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Reviewed

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Properties of 8-contraction-critical graphs with no K_7 minor. (English summary)[European J. Combin.](#) **110** (2023), Paper No. 103711, 13 pp.

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Review

A graph H is a minor of a graph G if H can be obtained from a subgraph of G by contracting edges. A graph G is k -contraction-critical if G has chromatic number $\chi(G) = k$ but $\chi(H) \leq k - 1$ for every proper minor H of G .

The paper under review gives a contribution to the well-known Hadwiger Conjecture [H. Hadwiger, *Vierteljschr. Naturforsch. Ges. Zürich* **88** (1943), 133–142; [MR0012237](#)]: Every graph with no K_t minor is $(t - 1)$ -colorable.

Despite receiving considerable attention over the years, this conjecture remains wide open for all $t \geq 7$. It is considered among the most important problems in graph theory and has motivated numerous developments in graph coloring and graph minor theory. Proving that graphs with no K_7 minor are 6-colorable is thus the first case of Hadwiger's Conjecture that is still open. It is not even known yet whether every graph with no K_7 minor is 7-colorable.

Partial results towards the conjecture for $t \leq 9$ are contained in the recent paper of M. Lafferty and the second author [“Every graph with no K_8^- minor is 7-colorable”, preprint, [arXiv:2208.07338](#)]. The reader is also referred to recent surveys [K. Cameron and K. Vušković, *Bull. Eur. Assoc. Theor. Comput. Sci. EATCS No. 131* (2020), 20–37; [MR4535143](#); K. Kawarabayashi, in *Topics in chromatic graph theory*, 73–93, *Encyclopedia Math. Appl.*, 156, Cambridge Univ. Press, Cambridge, 2015; [MR3380167](#); P. D. Seymour, in *Open problems in mathematics*, 417–437, Springer, [Cham], 2016; [MR3526944](#)] for further background on the conjecture.

The purpose of the paper under review is to study properties of 8-contraction-critical graphs G with no K_7 minor. The main result is about the minimum degree $\delta(G)$, the number n_i of i -vertices of G , $i \in \{8, 9\}$, and the subgraphs of G induced by $N[v]$, where v is a 9-vertex of G . This is one step in the effort to prove that every graph with no K_7 minor is 7-colorable.

Reviewer: [Ciampella, Adriana](#)

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This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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