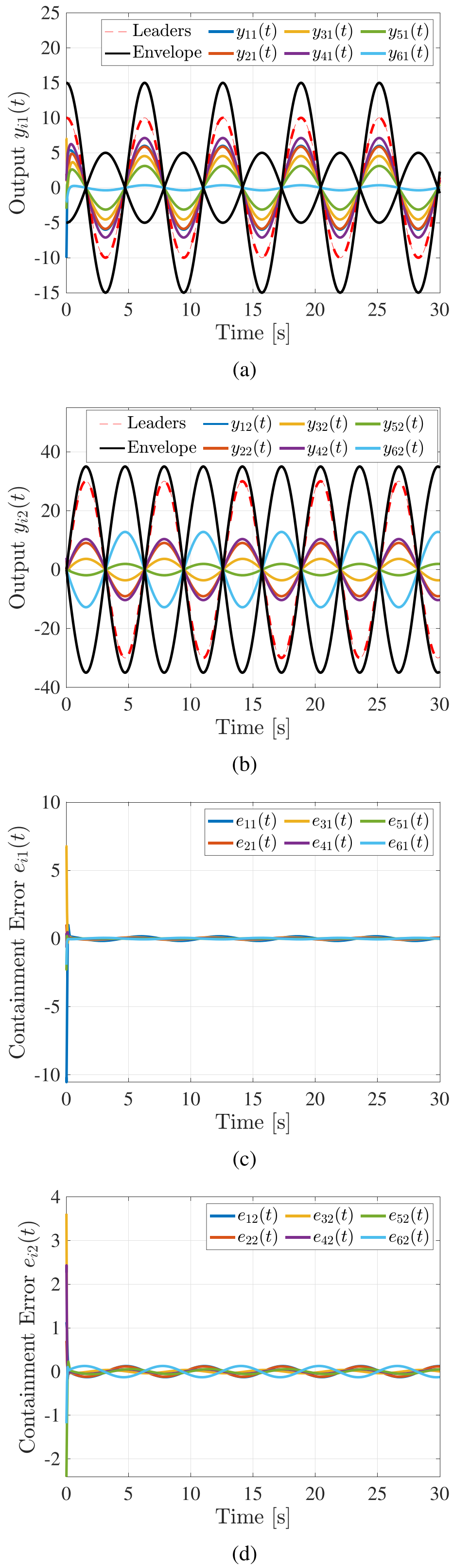

Fig. 6. Output containment control for MIMO MASs: a) Time history of the output $y_{i1}(t)$ $(i = 1, 2, \ldots, 6)$ and of the output $l_{k1}(t)$ $(k = 7, 8, 9)$; b) Time history of the output $y_{i2}(t)$ $(i = 1, 2, \ldots, 6)$ of the output $l_{k2}(t)$ $(k = 7, 8, 9)$; c) Time history of the output error $e_{i1}(t)$ $(i = 1, 2, \ldots, 6)$; d) Time history of the output error $e_{i2}(t)$ $(i = 1, 2, \ldots, 6)$.

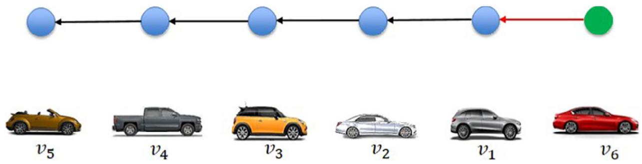

Fig. 7. Wireless vehicular network topology for the heterogeneous platoon of 5 vehicles plus a leader.

the black lines are related to the ones obtained via the purely diffusive control in [22]. Although each vehicle, under the action of both the controllers, is able to track the leader speed (see Fig. 8b), the purely diffusive controller can not ensure collision avoidance (see Fig. 8a). Conversely, the proposed PID protocol guarantees that no collisions occur (see Fig. 8a) and, moreover, ensures improved tracking capability in transient phase (see Fig. 8b). This behaviour is due to the benefits introduced by the integral and the derivative actions that increase both the steady-state and dynamic tracking performances.

5.5. Three-dimensional containment control for heterogeneous Unmanned Aerial Vehicles

To further disclose the effectiveness of the proposed PID approach in solving practical engineering applications, in this section, we also address the three-dimensional containment control for a MAS composed of $N = 5$ heterogeneous UAV and $M = 3$ leaders. According to [48], the UAV linear dynamical model can be described as the following system:

$$\dot{x}_i(t) = Ax_i(t) + B_i u_i(t) y_i(t) = Cx_i(t), \tag{51}$$

being $x_i(t) = \left[p_i^\top(t), v_i^{\top(t)}\right]^\top$, where $p_i(t) = [p_{xi}(t), p_{yi}(t), p_{zi}(t)]^\top$ and $v_i(t) = [v_{xi}(t), v_{yi}(t), v_{zi}(t)]^\top$ are the position and velocity of the i-th UAV along the x, y and z axis, respectively; the matrix A, B and C are:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_3, \quad B = \begin{bmatrix} 0 \\ \frac{1}{m_i} \end{bmatrix} \otimes I_3, \quad C = [1 \quad 0] \otimes I_3,$$

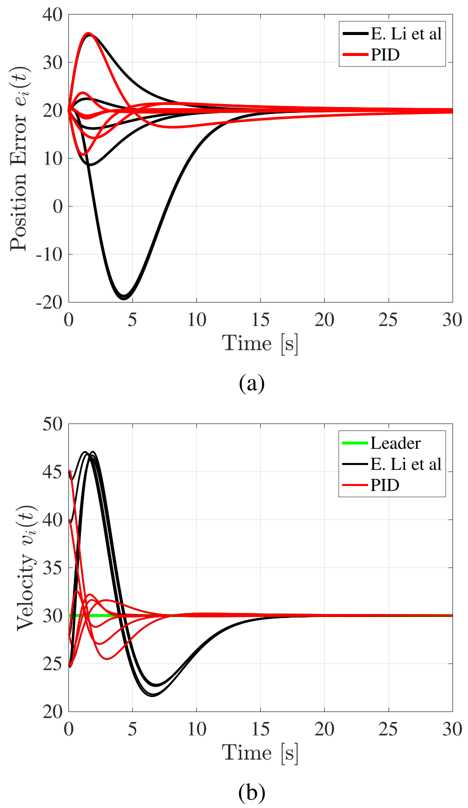

Fig. 8. Comparison Analysis between the distributed PID control protocol (9) and the purely diffusive one proposed in [22]. Leader tracking performances for the heterogeneous vehicle platoon of 5 vehicle plus a leader. Time history of: a) the inter-vehicle distance $e_i(t)$ computed as the difference between consecutive vehicles positions; b) vehicle speed $v_i(t), \forall . i = 1, \ldots, 6$.

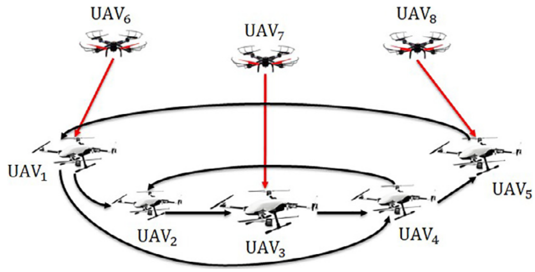

Fig. 9. Communication topology for three-dimensional containment control of 5 heterogeneous UAV plus 3 leaders.

where m_i is the mass of the i-th UAV.

The leaders are described as the autonomous dynamical system in (5) whose matrices are selected as

$$S = \begin{bmatrix} 0 & 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -b & 0 & 0 & 0 \end{bmatrix}, \quad R = [1 \quad 0] \otimes I_3.$$

The communication graph topology is shown in Fig. 9 and the simulation parameters are chosen as $a = 0.1$, $b = 0.01$ and $m_i = \{10.0\ 5.0\ 20.0\ 10.0\ 15.0\}$ $[kg]$ [48].

Under the action of the PID protocol in (9), each UAV closed-loop system takes the form of (17) for which it is possible to follow the design procedure as in Section 4. Solving the regulator equations in (11), we obtain $\Pi_i = I_6$ and

$$\Gamma_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -m_i b & 0 & 0 & 0 \end{bmatrix}, \forall i.$$

Once condition (12) in Theorem 1 is fulfilled, we tune the PID control gains following the procedure described in Algorithm 1. Specifically, a suitable choice of $\mathcal{Q}_i$ and $\mathcal{R}_i$ provide the following control gains matrices: $[K_{P1}\ K_{P2}\ K_{P3}\ K_{P4}\ K_{P5}] = [-131.7464I_3\ \ -26.5185I_3\ \ -34.3538I_3\ \ -54.1724I_3\ \ -40.6192I_3]$, $[K_{I1}\ K_{I2}\ K_{I3}\ K_{I4}\ K_{I5}] = [-70.7107I_3\ \ -12.9099I_3\ \ -12.9099I_3\ \ -26.4575I_3\ \ -17.3205I_3]$ and $[K_{D1}\ K_{D2}\ K_{D3}\ K_{D4}\ K_{D5}] = [-87.3781I_3\ \ -20.7810I_3\ \ -39.2532I_3\ \ -42.2309I_3\ \ -38.9689I_3]$.

Fig. 10 discloses the effectiveness of the proposed PID control strategy in ensuring that each follower reaches and does not leave the convex hull spanned by the multiple leaders. Now, we compare the performances of our controller w.r.t. the ones achievable via the purely diffusive output feedback protocol proposed in [25], where a Luenberger observer is also required

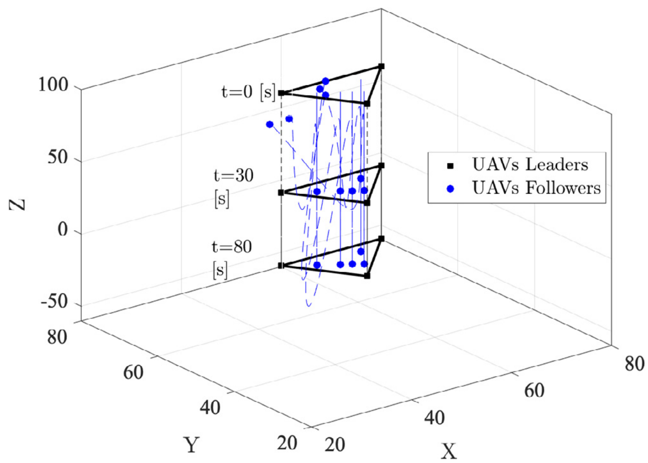

Fig. 10. Three-dimensional containment control for 5 heterogeneous UAV plus 3 leaders. State trajectories in the x-y-z plane for different simulation time instants.

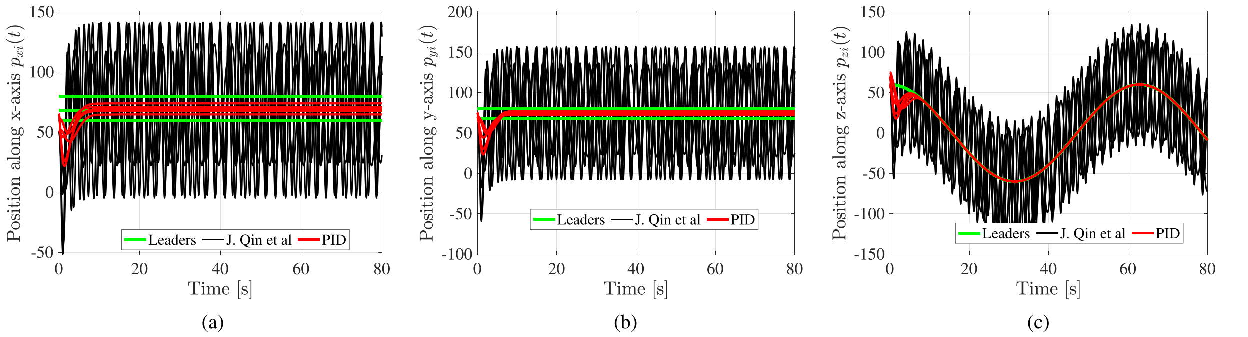

Fig. 11. Comparison Analysis between the distributed PID control protocol (9) and the purely diffusive output feedback controller proposed in [25]. Three-dimensional containment performance for 5 heterogeneous UAV plus 3 leaders. Time history of: a) position along x-axis $p_{xi}(t)$ $\forall i = 1, \ldots, 8$; b) position along y-axis $p_{yi}(t)$ $\forall i = 1, \ldots, 8$; c) position along z-axis $p_{zi}(t)$ $\forall . i = 1, \ldots, 8$.

for estimating the full state of each agent i. Note that, in order to disclose the advantages of the PID strategy w.r.t. the solely proportional action, for the comparative analysis we implement the purely diffusive control in [25] without any follower full state estimator. Numerical results are depicted in Fig. 11 where the red lines refer to the proposed optimal PID while the black lines are related to [25]. As it is possible to observe herein, while a classical proportional control strategy does not guarantee the achievement of the control objective, the adding of the integral and derivative actions allows the position $p_i(t)$ of each UAV to reach and do not leave the convex hull spanned by the 3 leaders. If the diffusive output-feedback controller [25] was equipped with state estimators, closed-loop performances similar to (9) would be obtained. However, this results in an increasing of the overall control design complexity. Conversely, our control approach is able to guarantee good containment performances without requiring any additional observers, hence reducing the architecture computational burden.

6. Conclusions

In this work, both output leader-tracking and output containment control problems for heterogeneous high-order MASs are achieved through a fully distributed PID strategy. Leveraging regulator equations for dealing with agents heterogeneity and adapting the Static Output Feedback transformation to the MASs framework, the stability of the closed-loop network is analytically derived. In addition, the design of a tuning procedure based on the LQR approach allows selecting optimal proportional, integral and derivative actions. Two illustrative MASs composed of 6 heterogeneous agents, both SISO and MIMO, are exploited to confirm the effectiveness of the theoretical derivation in the case of both single and multiple leader/s. Moreover, to better appreciate the advantages and the potential applications of the proposed approach, we apply our method for solving two practical problems into the engineering literature and comparison analyses w.r.t. alternative control approaches are presented.

Future works could include: *i*) the theoretical analysis of the proposed optimal PID-like control protocol in the presence of communication impairments originated by the wireless networks and/or external disturbances affecting the agent dynamics; *ii*) the tailoring of the proposed control framework to the challenging context of high-order nonlinear continuous MASs and high-order linear discrete-time ones; *iii*) the extension of the proposed approach to the constrained control field for nonlinear high-order MASs by leveraging, for example, the Barrier Lyapunov Function (BLF) method and the designing of anti-windup compensator (see e.g. [49,50]).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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