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The social value of overreaction to information ☆

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ABSTRACT

We study the welfare effects of overreaction to information in markets with asymmetric information as well as the impact of a simple intervention in the form of a tax or a subsidy on trade volume. A large enough level of overreaction is always welfare decreasing: in these situations, introducing a tax can improve welfare. However, a small degree of overreaction can increase welfare. This is because of the interplay of two competing externalities: an information externality, due to the informational role of prices, and a pecuniary externality, due to the allocative role of prices. Depending on the balance of these externalities, a trade volume subsidy may be optimal.

Introduction

Information aggregation is understood to be one of the fundamental roles of markets—particularly financial markets. Consequently, a large body of literature addresses the welfare properties and social value of information in markets, from Hayek (1945) to, e.g. Angeletos and Pavan (2007). Therefore, it is crucial to understand how agents make inferences from the information they receive: for example, traders in financial markets constantly update their beliefs about the valuations of financial assets as a consequence of changes in market prices, fundamentals, and the choices of other traders. There is growing evidence that agents' updating rules depart from Bayesian rationality in the form of over- or underreaction to information. In this paper, we ask: how do over or underreaction impact welfare and informational efficiency in financial markets? Can a simple intervention, such as a tax or a subsidy, mitigate inefficiencies?

To formalize departures from Bayesian rationality in a parsimonious way, we rely on the memory-based model of *diagnostic expectations*, introduced in Bordalo et al. (2018). When computing their posterior distribution after observing an informative signal, diagnostic agents adjust their prior in the correct direction. However, overreacting agents adjust it by an excessive amount with respect to the Bayesian posterior; underreacting agents adjust it by an insufficient amount. The model is a one-parameter deviation from Bayesian updating and is one of the simplest ways to reconcile anomalies in forecast data (Bordalo et al., 2020b) and experiments (Afrouzi et al., 2023).

We embed overreacting agents in a market game in which there is an asset of unknown valuation to be traded, and in which traders submit conditional bids, or schedules, that depend on the market price and a private signal. The price depends on private information via market clearing, and so, in equilibrium, the price is an endogenous signal of the value. Since the liquidity supply is stochastic, agents are not able to learn the value perfectly from the price. We adopt the tractable linear-quadratic Gaussian setting from (Vives, 2017). In this context, we compute the welfare effect of the diagnostic bias, showing that a moderate degree of over- or underreaction can be welfare improving (our first contribution), and we show that if the degree of overreaction is large enough, the introduction of a small tax is welfare-improving (our second contribution).

Diagnostic expectations have been used to rationalize several facts about macro-financial variables, such as credit cycles (Bordalo et al., 2018), stock return puzzles (Bouchaud et al., 2019 and Bordalo et al., 2018), interest rates (d'Arienzo, 2020), and the likelihood of a financial crisis (Maxted, 2024). This literature is reviewed in Gennaioli and Shleifer (2018). The majority of the papers above find that data are consistent with overreaction to information. However, some papers find that, in short time horizons, data display underreaction (Bouchaud et al., 2019), and more generally the level of overreaction may depend

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on the time horizon (d'Arienzo, 2020). Given that ours is an abstract setting, we allow for different parameter values representing both over and underreaction.[1]

In our market game, there are two sources of information: the private signal and the (public) market price. Formally, we adopt the *diagnostic expectations equilibrium* of Bordalo et al. (2020a), in which prices are formed in equilibrium given agents' trade choices, and agents correctly understand this mechanism, but their posterior expectation about the fundamental value is distorted due to over/under reaction to both private information (the private signal) and public information (the market price). In particular, in our context, the bias does not come from (possibly partially) failing to realize that other traders also understand the information contained in prices, as in the "cursed equilibrium" of Eyster et al. (2019) or the "partial equilibrium thinking" of Bastianello and Fontanier (2023). The main difference between (Bordalo et al., 2020a) and our work is that their focus is on bubbles rather than welfare and taxes.[2]

In a version of this model with standard Bayesian agents, Vives (2017) highlights two competing externalities: a *learning* externality and a *pecuniary* externality. The learning externality is present because agents do not internalize that their actions change the informativeness of the price as a signal of the underlying value, and thus reveal information. The pecuniary externality is present because agents submit schedules conditioning on the price, and so they also change how the equilibrium price reacts to the underlying value. As a consequence, the loading on private information can be either too high with respect to the efficient benchmark (if the pecuniary externality prevails) or too low (if the learning externality prevails). Both cases are possible for different values of the parameters.

We characterize the equilibrium in Proposition 1. In equilibrium, when agents display overreaction, agents trade higher volumes for the same private signal because they overweight the information it contains. As a consequence, they increase the informativeness of the price as a public signal of the value. However, this increase is not sufficient to offset the first-order effect on the private signal, and so the loading on the private signal in agents' actions is larger than it would be for Bayesian agents. As a consequence, the price reveals more information than in an economy with Bayesian agents, and the price volatility is higher (Corollary 2.1). Therefore, overreaction changes the relative importance of the learning and the pecuniary externalities with respect to the benchmark model: in particular, there are levels of overreaction such that the loading on private or on public information is at the efficient level, but never simultaneously (Proposition 2).

Having characterized the equilibrium, we study the effect of overreaction on welfare in Proposition 3. A small level of overreaction can either increase or decrease welfare. A small level of overreaction improves welfare when, in the Bayesian benchmark, the loading on private information is inefficiently low (the learning externality is stronger). Analogously, a small level of underreaction improves welfare when, in the Bayesian benchmark, the loading on private information is inefficiently high (the pecuniary externality is stronger). However, for a large enough level of over/underreaction, a further increase in the diagnostic bias in either direction always decreases welfare.

Finally, we explore whether introducing a small quadratic tax or subsidy can mitigate inefficiencies and improve welfare. Proposition 4 shows that a quadratic tax modifies the loadings on public and private information, offering an (imperfect) instrument to affect the equilibrium allocation. In Proposition 5 we show that, when the overreaction parameter is large enough, the introduction of a small tax is always welfare-improving. This result can offer a rationalization of a Tobin-type tax (Tobin, 1978) on financial transactions for reasons related to the interaction of a behavioral bias (diagnostic expectations) and informational efficiency. These reasons are complementary to but distinct both from arguments related to curbing speculation (as in Stiglitz, 1989 and Summers and Summers, 1989) and those arising from disagreement in agents' evaluations such as in Dávila (2023).[3] When instead agents underreact strongly enough, a small subsidy is optimal.[4] When overreaction is close to zero, the welfare effect of a tax depends on the balance between the learning and pecuniary externalities. Consequently, the model implications for the optimality of a tax depend on the degree of agents' overreaction to information.

Our work is related to three strands of literature: the literature on overreaction and related biases in information processing; the literature studying taxes in the presence of behavioral biases, especially on financial transactions; and the literature on the social value of information. Our contribution is to show how overreaction can be welfare-improving because it mitigates the learning externality: that, is, overreaction can have a "social value". However, when overreaction is large enough, it can rationalize a tax on financial transactions, even in the presence of the learning externality. The literature on overreaction in finance and macroeconomics has mostly focused on identifying and measuring overreaction and its explanatory power for rationalizing various macroeconomic phenomena (Bordalo et al., 2022). Some papers have explored macroeconomic policy under overreaction or exuberance, such as (Maxted, 2024), which also finds a positive welfare effect, albeit one that works through a balance sheet mechanism rather than through the learning externality. Dávila and Walther (2023) explore macro-prudential policy implications with extrapolative beliefs. The fact that overreaction mitigates the learning externality is similar to the effect of overconfidence in the social learning model of Bernardo and Welch (2001): they study a simple sequential learning model instead of a financial market, and therefore, only the learning externality is present in their setting, not the pecuniary externality.

The literature on behavioral finance has studied models that incorporate related biases in information processing. In the cursed equilibrium of Eyster et al. (2019) and Bayona and Manzano (2022), agents neglect the informational content of the price. This can be seen as an extreme form of underreaction to the price signal. Instead, in the diagnostic expectation model we use, agents overreact or underreact to all information in the same way. Eyster et al. (2019) did not study welfare; while (Bayona and Manzano, 2022) showed that cursedness can improve welfare. Mondria et al. (2022) studied costly information processing, which has similar implications to underreaction to public information in that agents do not consider adequately the information in the price signal; however, while they show that this can give rise to excess volatility, we imply the opposite because in our case, when agents underreact, they do so with respect to both public and private information. None of these papers focus on the effect of tax/subsidy schemes.

Another related bias is overconfidence. The main difference between overreaction and overconfidence is that overconfident agents overestimate the precision of their information, but their updating is still Bayesian, as in: Kyle and Wang (1997), Bernardo and Welch (2001), Sandroni and Squintani (2007), Daniel et al. (2001,?). Therefore, overconfidence cannot explain the predictability of forecast errors observed in the data (Bordalo et al., 2020b; Afrouzi et al., 2023).

[1] See also (Ba et al., 2022) for a discussion of over and underreaction and a unifying modeling approach.

[2] Moreover, they use a model with CARA utility and inelastic supply, whereas, for tractability, we follow (Vives, 2017) by using a model with elastic supply and quadratic utility.

[3] The tax on financial transactions has been the subject of a long debate and is still an important issue in economic policy: first advocated by Keynes, it is currently in place in multiple countries (such as the UK and Sweden), and it has been the object of a European Commission official proposal since 2011.

[4] These considerations stem purely from efficiency reasons: if there are other rationales for a tax, such as redistribution, then the case in which a subsidy is optimal could be interpreted as a case in which a *smaller tax* is optimal.

Moreover, even if the posterior is biased in the same direction in both models, the posterior expectation is still a convex combination of the prior and the signals, whereas the expectation of overreacting agents can overshoot and lie outside of the convex combination. Bordalo et al. (2022) use this observation to argue that overreaction can rationalize facts about the behavior of bubbles that overconfidence cannot. Again, none of these papers focus on the effect of tax/subsidy schemes.

While the taxation literature has studied various behavioral biases, for example, related to attention and salience as in Goldin (2015), Moore and Slemrod (2021), Farhi and Gabaix (2020), the literature specifically on taxation *of financial transactions* has mostly focused on rational models: Auerbach and Bradford (2004), Rochet and Biais (2023), Adam et al. (2017), Buss et al. (2016), and at most with heterogeneous priors as in Dávila (2023). The literature on the social value of information has also mainly focused on Bayesian agents, e.g. Angeletos and Pavan (2007, 2009), Bayona (2018), Colombo et al. (2014). An exception is (Ostrizek and Sartori, 2021), who study a strategic setting in which agents follow the cursed equilibrium model of Eyster and Rabin (2005), Eyster et al. (2019), showing that cursedness can improve welfare: their mechanism works through information acquisition and not through the pecuniary externality like ours.

The next section introduces the model, Section 2 describes the equilibrium characterization, Section 3 describes our results, and Section 4 concludes. All proofs are in Appendix.

1. The model

Our model closely follows (Vives, 2017) in its financial market interpretation, except for the behavioral bias due to diagnostic expectations.[5] We consider a financial market to be populated by informed speculators and liquidity suppliers. There is only one asset traded.

Informed agents. There is a continuum of informed speculators indexed by $i \in [0,1]$, distributed uniformly. Informed speculators face quadratic transaction costs. Each of them can decide their position D_i with respect to the only asset exchanged, where short sales are allowed (D_i can be negative).

The profit of an informed agent i holding D_i units of the asset when the market price is p is:

$$u_i(D_i, p, V) = (V - p)D_i - \frac{1}{2}\gamma D_i^2,$$

where V is the (unobservable) fundamental value of the asset, and the quadratic term represents transaction costs. Equivalently, it can be considered a form of (non-constant) risk aversion.[6] Informed speculators have a prior over the fundamental value V, which is Gaussian: $V \sim \mathcal{N}(0, \tau_0^{-1})$. They also have access to a private signal s_i that, conditional on V, follows a Gaussian distribution: $s_i \mid V \sim \mathcal{N}\left(V, \tau_\varepsilon^{-1}\right)$. Moreover, s_i is independent of s_j for $i \neq j$, conditionally on V: $s_i \perp s_j \mid V$.

In the following, various steps involve the integration of a continuum of random variables over $[0,1]$. We follow the literature *defining* the integral over a continuum of independent random variables $(X_i)_{i\in[0,1]}$ as $\int X_i \mathrm{d}i := \int \mathbb{E}(X_i)\mathrm{d}i$ whenever the map $\mathbb{E}(X_i)$ is integrable (which is always the case in our setting).[7] So, a form of the Law of Large Numbers holds, so that, conditionally on V, we have $\int s_i \mathrm{d}i = V$. The Law of Large Numbers is going to be the only property of this integral we need.[8] We denote the total demand from all informed agents as $\overline{D} = \int D_i \mathrm{d}i$.

Liquidity suppliers. As in Vives (2017), liquidity suppliers have an elastic supply function. In particular, they trade according to the aggregate (inverse) supply function $p = -\mu_S - S + \beta\overline{D}$. S is a random variable distributed as $S \sim \mathcal{N}(0, \tau_S^{-1})$, representing the noise in the demand. The parameter μ_S is a constant that we can think of as a shifter of the random variable S, which we include for generality but which has little effect on the efficiency properties. Instead, the slope of the supply $\beta > 0$ is going to be important because it regulates how prices react to quantities and the strength of both the learning and the pecuniary externalities. Classic noise traders, as in Grossman and Stiglitz (1980), are a special case of this specification in which $\beta \to \infty$, $\tau_S \to \infty$, and $\tau_S\beta^2 = \tau_S' > 0$. In this case, the aggregate supply is independent of prices and is simply a random variable with precision τ_S'.

Diagnostic expectations. To trade, agents form posteriors on the asset value V by updating their prior, using the private signal s_i and also the information contained in the price p. Crucially, their posteriors are not Bayesian but instead follow the diagnostic expectations model of Bordalo et al. (2018). The model is a parsimonious characterization of Kahneman and Tversky (1972) "representativeness heuristic". When forming posterior beliefs, agents overweight representative "traits" of the signals observed, which are traits that are objectively more likely for the signal observed with respect to a benchmark. This idea can be applied to any context in which agents form beliefs after observing information. Indeed, the representativeness bias is consistent with biased beliefs in a variety of domains, from stereotypes (Bordalo et al., 2016), to race (Arnold et al., 2018). In our context, as explained in detail below, agents update their beliefs over the fundamental value of an asset when observing some signal of its value. They have as a benchmark the ex-ante expected value of the signal, and when they observe a signal higher than the benchmark, they find this representative of a high fundamental value. Therefore, following the representativeness bias, they overestimate the posterior probability of a high fundamental with respect to a pure Bayesian agent.

The model can be microfounded based on friction in memory retrievals (Bordalo et al.,2023a), costly information processing (Afrouzi et al., 2023), or rational inattention (Gabaix, 2019). The details depend on the specific case, but a general idea is that failure to properly take into consideration all the past information can generate overreaction to the most recent information.

We follow the formalization of Bordalo et al. (2016), and especially (Bordalo et al., 2018), which applies it to financial markets and introduces the so-called diagnostic expectations model. In the setting just introduced, the representativeness heuristic consists in the fact that, when estimating the posterior distribution of the asset value V, after observing the information $G = (s_i, p)$, the posterior density $f(V|G)$ is inflated/deflated by (an increasing function of) the likelihood ratio $\frac{f(V|G)}{f(V|G_0)}$, measuring how much the realization G is representative of high values of V with respect to a *benchmark* value G_0. In particular, the distorted posterior density $f^\theta(V \mid G)$ is, up to a normalization constant, equal to:

$$f^\theta(V \mid G) \propto f(V \mid G)\left(\frac{f(V|G)}{f(V|G_0)}\right)^\theta. \tag{1}$$

The parameter $\theta \in (-1, \infty)$ modulates the strength of the effect. The case of Bayesian agents corresponds to $\theta = 0$. For $\theta > 0$ when $f(V|G) > f(V|G_0)$, agents overestimate $f(V|G)$: this is the case of *overreaction*. For $\theta \to \infty$, agents completely neglect the prior. For $\theta < 0$, we instead

[5] Vives (2017) studies different interpretations of the same abstract model, one being agents in a financial market, and another firms competing in schedules. For our purposes, we stick to the interpretation of agents trading in a financial market.

[6] The quadratic functional form makes the model very tractable. A similar approach is followed in Vives (2014).

[7] See Vives (2010).

[8] The most commonly used approach to formalize the integral over a continuum of random variables is the one of Uhlig (1996). Since the only property we are going to need is the Law of Large Numbers, we avoid these technical issues and directly assume it.

obtain *under*-reaction: agents revise their priors *less* than a Bayesian would; for $\theta \to -1$, agents do not revise their prior at all. Accordingly, it is common to consider the meaningful range of θ as $(-1, \infty)$. We allow for both overreaction and underreaction, since both have been found to be consistent with the data (even if underreaction occurs only within a very short time horizon, Bouchaud et al., 2019).

In the financial market application, the diagnostic expectations model uses as the benchmark G_0 a pair of signals that exactly confirms the prior expectation: $(\mathbb{E}s_i, \mathbb{E}p)$. The idea is that an above-average signal is more representative of a good underlying fundamental V than a signal equal to the ex-ante expectation.

Bordalo et al. (2020a) show that when the prior and the signal distribution follow a Gaussian distribution (as in our setting), the diagnostic expectation bias yields posterior beliefs that follow a Gaussian with the same variance as the Bayesian posterior, but expectation equal to:

$$\mathbb{E}^{\theta}(V \mid G) := \mathbb{E}(V \mid G) + \theta(\mathbb{E}(V \mid G) - \mathbb{E}(V)). \tag{2}$$

If agents overreact ($\theta > 0$) when the information leads them to revise their prior expectation upwards ($\mathbb{E}(V \mid s_i, p) > \mathbb{E}(V)$), they revise it upwards more than a Bayesian would: $\mathbb{E}^{\theta}(V \mid s_i, p) > \mathbb{E}(V \mid s_i, p)$. However, if the information leads to a downward revision ($\mathbb{E}(V \mid s_i, p) < \mathbb{E}(V)$), they revise it downwards more than a Bayesian would: $\mathbb{E}^{\theta}(V \mid s_i, p) < \mathbb{E}(V \mid s_i, p)$. If agents underreact ($\theta < 0$), agents instead revise their priors *less* than a Bayesian would.

Equilibrium. Agents compete choosing demand schedules—that is, functions D_i, which map values of the private signal s_i and the price p into real numbers $D_i(s_i, p)$, represent the net demand of agent i.

We follow (Bordalo et al., 2020a) in looking for a *diagnostic expectations equilibrium* that is analogous to the Bayesian Nash equilibrium of the game in schedules of Vives (2017), except that agents are not Bayesians but have diagnostic expectations. Namely, we look for a set of demand schedules D_i and a pricing function P that satisfy:

1. Individual optimization: the demand function D_i maximizes the (diagnostic) expected utility of the trader i given the observation of the private signal s_i and the price p, formally: $D_i(s_i, p) \in \arg\max_{x_i} \{\mathbb{E}^{\theta,i}[u_i(x_i, p, V) \mid s_i, P(S, V) = p]\}$;
2. Market clearing: the pricing function clears the market; that is, the relation $P(S, V) = -\mu_S - S + \beta \int D_i(P(S, V), s_i)\mathrm{d}i$ holds for any realization of S, V, and each s_i.

Similar to Vives (2017), we restrict attention to linear equilibria, namely equilibria where the function P is linear (or, more exactly, affine).

The welfare measure. We follow (Vives, 2017) in expressing our welfare measure as the expected total surplus and, moreover, in expressing welfare evaluations in terms of *welfare loss* relative to the first-best allocation. The surplus is the informed trader surplus plus the surplus of the liquidity suppliers (defined as is standard as the area below the supply curve).[9]

$$W = \mathbb{E}\left(\left(\mu_S + S - \beta\frac{1}{2}\overline{D}\right)\overline{D} + \int \left(V D_i - \frac{\gamma}{2}D_i^2\right)\mathrm{d}i\right). \tag{3}$$

In this context, if agents could pool their information, they would learn V perfectly, since by the law of large numbers $\int s_i \mathrm{d}i = V$. Therefore, if agents are allowed to pool information, the first-best allocation is the complete information allocation. The first-best allocation solves: $\max_{D_i} W$. Since the agents are ex-ante identical the first-best allocation is the same across all agents, and we denote it D^o. We denote W^o as the aggregate welfare in this allocation; and we denote the *welfare loss* of some allocation $(D_i)_{i\in[0,1]}$ from the first best as $WL = W^o - W$, where W is the welfare in allocation $(D_i)_{i\in[0,1]}$.

The following lemma from Vives (2017) characterizes the welfare loss relative to the first best:

Lemma 1.1. *At the allocation $(D_i)_{i\in[0,1]}$ the welfare loss relative to the first best allocation D^o is*

$$WL = \mathbb{E}(W^o - W) = (\beta + \gamma)\frac{1}{2}\mathbb{E}(\overline{D} - D^o)^2 + \frac{\gamma}{2}\mathbb{E}\int (D_i - \overline{D})^2 \mathrm{d}i. \tag{4}$$

Note that the expectations that appear in the expression are all taken from the perspective of Bayesian agents. In doing this, we interpret the agents' deviation from the Bayesian benchmark as a proper "mistake", not as a taste or preference feature, following a standard approach in the behavioral economics literature (e.g. O'Donoghue and Rabin (2006), Spinnewijn (2015)) and in the survey by Mullainathan et al. (2012).[10]

The interpretation of the above expression is that the welfare loss results from two parts that Angeletos and Pavan (2007) call, respectively, "variance" and "dispersion". The first part represents the departure of the aggregate demand from its first-best level; the second part represents the cross-sectional dispersion of trades across agents. The effect of information (and thus overreaction to information) results from this trade-off: precise information means a small aggregate deviation from the first best, but a large dispersion, because precise information means traders trade more aggressively. The welfare impact of overreaction will result from this fundamental trade-off.

2. Equilibrium characterization

In this section, we illustrate the equilibrium and the welfare benchmark.

The optimal trade of agent i is:

$$D_i(s_i, p) = \frac{1}{\gamma}\left(\mathbb{E}^{\theta}\left(V \mid s_i, p\right) - p\right). \tag{5}$$

We focus on the unique equilibrium with a linear pricing function. In this equilibrium, the equilibrium strategy D_i is an affine function of s_i and p. To highlight the different roles that private information and public information play, a convenient representation of the net trade (5) is:

$$D_i(s_i, p) = \alpha s_i + \eta\mathbb{E}(V \mid p) - \eta_p p, \tag{6}$$

where $\mathbb{E}(V \mid p)$ is the Bayesian posterior after observing only the price p, α is the *loading on private information*, η is the *loading on public information*, and η_p is the *loading on the price*. Here and in the following, it will be useful to define the precision of *public information* as $\tau(\alpha) = \tau_0 + \alpha^2\beta^2\tau_S$. The Proposition below shows the equilibrium value of the loadings in the diagnostic expectations equilibrium.

Proposition 1. *There is a unique diagnostic expectation equilibrium with linear pricing function $P(S, V) = A + BV - CS$. In this equilibrium, trades chosen by each agent have the form of Eq. (6), where:*

$$\alpha = a(\theta + 1) \quad \eta = (\theta + 1)\frac{1 - \gamma a}{\gamma} \quad \eta_p = \frac{1}{\gamma} \tag{7}$$

and a is the unique real solution of the equation:

$$\gamma a = \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau(a(\theta + 1))}. \tag{8}$$

[9] If we were to exclude the liquidity traders from welfare calculations, there would still be a scope for intervention, since (Vives, 2017) shows that the learning and pecuniary externality would still be present, even if the precise expression would change.

[10] There is another, more conceptual reason. To compute the ex-ante welfare from the perspective of a diagnostic decision-maker would require specifying how the decision-maker predicts her future behavior *once she receives the information*: is she aware of her bias or not? This would require considerably more assumptions than simply computing the welfare from the perspective of a Bayesian agent, so we follow the latter approach.

The expressions for the coefficients A, B, C and for $\mathbb{E}(V \mid p)$ *are given by Eqs. (14) and (15) in the Proof in Appendix.*

2.1. Properties of the equilibrium with diagnostic expectations

We collect some of the positive properties of the equilibrium in the next Corollary.

Corollary 2.1. *In equilibrium, the following properties hold:*

1. *The sensitivity to private information* α *is increasing in* θ*;*
2. *The precision of the price as a signal of the value* $B^2/C^2\tau_S$ *is increasing in* θ*;*
3. *The volatility of the price* $Var(p)$ *is increasing in* θ*.*

Point 1 yields the fundamental mechanism of what follows: overreaction increases the sensitivity to private information. This is immediate by construction when fixing the precision of the public signal, but, in equilibrium, overreaction also affects its precision because more information is revealed. This indirect effect on the precision of the price, however, is not strong enough to counteract the main effect, and so the loading α increases in θ.

Point 2 shows that the price reacts more to the true value than it would in the Bayesian case. This is because with overreaction the sensitivity to private information is higher, and therefore, the precision of the price as a signal of the value is higher: this is analogous to what happens in the model of Bordalo et al. (2020a).

Point 3 shows that the price displays *excess volatility* under overreaction. This is because overreaction induces agents to trade more aggressively, thereby generating larger price movements. Excess volatility of financial markets is a well-known empirical regularity. This result shows that, in our setting, excess volatility can be rationalized by overreaction to information, as in Bordalo et al. (2023b, 2022).

3. The effect of overreaction on welfare

In this section, we study the effect of overreaction. First, as a benchmark, we illustrate the welfare analysis of the Bayesian model with $\theta = 0$.

3.1. The Bayesian benchmark

Define a^* as the loading on the private signal at the market solution in the Bayesian benchmark: that is the solution of Eq. (8) for $\theta = 0$. Define a^T as the solution of:

$$a^T = \frac{\tau_\varepsilon}{\gamma(\tau(a^T) + \tau_\varepsilon) + \beta\tau(a^T) - \Delta(a^T)},$$

where $\Delta(a^T) = \frac{(1-\gamma a^T)^2\beta^2\tau_S\tau_\varepsilon}{\gamma\tau(a^T)}$. Vives (2017) shows that the market solution is second-best efficient if and only if $a^* = a^T$, where second-best efficient means the optimal *decentralized* linear strategy, assuming that agents cannot pool their private information. We also call it team solution. In particular, using the fact that in the market solution when the loading on private info is a and the loading on public information is $\eta = (1-\gamma a)/\gamma$, we can think of the welfare loss as a function of a: $\frac{\mathrm{d}WL}{\mathrm{d}a} > 0 \iff a^* > a^T$. In particular, the loading on private information at the market equilibrium a^* can either be too high or too low from a welfare perspective. This is because of the interplay between a *learning externality* and a *pecuniary externality*. The learning externality derives from the informational role of the price and is well understood: agents' decisions to trade reveal information to other agents through the price, but agents do not internalize this effect in the market equilibrium. This force pushes the sensitivity a^* to be too low with respect to the second best. The pecuniary externality derives from the allocative role of the price, and it derives from the fact that agents' decisions affect how the price correlates to the true value V, but they do not internalize this in the market equilibrium. This externality pushes the sensitivity a^* to be *too large*. In summation:

1. if $a^T > a^*$, the learning externality is stronger and the market equilibrium is inefficient;
2. if $a^T < a^*$, the pecuniary externality is stronger and the market equilibrium is inefficient;
3. if $a^* = a^T$ the two externalities exactly balance each other and the market equilibrium maximizes welfare.

3.2. Overreaction and the information loadings

The reason why it is sufficient to look at the loading on private information in the Bayesian case is that in the second-best (team) solution, the loading on *public* information has the same relation with the loading on private information, as in the market equilibrium: $\eta = \frac{1}{\gamma} - a^T$. As a consequence, the loading on public information is at the second-best level if and only if the loading on private information is at the second-best level; and when the loading on private information is higher than the efficient level, the loading on public information is too low and vice versa. This breaks down with diagnostic expectations: it is possible that both loadings are too high or too low with respect to the efficient benchmark. The next Proposition characterizes this behavior.

Proposition 2.

1. *There is a unique value* θ' *such that the loading on private information is at the efficient level:* $\alpha(\theta') = a^T$*. Moreover,* $\theta' > 0$ *if and only if in the Bayesian benchmark* $a^* < a^T$*.*
2. *There is a unique value* θ'' *such that the loading on public information is at the efficient level:* $\eta(\theta'') = \frac{1}{\gamma} - a^T$*. Moreover,* $\theta'' > 0$ *if and only if in the Bayesian benchmark* $a^* > a^T$*;*
3. *The two values are the same,* $\theta' = \theta''$*, if and only if agents are Bayesians:* $\theta' = \theta'' = 0$*, and the Bayesian benchmark is efficient:* $a^* = a^T$*.*

The Proposition clarifies the key trade-off of an increase in overreaction: the welfare effect depends on the balance of the effect on the loading on private and public information. If the learning externality is stronger in the Bayesian benchmark, so that $a^* < a^T$, then a sufficiently strong level of overreaction is always sufficient to reproduce the efficient loading on private information. When the pecuniary externality is stronger ($a^* > a^T$), a sufficiently high level of underreaction can reproduce the efficient loading on private information. An analogous fact is true for the loading on public information, but crucially, part (3) clarifies that no distortion θ can reproduce the efficient level for both.

3.3. Welfare decomposition

The endogenous loadings on private and public information α and η are critical to understanding the efficiency properties of the equilibrium. In the following Lemma, we provide a decomposition of the welfare loss that is going to be useful in the following.

Lemma 3.1. *In equilibrium, we can decompose the welfare loss (3) as* $WL = WL^B + WL^D$*:*

$$WL^B = \frac{1}{2}\frac{(1-\gamma\alpha)^2}{(\beta+\gamma)}\frac{1}{\tau} + \frac{\gamma\alpha^2}{2\tau_\varepsilon} \tag{9}$$

$$WL^D = \frac{(1-\gamma\alpha-\gamma\eta)^2}{2(\beta+\gamma)}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right), \tag{10}$$

where $1 - \gamma\alpha - \gamma\eta = \theta$*.*

The first term, WL^B, is the welfare loss that would occur for Bayesian agents having loading on private information equal to α. The second term, WL^D, represents the additional bias that diagnostic expectations add *beyond* the change in α. It represents the welfare loss due to the inefficient relation between the loading on private information and the loading on public information. In the Bayesian benchmark and team solution, $\eta = \frac{1}{\gamma} - \alpha$, so the term WL^D vanishes. Instead, with diagnostic expectations, we have $1 - \gamma\alpha - \gamma\eta = \theta$. In particular, this term comes from the fact that the weight of public information will overshoot or undershoot with respect to the optimal value, depending on whether $\theta > 0$ or $\theta < 0$. This is useful to separate the direct effect of overreaction from the effect on the loading α.

3.4. Welfare effect

The following proposition characterizes the effect of overreaction on welfare.

Proposition 3.

1. *A small overreaction improves welfare if and only if the loading on private information is too small in the Bayesian benchmark. Formally: in $\theta = 0$ we have:* $\frac{dWL}{d\theta}\,|_{\theta=0} > 0 \iff a^* > a^T$.
2. *If overreaction is large enough, a further increase hurts welfare; and the analogous is true for underreaction. Formally: there are thresholds θ^*, θ_* such that for $\theta > \theta^*$ we have $\frac{dWL}{d\theta} > 0$, and for $\theta < \theta_*$ we have $\frac{dWL}{d\theta} < 0$.*

The proposition shows that, when overreaction is close to zero, its welfare impact depends solely on the balance of externalities in the Bayesian case: in particular, if $a^* < a^T$, so that the learning externality prevails, overreaction is welfare improving. The key mechanism driving the result is that overreaction increases the sensitivity to private information $\alpha = a(\theta + 1)$, and it also increases the sensitivity to public information η (as Proposition 2 describes):

$$\frac{dWL}{d\theta} = \frac{\partial WL}{\partial \alpha}\frac{d\alpha}{d\theta} + \frac{\partial WL^D}{\partial \eta}\frac{d\eta}{d\theta}.$$

The increase of α has the effect of making the price more sensitive to the true value, which has two implications. First, this makes the price a better signal of the value, mitigating the information externality. Second, it exacerbates the pecuniary externality. The increase in η, instead, has only the effect of increasing the term related to the over/undershooting of expectations WL^D. From Lemma 3.1, we can conclude that the loading on public information affects only WL^D, and indeed the term WL^D is minimized for $\eta = 1 - \gamma\alpha$, which is true only when $\theta = 0$.[11] This is because the precision of public information is only affected by the loading α, not η. As a consequence, $\frac{\partial WL^D}{\partial \eta}\,|_{\theta=0} = 0$ and also $\frac{\partial WL^D}{\partial \alpha}\,|_{\theta=0} = 0$; additionally, since $\frac{d\alpha}{d\theta} > 0$ by Corollary 2.1, we have:

$$sgn\left(\frac{dWL}{d\theta}\right) = sgn\left(\frac{\partial WL}{\partial \alpha}\frac{d\alpha}{d\theta} + \frac{\partial WL^D}{\partial \eta}\frac{d\alpha}{d\theta}\right) = sgn\left(\frac{\partial WL^B}{\partial \alpha}\right).$$

The sign of the welfare impact is given by the sign of $\frac{\partial WL}{\partial \alpha}$, which is positive if and only if the pecuniary externality is stronger at the Bayesian benchmark from Proposition 2.

When the overreaction parameter is far from 0, the term WL^D instead becomes important. This term incorporates the expected mistake that agents make in overestimating (underestimating) V when they get positive (negative) information. The second part of the Proposition says that if the overreaction parameter θ and the consequent expected error is large enough, positive or negative, then moving further from the Bayesian benchmark can only reduce welfare. To sum up: a limited amount of overreaction can have a positive effect, depending on the interplay of prediction error, information externality, and pecuniary externality.

[11] This can also be seen from the fact that the term WL^D is second order in θ.

In Fig. 1, we can see a graphical representation of the welfare loss as a function of θ for different values of the parameters. In both subfigures, the blue line represents the total welfare loss WL, the orange line represents WL^B, the horizontal green line represents the welfare loss at the decentralized market equilibrium for $\theta = 0$, and the red line represents the welfare loss at the second-best (team) solution. The optimal value of θ is denoted θ^*. In both the above figures, the welfare loss is convex, so we find that the two thresholds identified in Proposition 3 are the same: $\theta^* = \theta_*$. The minimum of the welfare loss WL^B is reached for the value θ' of Proposition 2, such that $a^*(\theta') = a^T$. In the left panel (Fig. 1(a)), the parameters are such that $a^* < a^T$: consistent with Proposition 3, the graph shows that around $\theta = 0$, a small increase in overreaction decreases the welfare loss. Actually, the graph shows more: there is an optimal overreaction level $\theta^* > 0$. As explained above, the mechanism works through overreaction increasing the loading on private information. However, since also the loading on public information increases, we have that the value of θ', where the loading on private information is at the efficient level, is too large: at that level, the welfare loss is increasing again. Indeed, the optimal value of overreaction is reached for a value θ^* smaller than θ'. Analogously, in the right panel (Fig. 1(b)), the parameters are such that $a^* > a^T$, so the pecuniary externality prevails, and indeed around $\theta = 0$, a small decrease in θ improves welfare.

3.5. Policy

We have seen that, in this economy, there are multiple inefficiencies due to the fact that agents might trade too much or too little relative to what would be the optimum, given their private signals. These inefficiencies are already present in the Bayesian case: moreover, the diagnostic bias can exacerbate (or not) these inefficiencies. Since the inefficiencies stem from the departures of the amounts traded from the second best, we now explore whether a tax (or subsidy) on quantities exchanged can be used to correct the inefficiencies and provide higher welfare. Vives (2017) shows that, in the Bayesian case, a quadratic tax/subsidy can implement the second-best level of the loading on private information a^T. In this section, we ask a related question: when does the introduction of a small tax improve welfare, and when does a small subsidy do so instead?[12]

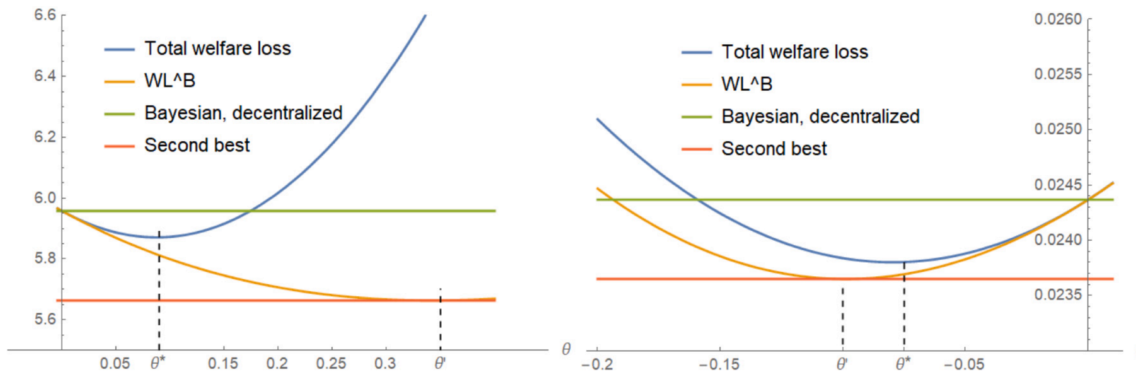
A linear tax/subsidy here cannot improve welfare. Indeed, a linear tax would simply shift uniformly all the demands, but would leave the loading on private and public information unaffected. Therefore, it would simply add an additional term t^2 to the welfare loss, thereby increasing the volatility term: as a consequence, the introduction of a linear tax/subsidy would never be optimal. The natural next step is to explore a quadratic tax/subsidy δ.

Formally, we assume that when agents trade a volume $|D_i|$, they have to pay an additional amount $\frac{1}{2}\delta D_i^2$, where if $\delta < 0$, this is understood to be a subsidy. Both buyers and sellers have to pay the tax. So, the payoff of the informed speculators becomes:

$$u_i = (V - p)D_i - \frac{1}{2}(\gamma + \delta)D_i^2.$$

We assume $\delta > -\gamma$ so that the problem of the agents remains concave. Since the tax is levied also on the liquidity suppliers, the inverse demand becomes: $p = -\mu_S - S + (\beta + \delta)\overline{D}$. In the next subsection, we show that the results are qualitatively the same if the tax is levied on informed speculators only.

[12] As discussed in the introduction, a tax on financial transactions is a well-known idea. An example of a policy that can be compared to a subsidy is the tax incentive for investment in retirement plans and pension funds, present in many countries: for example, the tax-deductibility of 401(k) plan contributions in the USA (Engen and Gale, 2000); similar policies are present in many countries, such as Italy and the UK (Whitehouse, 2005). These are not exactly subsidies on trading volume, but on investment: however, our setting is one where we cannot distinguish investment from financial speculation.



(a) The parameters are: $\gamma = 3$, $\beta = 0.1$, $\tau_0 = \tau_\varepsilon = 0.01$, $\tau_S = 50$. The loadings satisfy $a^* = 0.079 < a^T = 1.12$, and the thresholds $\theta^* = \theta_* = 0.09 < \theta' = 0.35$.

(b) The parameters are: $\gamma = 3$, $\beta = 2$, $\tau_0 = \tau_S = 1$, $\tau_\varepsilon = 5$. The loadings satisfy $a^* = 1.64 > a^T = 1.54$, and the thresholds $\theta^* = \theta_* = -0.075 > \theta' = -0.1$.

Fig. 1. A graphical representation of the welfare loss as a function of θ for different values of the parameters.

We follow the assumption in Vives (2017) that the revenues/payments from this tax/subsidy are rebated in a lump-sum amount T to satisfy the budget balance. As such, the rebate T does not affect the optimal choice of the agents. In the model with a tax/subsidy, to obtain the demand of agent i we simply have to substitute $\gamma+\delta$ to γ and $\beta+\delta$ to β in Eq. (7): the expressions can be found in Eq. (16) in the Appendix.

The total amount paid by informed speculators is $\frac{\delta}{2}\int D_i^2 \mathrm{d}i$, the one paid by the liquidity suppliers is $\frac{\delta}{2}\overline{D}^2$, and the total revenues collected must equal the rebate, so: $T = \frac{\delta}{2}\int D_i^2 \mathrm{d}i + \frac{\delta}{2}\overline{D}^2$. The welfare loss with respect to the first best is:

$$W^o - \left(W - \frac{\delta}{2}\overline{D}^2 - \frac{\delta}{2}\int D_i^2 \mathrm{d}i + T \right) = W^o - W$$

because the additional terms cancel out thanks to the budget balance condition. Therefore, the welfare loss satisfies the *same* expression as Eq. (4): this is because the welfare loss is computed from the perspective of a Bayesian agent.[13]

The effect of the tax is to reduce the incentive to trade: this means that a higher tax affects *both* the loading on private information and the loading on public information. In the following, the loadings are always functions of δ, so we suppress the functional dependence to lighten the notation. The next Proposition characterizes the effect of the tax on the loadings.

Proposition 4. *In the diagnostic expectation equilibrium of the model with the tax/subsidy δ, we have:*

1. *The loading on private information α is decreasing in the tax: $\frac{\mathrm{d}\alpha}{\mathrm{d}\delta} < 0$. Moreover, there always exist a unique δ^* such that $\alpha(\delta^*) = a^T$.*
2. *The loading on public information η can be both increasing or decreasing in δ.*
3. *The loading on the price η_p is decreasing in δ: $\frac{\mathrm{d}\eta_p}{\mathrm{d}\delta} < 0$.*
4. *The equilibrium is second-best efficient if and only if $\theta = 0$, $a^* = a^T$ and $\delta = 0$.*

The tax tends to decrease the loadings because it tends to decrease trade. Indeed, the loading on private information α is decreasing in the tax. However, the tax has an ambiguous effect on the informativeness of the price, $B^2/C^2 = \alpha^2(\beta+\delta)^2$, because it increases the slope of the demand $\beta+\delta$. Therefore, since it can increase the precision of public information, it has an ambiguous effect on the loading on public information η.

Point (1) shows that it is always possible to find a tax level δ^* that implements the second-best level of the loading on private information, meaning that $\alpha(\delta^*) = a^T$. However, since the tax distorts all the loadings, including the price loading, there is no tax level that can achieve second-best efficiency. This is easiest to see when noting that the second-best efficient loading on the price is equal to $1/\gamma$, so the only tax that can achieve it is $\delta^* = 0$, even in the Bayesian case $\theta = 0$, since the price loading does not depend on θ. Then point 4 follows from the case without tax studied in Proposition 2.

If we cannot achieve the second best, can we at least improve welfare with a tax/subsidy? The next Proposition affirms that we can. It shows the expression of the welfare loss in the equilibrium with the tax, shows that there is always a finite optimal level of tax/subsidy, and studies the welfare effect of the introduction of a *small tax*, formally characterized as the derivative of the welfare loss, computed at $\delta = 0$, $\frac{\mathrm{d}WL}{\mathrm{d}\delta}\,|_{\delta=0}$. When $\frac{\mathrm{d}WL}{\mathrm{d}\delta}\,|_{\delta=0} < 0$, a small positive tax decreases the welfare loss, and so we say that *a small tax is welfare improving*. When the opposite is true, we say that *a small subsidy is welfare improving*. The formal expression of the welfare loss in this case is Eq. (17) in Appendix.

Proposition 5. *In the diagnostic expectation equilibrium with a tax/subsidy δ:*

1. *If θ is large enough (overreaction strong enough), the introduction of a small tax is welfare improving: $\frac{\mathrm{d}WL^\delta}{\mathrm{d}\delta}\,|_{\delta=0} < 0$;*
2. *If θ is small enough (underreaction strong enough), the introduction of a small subsidy is welfare improving: $\frac{\mathrm{d}WL^\delta}{\mathrm{d}\delta}\,|_{\delta=0} > 0$;*
3. *If $\theta = 0$, a tax could be either welfare improving or decreasing depending on the parameters. For $a^* = a^T$, a small tax is welfare decreasing if and only if $\alpha\beta\tau_S(\alpha(\beta+\gamma)-1)+\tau_0 > 0$.*

A tax δ decreases the total amount traded, and in so doing it also changes the loadings: an increase in δ decreases α, η, and η_p. The expression of the welfare loss above sums up these direct and indirect effects. When θ is large enough, we obtain $\frac{\mathrm{d}WL^\delta}{\mathrm{d}\delta}\,|_{\delta=0} < 0$. This is because when θ goes to infinity, α and η do so as well. Therefore, the amount traded is larger than at the efficient level, and a tax partially corrects this distortion, and thus it is welfare improving. When θ is small enough, the reasoning is analogous, resulting in a subsidy instead of a tax.

When $\theta = 0$ and $\delta = 0$, the indirect effect $\frac{\partial WL^\delta}{\partial \alpha}$ is the same as without the tax: so it is positive or negative according to whether

[13] Notice that this is a key difference with respect to the equilibrium trades, in which the loadings are obtained substituting $\gamma+\delta$ to γ and $\beta+\delta$ to β.

$a^* > a^T$ or vice versa. However, here this is not the only first-order effect. The effect of the tax here works not only through the demand of the informed traders (and their loadings), but also through the slope of supply of the liquidity suppliers $\beta+\delta$. Moving δ away from zero here has two effects: first, it distorts (downward) the amount traded, creating an average discrepancy between the first best and the equilibrium; second, it affects both the strength of the learning externality via the precision of public information ($\tau = \tau_0 + \alpha^2(\beta+\delta)^2\tau_S$), and the strength of the pecuniary externality because it directly changes how the price reacts to quantity. Accordingly, even when $a^* = a^T$, the tax/subsidy can be welfare improving depending on the interplay of these effects. Indeed, in the next paragraph, where we explore the case of a tax that affects only the informed traders, these additional effects are absent, and for $\delta = 0$, the welfare effect of the tax is solely determined by whether $a^* > a^T$ or vice versa.

Proposition 5 addresses the problem of the introduction of a small tax. In Appendix A, we show numerical solutions for the optimal tax level for the two sets of parameters of Figs. 1, showing that they follow the intuition of Proposition 5: with overreaction, the optimal tax is positive, while for low values (large enough underreaction), the optimal tax may be negative, i.e. a subsidy.

3.5.1. Tax affecting only informed traders

Here, we explore a variation in which it is possible to levy the tax only on informed speculators, and we show that the qualitative results are very similar.

If the tax affects only the informed speculators, the liquidity suppliers inverse demand remains $p = -\mu_S - S + \beta\overline{D}$, as in the baseline model. Instead, the loadings in the informed traders' strategies are given by expressions (16), and the coefficient $a(\delta)$ solves the equation:

$$(\gamma+\delta)a(\delta) = \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau(a(\delta))},$$

with the difference that now the precision of public information does not depend directly on δ: $\tau(a(\delta)) = a(\delta)^2(\theta+1)^2\beta^2\tau_S$. As a consequence, δ decreases the loading on both private and public information.

The results are collected in the following Proposition. The expression of the welfare loss in this case is Eq. (19) in Appendix.

Proposition 6. *In the diagnostic expectation equilibrium with a tax only on informed speculators:*

1. *If θ is large enough, the introduction of a small tax is welfare improving.*
2. *If θ is small enough, the introduction of a small subsidy is welfare improving.*
3. *if $\theta = 0$, the introduction of a small tax is welfare-improving if and only if $a^* > a^T$.*

The only qualitative difference from Proposition 5 is point 3, saying that the first order effect of the tax when $\theta = 0$ is determined by whether the learning or the pecuniary externality dominates in the Bayesian benchmark. This is true in this case because the effect of the tax acts only through the loadings of the demand of the informed traders, and the loadings are all at the optimal level exactly when $a^* = a^T$.

4. Conclusion

We show that overreaction to information in the form of diagnostic expectations can improve welfare in markets where there is a strong enough information externality. When the information externality is not strong enough, overreaction can rationalize a tax on financial transactions on efficiency grounds. These results highlight that understanding the *degree* of overreaction is crucial for understanding its welfare effect and the sign of the optimal intervention. The interactions of these effects with other rationales for trading, such as hedging or heterogeneity, and other biases such as cursedness, are interesting avenues for further research.

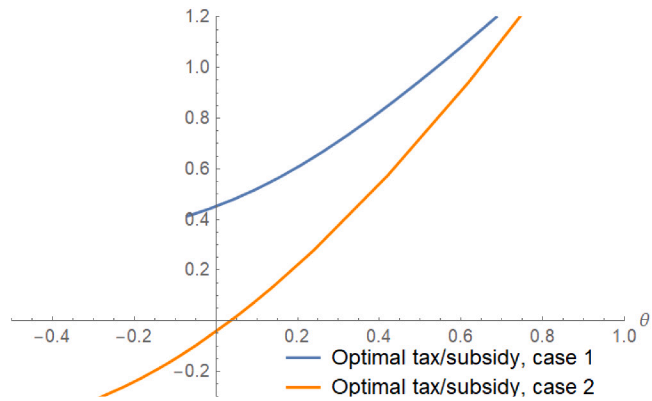


Fig. 2. The optimal tax for different levels of overreaction θ and the two sets of parameters of Figs. 1: case 1 corresponds to Fig. 1(a), case 2 corresponds to Fig. 1(b). In particular, the range of θ for case 1 is shorter because for θ smaller than −0.1 the optimal tax would have been smaller than $-\beta$, and so not feasible.

CRediT authorship contribution statement

Matteo Bizzarri: Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization. **Daniele d'Arienzo:** Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization.

Declaration of competing interest

We declare that we have no relevant or material financial interests that relate to the research described in this paper.

Appendix A. Some numerical representation of the optimal tax/subsidy

Fig. 2 represents the numerical calculation of the *optimal* tax level (not infinitesimal) for different values of θ, and for the two sets of parameters of Figs. 1 (labeled here case 1 and case 2). We can see that, consistently with the intuition, the tax is increasing with θ and, in case 2, for low values of θ (large enough underreaction), the optimal tax is negative, i.e. a subsidy.

Appendix B. Alternative interpretation for the liquidity suppliers

In this section we illustrate an alternative interpretation for the origin of the elastic inverse demand, originating from a simple reduced form model of an entrepreneur issuing equity. There is an entrepreneur that has a project with dividend value V, that is not ex-ante known. The entrepreneur has preferences for remaining in control of the firm, measured by the random variable $\mu_S + S$, that represents the disutility per share sold for the entrepreneur. If she sells an amount $\overline{D}$ of equity, she can raise $p\overline{D}$, at the utility cost $-(\mu_S + S)\overline{D}$, paying the transaction costs $\frac{\beta}{2}\overline{D}^2$. So, in total, the profit of the entrepreneur is:

$$u_i^e = (p + \mu_S + S)\overline{D} - \frac{\beta}{2}\overline{D}^2$$

that gives rise exactly to the inverse demand in the main text.

Appendix C. Proofs

C.1. Proof of Proposition 1

We follow the standard proof method to show the existence of a linear equilibrium (e.g. Vives (2017), Online Appendix), namely, we show that if the pricing function has the form $p = A + BV - CS$, then the equilibrium conditions of individual updating, optimization, and market clearing, give conditions on the coefficients A, B, C under which $p = A + BV - CS$ is indeed an equilibrium.

If the price depends on the fundamental V and the noise S according to $p = A + BV - CS$, then $(p-A)/B$ is a Gaussian random variable of mean V and precision $\tau_{p|V} = B^2/C^2\tau_S$: the agents understand this dependence and use it for their updating. So, after observing private signal s_i and the price p, the Bayesian posterior distribution of the belief on the fundamental V is a Normal with parameters:

$$\mathbb{E}(V \mid s_i, p) = \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S} s_i + \frac{B^2/C^2\tau_S}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S}\frac{p-A}{B}$$
$$Var(V \mid s_i, p) = \left(\tau_0 + \tau_\varepsilon + B^2/C^2\tau_S\right)^{-1}$$

Then, Eq. (2) pins down the distorted posterior expectation of diagnostic agents $\mathbb{E}^\theta(V \mid s_i, p)$, while the variance is the same as the Bayesian one.

The posterior after public information alone is:

$$\mathbb{E}(V \mid p) = \frac{B^2/C^2\tau_S}{\tau_0 + B^2/C^2\tau_S}\frac{p-A}{B}$$

Solving the optimization, the optimal choice for agents is:

$$D_i = \frac{1}{\gamma}\left(\frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S} s_i + \frac{(\theta+1)B^2/C^2\tau_S}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S}(p-A)/B - p\right)$$

So, the loadings are:

$$\alpha = \frac{1}{\gamma}\frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S} \tag{11}$$

$$\eta = \frac{1}{\gamma}\frac{\frac{(\theta+1)B^2/C^2\tau_S}{\tau_\varepsilon+\tau_0+B^2/C^2\tau_S}\frac{1}{B}}{\frac{B^2/C^2\tau_S}{\tau_0 + B^2/C^2\tau_S}\frac{1}{B}} = \frac{(\theta+1)}{\gamma}\frac{\tau_0 + B^2/C^2\tau_S}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S} \tag{12}$$

$$\eta_p = \frac{1}{\gamma} \tag{13}$$

By the Law of Large Numbers, $\int s_i di = V$. So $\overline{D} = \alpha V + \eta\mathbb{E}(V \mid p) - p/\gamma$. Defining $k = \frac{B^2/C^2\tau_S}{\tau_0 + B^2/C^2\tau_S}$, we have $\overline{D} = \alpha V + \eta k(p-A)/B - p/\gamma$. So the market clearing reads:

$$p = -\mu_S - S + \beta/\gamma(\gamma\alpha V + \gamma\eta k(p-A)/B - p)$$

Solving for p:

$$p = \frac{-\gamma\mu_S - \gamma S + \beta(\gamma\alpha V - \gamma\eta k A/B)}{\gamma + \beta(1 - \gamma\eta k/B)}$$

So:

$$B = \frac{\beta\gamma\alpha}{\gamma + \beta(1-\gamma\eta k/B)}$$

$$1 = \frac{\beta\gamma\alpha}{B(\gamma+\beta) - \beta\gamma\eta k}$$

$$B = \beta\gamma\frac{\alpha + \eta k}{\gamma + \beta}$$

$$C = \frac{\gamma}{\gamma + \beta(1-\gamma\eta k/B)}$$

$$C = \frac{\gamma B}{(\gamma+\beta)B - \gamma\beta\eta k} = \frac{\gamma\beta\gamma\frac{\alpha+\eta k}{\gamma+\beta}}{(\gamma+\beta)\beta\gamma\frac{\alpha+\eta k}{\gamma+\beta} - \gamma\beta\eta k} = \frac{\gamma(\alpha+\eta k)}{(\gamma+\beta)\alpha}$$

so that: $B^2/C^2 = \beta^2\alpha^2$, and so: $\tau_{p|V} = \alpha^2\beta^2\tau_S$. Now define $a = \alpha/(\theta+1)$. It must satisfy the equation:

$$\gamma a = \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S} = \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_0 + \beta^2 a^2(\theta+1)^2\tau_S}$$

that is Eq. (8) in the text of the Proposition.

From this, we find that in equilibrium, α and η must satisfy:

$$\alpha = \frac{1}{\gamma}\frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon + \tau_0 + B^2/C^2\tau_S} = \frac{1}{\gamma}\frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon + \tau_0 + \beta^2\alpha^2\tau_S}$$
$$\eta = \frac{1}{\gamma}\frac{(\theta+1)\beta^2\alpha^2\tau_S}{\tau_\varepsilon + \tau_0 + \beta^2\alpha^2\tau_S}\frac{1}{B}$$
$$= \frac{1}{\gamma}\frac{(\theta+1)\beta^2\alpha^2\tau_S}{\tau_\varepsilon + \tau_0 + \beta^2\alpha^2\tau_S}\frac{1}{B}$$
$$= (\theta+1)(1-\gamma a)/\gamma$$

generating expression (7) in the main text.

Since the RHS is monotone decreasing and the LHS is monotone increasing (from 0 to ∞), there is a unique positive solution.

Using the expressions for α and η, we get the equilibrium coefficients:

$$A = \frac{-\gamma\mu_S}{\gamma+\beta}$$
$$B = \beta\frac{(\theta+1)}{\gamma+\beta}$$
$$C = \frac{\theta+1}{\alpha(\gamma+\beta)} \tag{14}$$

where the expression for A comes from:

$$A = \frac{-\gamma\mu_S - \beta((1-\gamma a)(\theta+1)A/B)}{\gamma + \beta(1-(1-\gamma a)(\theta+1)/B)}$$

$$A + \frac{\beta((1-\gamma a)(\theta+1)A/B)}{\gamma + \beta(1-(1-\gamma a)(\theta+1)/B)} = \frac{-\gamma\mu_S}{\gamma + \beta(1-(1-\gamma a)(\theta+1)/B)}$$

$$A = \frac{-\gamma\mu_S}{\gamma+\beta}$$

Finally, define $\tau = \tau_0 + \beta^2 a^2(\theta+1)^2\tau_S$ the precision of public information. The posterior after public information alone is:

$$\mathbb{E}(V \mid p) = \frac{\beta^2\alpha^2\tau_S}{\tau_0 + \beta^2\alpha^2\tau_S}(p-A)/B \tag{15}$$

Using the Law of the Large Numbers, we can express the total demand as:

$$\overline{D} = \frac{\gamma\alpha V + \gamma\eta\mathbb{E}(V \mid p) + S + \mu_S}{\beta+\gamma}$$

or:

$$\overline{D} = \frac{\gamma a(\theta+1)V + (1-\gamma a)(\theta+1)\mathbb{E}(V \mid p) + S + \mu_S}{\beta+\gamma}$$ □

C.2. Proof of Corollary 2.1

1. The first point follows from the implicit function theorem. Indeed, we have:

$$\frac{\mathrm{d}a}{\mathrm{d}\theta} = -\frac{\frac{2a^2\beta^2(\theta+1)\tau_S\tau_\varepsilon}{(a^2\beta^2(\theta+1)^2\tau_S+\tau_0+\tau_\varepsilon)^2}}{\frac{2a\beta^2(\theta+1)^2\tau_S\tau_\varepsilon}{(a^2\beta^2(\theta+1)^2\tau_S+\tau_0+\tau_\varepsilon)^2} + \gamma} = -\frac{2\gamma a(1-\gamma a)\frac{\tau-\tau_0}{\tau(\theta+1)}}{2\gamma(1-\gamma a)\frac{\tau-\tau_0}{\tau} + \gamma} = -\frac{2a(1-\gamma a)\frac{\tau-\tau_0}{\tau(\theta+1)}}{2(1-\gamma a)\frac{\tau-\tau_0}{\tau} + 1}$$

   so $\frac{\mathrm{d}a}{\mathrm{d}\theta} < 0$. But:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\theta} = \frac{\mathrm{d}a}{\mathrm{d}\theta}(\theta+1) + a = -a\frac{2\gamma(1-\gamma a)\frac{\tau-\tau_0}{\tau}}{2\gamma(1-\gamma a)\frac{\tau-\tau_0}{\tau} + \gamma} + a > 0$$

2. from the proof of Proposition 1 we get that $B^2/C^2 = a^2(\theta+1)^2\beta^2\tau_S = \alpha^2\beta^2\tau_S$, hence it is increasing in θ.
3. the volatility of the price is given by:

$$Var(p) = B^2 + C^2 = \frac{1}{(\gamma+\beta)^2}\left(\beta^2(\theta+1)^2\frac{1}{\tau_0} + \frac{1}{a^2\tau_S}\right)$$

   that is increasing in θ. □

C.3. Proof of Proposition 2

The results follow from monotonicity of the loadings, and the fact that they touch every nonnegative value for the relevant θ. To see this precisely, first, we compute the limits of α at the extreme of the domain. For $\theta \to -1$ we have that a goes to its maximum, $\underline{a} = \frac{\tau_\varepsilon}{\tau_\varepsilon+\tau_0}$, and $\alpha \to 0$, as τ does. For $\theta \to \infty$ instead we have $a \to 0$ but $\alpha \to \infty$. Indeed, both a and α are monotonic so they have a limit. Indeed, if $\lim_{\theta\to\infty} a = a' > 0$ (possibly infinite) we would have:

$$\lim_{\theta\to\infty} a = \lim_{\theta\to\infty} \frac{\tau_\varepsilon}{\gamma(\tau_\varepsilon + \tau_0(a')^2\beta^2(\theta+1)^2)} = 0$$

and if $\lim_{\theta\to\infty} \alpha = \alpha' < \infty$ (possibly zero), we would have:

$$\lim_{\theta\to\infty} \alpha = \lim_{\theta\to\infty} \frac{\tau_\varepsilon(\theta+1)}{\gamma(\tau_\varepsilon + \tau_0(\alpha')^2\beta^2)} = \infty$$

that would be contradictions.

Now, for part 1, the limits computed above show that α increases from 0 to infinity, so there is at least a value θ' satisfying the condition. Moreover, Corollary 2.1 shows that α is monotonically increasing in θ, so there can be only one. Finally, since α is monotonically increasing, we have that $\theta' > 0$ if and only if $a^T = \alpha(\theta') > \alpha(0) = a^*$, proving the last part of the statement.

For part 2, the reasoning is analogous: the derivative of the loading is:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}(\eta) = \frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{\theta+1}{\gamma} - \alpha\right) = \frac{1}{\gamma} - \frac{\mathrm{d}\alpha}{\mathrm{d}\theta} = \frac{1}{\gamma} - a + a\frac{2\gamma(1-\gamma a)\frac{\tau-\tau_0}{\tau}}{2\gamma(1-\gamma a)\frac{\tau-\tau_0}{\tau} + \gamma} > \frac{1-\gamma a}{\gamma} > 0$$

so it is monotonically increasing. Moreover, for $\theta \to -1$ the loading goes to zero. Instead, for $\theta \to \infty$ we have that $\alpha \to \infty$, so:

$$\frac{\theta+1}{\gamma} - \alpha = \frac{\theta+1}{\gamma}\left(1 - \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_0(\alpha)^2\beta^2}\right)$$

the term in the parenthesis goes to 1 as $\alpha \to \infty$, so the loading diverges. So, the equation has one and only one solution. Finally, since η is monotonically increasing, we have that $\theta'' > 0$ if and only if $\frac{1}{\gamma} - a^T = \frac{\theta''+1}{\gamma} - \alpha(\theta'') > \frac{1}{\gamma} - \alpha(0) = \frac{1}{\gamma} - a^*$, that is equivalent to $a^* > a^T$, proving the last part of the statement.

For part 3, we have that:

$$\frac{1}{\gamma} - \alpha(\theta') = \frac{\theta''+1}{\gamma} - \alpha(\theta'') \iff$$

$$\alpha(\theta'') - \alpha(\theta') = \frac{\theta''}{\gamma}$$

from which the thesis follows. □

C.4. Proof of Lemma 3.1

The expression for the welfare loss is:

$$WL = W^* - W = (\beta+\gamma)\frac{1}{2}\mathbb{E}(D^o - \overline{D})^2 + \frac{\gamma}{2}\mathbb{E}Var(D_i)$$

The second term is:

$$\mathbb{E}(VarD_i) = \mathbb{E}\int(-\alpha s_i + \alpha V)^2 = \frac{\alpha^2}{\tau_\varepsilon}$$

The first is:

$$\begin{aligned} D^o - \overline{D} &= \frac{V + \mu_S + S}{\beta+\gamma} - \frac{1}{\beta+\gamma}\left(\mu_S + S + \gamma\alpha V + \gamma\eta\mathbb{E}(V \mid p)\right) \\ &= \frac{(1-\gamma\alpha)}{\beta+\gamma}(V - \mathbb{E}(V \mid p)) + \frac{1-\gamma\alpha-\gamma\eta}{\beta+\gamma}\mathbb{E}(V \mid p) \end{aligned}$$

Now we want to compute the expectation of the square. This is equivalent to the variance since all the variables involved have zero expectation:

$$\begin{aligned} \mathbb{E}(D^o - \overline{D})^2 = &\frac{(1-\gamma a)^2}{(\beta+\gamma)^2}\mathbb{E}(V - \mathbb{E}(V \mid p))^2 + \frac{(1-\gamma\alpha-\gamma\eta)^2}{(\beta+\gamma)^2}\mathbb{E}(\mathbb{E}(V \mid p))^2 \\ &+ 2\frac{(1-\gamma\alpha)(1-\gamma\alpha-\gamma\eta)}{(\beta+\gamma)^2}Cov((V - \mathbb{E}(V \mid p))\mathbb{E}(V \mid p)) \end{aligned}$$

We are going to use the following facts:

$$\mathbb{E}(V - \mathbb{E}(V \mid p))^2 = \mathbb{E}(\mathbb{E}((V - \mathbb{E}(V \mid p))^2 \mid p)) = \mathbb{E}(Var(V \mid p)) = \mathbb{E}\left(\frac{1}{\tau}\right) = \frac{1}{\tau}$$

$$\mathbb{E}(\mathbb{E}(V \mid p)^2) = \frac{(\tau-\tau_0)^2}{\tau^2}\mathbb{E}\left(V - \frac{C}{B}S\right)^2 = \frac{(\tau-\tau_0)^2}{\tau^2}\left(\frac{1}{\tau_0} + \frac{C^2}{B^2}\frac{1}{\tau_S}\right) = \frac{1}{\tau_0} - \frac{1}{\tau}$$

and:

$$Cov((V - \mathbb{E}(V \mid p))\mathbb{E}(V \mid p)) = \mathbb{E}(\mathbb{E}(V \mid p)V) - \mathbb{E}(\mathbb{E}(V \mid p)^2) = 0$$

So:

$$\mathbb{E}(D^o - \overline{D})^2 = \frac{(1-\gamma a)^2}{(\beta+\gamma)^2\tau} + \frac{(1-\gamma\alpha-\gamma\eta)^2}{(\beta+\gamma)^2}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right)$$

So, the total welfare loss is:

$$WL = \frac{1}{2}\frac{(1-\gamma\alpha)^2}{(\beta+\gamma)}\frac{1}{\tau} + \frac{1}{2}\frac{(1-\gamma\alpha-\gamma\eta)^2}{(\beta+\gamma)}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right) + \frac{\gamma\alpha^2}{2\tau_\varepsilon}$$

where from Proposition 1, we have that $1 - \gamma\alpha - \gamma\eta = \theta$. So it can be decomposed as:

$$WL^B(\alpha) = \frac{1}{2}\frac{(1-\gamma\alpha)^2}{(\beta+\gamma)}\frac{1}{\tau} + \frac{\gamma\alpha^2}{2\tau_\varepsilon}$$

$$WL^D = \frac{1}{2}\frac{(1-\gamma\alpha-\gamma\eta)^2}{(\beta+\gamma)}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right) = \frac{1}{2}\frac{\theta^2}{(\beta+\gamma)}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right) \quad \square$$

C.5. Proof of Proposition 3

The proof proceeds by computing the derivatives of the welfare loss, and using the decomposition of Lemma 3.1.

From Lemma 3.1, we have that the welfare loss has two components:

$$WL = WL^B + WL^D$$

where WL^B depends on θ only via α, and WL^D is second order in θ. Hence, in $\theta = 0$:

$$\frac{\mathrm{d}WL}{\mathrm{d}\theta}\Big|_{\theta=0} = \frac{\partial WL^B}{\partial\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\theta}\Big|_{\theta=0}$$

Moreover, from Corollary 2.1 we know that α is increasing in θ, so we conclude that, in $\theta = 0$, $\frac{\mathrm{d}WL}{\mathrm{d}\theta}$ this has the same sign as $\frac{\partial WL^B}{\partial\alpha}$. Since this is the Bayesian welfare loss, this is positive if and only if $a^* > a^T$.

The derivatives are:

$$\frac{\partial WL^B}{\partial\alpha} = -\frac{(1-\alpha\gamma)\left(\gamma\tau_0 + \alpha\beta^2\tau_S\right)}{(\beta+\gamma)\left(\alpha^2\beta^2\tau_S + \tau_0\right)^2} + \frac{\alpha\gamma}{\tau_\varepsilon}$$

$$\frac{\partial WL^D}{\partial\theta} = \frac{\theta}{\beta+\gamma}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right)$$

$$\frac{\partial WL^D}{\partial\alpha} = \frac{\alpha\beta^2\theta^2\tau_S}{(\beta+\gamma)\left(\alpha^2\beta^2\tau_S + \tau_0\right)^2}$$

Using the limits computed in the proof of Proposition 2, it follows that for $\theta \to -1$ $\frac{\mathrm{d}\alpha}{\mathrm{d}\theta}$ goes to the finite value $\underline{a} > 0$, while for $\theta \to \infty$ it goes to zero.

Now for $\theta \to \infty$ we have that $\frac{\partial WL^D}{\partial\theta}$ goes to infinity for $\theta \to \infty$, $\frac{\partial WL^D}{\partial\alpha} > 0$ and $\frac{\partial WL^B}{\partial\alpha} > 0$ because WL^B has a finite minimum. So we conclude that $\lim_{\theta\to\infty}\frac{\mathrm{d}WL}{\mathrm{d}\theta} = +\infty$.

Instead, for $\theta \to -1$ $\frac{\partial WL^D}{\partial\theta} < 0$, $\frac{\partial WL^D}{\partial\alpha}$ goes to zero, and $\frac{\mathrm{d}\alpha}{\mathrm{d}\theta}$ goes to the finite value $\underline{a} > 0$. So only $\frac{\partial WL^D}{\partial\theta} < 0$ survives, and the limit is negative: $\lim_{\theta\to-1}\frac{\mathrm{d}WL^D}{\mathrm{d}\theta} < 0$

Now for $\theta \to \infty$ the welfare loss diverges: hence the optimal value of θ has to be finite. (take any finite value $t = WL(\theta')$, there is a θ'' such that $WL > t$ for all $\theta > \theta''$ and so the optimum is smaller than θ''). Hence, for θ large enough, $\frac{\mathrm{d}WL}{\mathrm{d}\theta} > 0$. □

C.6. Proof of Proposition 4

All the equilibrium expressions are analogous to what derived in Proposition 1, with $\gamma+\delta$ in place of γ and $\beta+\delta$ in place of β. In particular we obtain the following expressions for the equilibrium loadings:

$$\alpha(\delta) = a(\delta)(\theta+1)$$
$$\eta(\delta) = \left(\frac{1}{\gamma+\delta} - a(\delta)\right)(\theta+1) = \frac{\theta+1}{\gamma+\delta} - \alpha(\delta)$$
$$\eta_p(\delta) = \frac{1}{\gamma+\delta} \tag{16}$$

where $a(\delta)$ solves:

$$(\gamma+\delta)a(\delta) = \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau(a(\delta))}$$

and $\tau(a(\delta)) = a(\delta)^2(\theta+1)^2(\beta+\delta)^2\tau_S$. Now we use these expressions and compute their derivatives to see the behavior of the loadings as functions of δ.

Using the implicit function theorem, the effect of δ on the loading on private information is:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\delta} = -\frac{\alpha\left(\alpha^2(\beta+\delta)\tau_S(\beta+2(\gamma+\delta)+\delta)+\tau_0+\tau_\varepsilon\right)}{(\gamma+\delta)\left(3\alpha^2(\beta+\delta)^2\tau_S+\tau_0+\tau_\varepsilon\right)} < 0$$

So it is monotonic in δ. Moreover, the limit of α for $\delta \to \infty$ is zero, and the limit for $\delta \to -\gamma$ is $+\infty$ (the proof of these two statements is below). So, there always is a unique δ^* such that $\alpha(\delta^*) = a^T$, proving point 1.

If the limit for $\delta \to \infty$ was a finite or infinite value $\alpha' > 0$ we would have:

$$\lim_{\delta\to\infty} \alpha = \lim_{\delta\to\infty} \frac{1}{\gamma+\delta}\frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon+\tau(\alpha')} = \frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon+\tau(\alpha')}\lim_{\delta\to\infty}\frac{1}{\gamma+\delta} = 0 = \alpha'$$

that would contradict $\alpha' > 0$.

The limit for $\delta \to -\gamma$ is $+\infty$. Indeed, if it was a finite value α', as above:

$$\lim_{\delta\to-\gamma} \alpha = \lim_{\delta\to-\gamma} \frac{1}{\gamma+\delta}\frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon+\tau(\alpha')} = \frac{(\theta+1)\tau_\varepsilon}{\tau_\varepsilon+\tau(\alpha')}\lim_{\delta\to-\gamma}\frac{1}{\gamma+\delta} = +\infty = \alpha'$$

that would contradict the fact that α' is finite.

The effect of the tax on the loading on public information is, instead:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\delta}\left(\frac{\theta+1}{\gamma+\delta}-\alpha\right) &= -\frac{\theta+1}{(\gamma+\delta)^2} + \frac{\alpha\left(\alpha^2(\beta+\delta)\tau_S(\beta+2(\gamma+\delta)+\delta)+\tau_0+\tau_\varepsilon\right)}{(\gamma+\delta)\left(3\alpha^2(\beta+\delta)^2\tau_S+\tau_0+\tau_\varepsilon\right)}\\ &= -\frac{\theta+1}{(\gamma+\delta)^2}\left(1-\frac{\tau_\varepsilon}{\tau_\varepsilon+\tau}\frac{\left(\alpha^2(\beta+\delta)\tau_S(\beta+2(\gamma+\delta)+\delta)+\tau_0+\tau_\varepsilon\right)}{\left(3\alpha^2(\beta+\delta)^2\tau_S+\tau_0+\tau_\varepsilon\right)}\right)\\ &= -\frac{\theta+1}{(\gamma+\delta)^2}\left(\frac{\tau}{\tau_\varepsilon+\tau}-\frac{2\alpha^2(\beta+\delta)\tau_S(\gamma-\beta)}{\left(3\alpha^2(\beta+\delta)^2\tau_S+\tau_0+\tau_\varepsilon\right)}\right)\end{aligned}$$

If $\gamma < \beta$ we have that the derivative is negative. Instead, for $\gamma > \beta$, we have that the derivative is positive if and only if:

$$\frac{\tau}{\tau_\varepsilon+\tau} - \frac{2\alpha^2(\beta+\delta)\tau_S(\gamma-\beta)}{\left(3\alpha^2(\beta+\delta)^2\tau_S+\tau_0+\tau_\varepsilon\right)} \le \frac{\tau}{\tau_\varepsilon+\tau} - \frac{2(\tau-\tau_0)(\gamma-\beta)}{3(\tau+\tau_\varepsilon)}$$

that is negative if and only if $\gamma > \beta + \frac{3\tau}{2(\tau-\tau_0)}$. The LHS grows from zero to ∞ and the RHS decreases from ∞ to zero, so it follows that for γ large enough this is satisfied, proving point 2.

Part 3 and 4 are immediate from $\eta_p = \frac{1}{\gamma+\delta}$ □

C.7. Proof of Proposition 5

All the equilibrium expressions are analogous to what derived in Proposition 1, with $\gamma+\delta$ in place of γ and $\beta+\delta$ in place of β. In particular, the level of trade for agent i is:

$$D_i = (\theta+1)as_i + \frac{(1-(\gamma+\delta)a)(\theta+1)\mathbb{E}(V \mid p) - p}{\gamma+\delta}$$

where a solves:

$$(\gamma+\delta)a = \frac{\tau_\varepsilon}{\tau_\varepsilon+\tau(a)}$$

and $\tau(a) = a^2(\theta+1)^2(\beta+\delta)^2\tau_S$, and:

$$\overline{D} = \frac{1}{\beta+\gamma+2\delta}\left(S+\mu_S+(\gamma+\delta)a(\theta+1)V+(1-(\gamma+\delta)a)(\theta+1)\mathbb{E}(V \mid p)\right)$$

We now use these expressions to compute the expression of the new Welfare loss, and compute its derivatives.

From Lemma 1.1, we know that the expression for the welfare loss is:

$$\frac{1}{2}\left((\beta+\gamma)\mathbb{E}(D^o-\overline{D})^2 + \gamma\mathbb{E}Var(D_i)\right)$$

this is not affected, because the lump-sum rebate means that the tax terms cancel out.

The first best solution D^o is of course not affected by the tax. We have to compute the two terms using the individual demands under a tax δ. The dispersion term has the same form as a function of a as would without the tax:

$$\begin{aligned}\mathbb{E}Var(D_i) &= \mathbb{E}\int \alpha^2(s_i-V)^2\mathrm{d}i = \alpha^2\int \mathbb{E}(s_i-V)^2\mathrm{d}i\\ &= \alpha^2\int \mathbb{E}(\mathbb{E}((s_i-V)^2 \mid V))\mathrm{d}i = \frac{\alpha^2}{\tau_\varepsilon}\end{aligned}$$

Instead, for the volatility term:

$$\begin{aligned}&\overline{D}^o - \overline{D}\\ &= \frac{\mu_S+S+V}{\beta+\gamma} - \frac{1}{\beta+\gamma+2\delta}\left(S+\mu_S+(\gamma+\delta)a(\theta+1)V+(1-(\gamma+\delta)a)(\theta+1)\mathbb{E}(V \mid p)\right)\\ &= \frac{(\beta+\gamma)((1-(\gamma+\delta)a(\theta+1)V)+(1-(\gamma+\delta)a)(\theta+1)\mathbb{E}(V \mid p))+2\delta(\mu_S+S+V)}{(\beta+\gamma)(\beta+\gamma+2\delta)}\\ &= \frac{1}{(\beta+\gamma)(\beta+\gamma+2\delta)}\left((\beta+\gamma)(1-(\gamma+\delta)\alpha)(V-\mathbb{E}(V \mid p))+\right.\\ &\left.+(\beta+\gamma)\theta\mathbb{E}(V \mid p)+2\delta(\mu_S+V+S)\right)\end{aligned}$$

Taking the square and the expectation we get:

$$\begin{aligned}\mathbb{E}(D^o-\overline{D})^2 &= \frac{(1-(\gamma+\delta)\alpha)}{(\beta+\gamma+2\delta)^2\,\tau}\left((1-(\gamma+\delta)\alpha)+\frac{4\delta}{\beta+\gamma}(1+(\beta+\delta)\alpha)\right)\\ &+\frac{4\delta^2(\mu_S^2+\tau_S^{-1}+\tau_0^{-1})}{(\beta+\gamma)^2(\beta+\gamma+2\delta)^2}\\ &+\left(\frac{1}{\beta+\gamma+2\delta}\right)^2\left(\theta^2-\frac{4\delta\theta}{\beta+\gamma}\left(1-\frac{\tau_0}{(\beta+\delta)\alpha\tau_S}\right)\right)\left(\frac{1}{\tau_0}-\frac{1}{\tau}\right)\end{aligned}$$

So the total welfare loss is:

$$\begin{aligned}WL^\delta = \frac{1}{2}&\left(\frac{(1-(\gamma+\delta)\alpha)}{(\beta+\gamma+2\delta)^2\,\tau}((1-(\gamma+\delta)\alpha)(\beta+\gamma)+4\delta(1+(\beta+\delta)\alpha))\right.\\ &+\frac{4\delta^2(\mu_S^2+\tau_S^{-1}+\tau_0^{-1})}{(\beta+\gamma)(\beta+\gamma+2\delta)^2}\\ &\left.+\left(\frac{1}{\beta+\gamma+2\delta}\right)^2\left(\theta^2(\beta+\gamma)-4\delta\theta\left(1-\frac{\tau_0}{(\beta+\delta)\alpha\tau_S}\right)\right)\left(\frac{1}{\tau_0}-\frac{1}{\tau}\right)+\frac{\gamma\alpha^2}{\tau_\varepsilon}\right)\end{aligned} \tag{17}$$

Calculating the derivatives in $\delta = 0$ we get:

$$\begin{aligned}\frac{\partial WL^\delta}{\partial\alpha} &= -\frac{(1-\alpha\gamma)\left(\gamma\tau_0+\alpha\beta^2\tau_S\right)+\theta^2\alpha\beta^2\tau_S}{(\beta+\gamma)\left(\alpha^2\beta^2\tau_S+\tau_0\right)^2}+\frac{\alpha\gamma}{\tau_\varepsilon}\\ \frac{\partial WL^\delta}{\partial\delta} &= \frac{\alpha\left(\tau_0^2(1-(\alpha\gamma)(\beta+\gamma)+2\beta\theta)-2\alpha^3\beta^4\theta(\theta+1)\tau_S^2\right.}{\tau_0(\beta+\gamma)^2\left(\alpha^2\beta^2\tau_S+\tau_0\right)^2}\\ &+\frac{-\alpha\beta\tau_0\tau_S\left((\alpha\gamma-1)(\beta+\gamma)(\alpha(\beta+\gamma)-1)-2\beta\theta(\alpha\beta-1)+\theta^2(\beta-\gamma)\right)}{\tau_0(\beta+\gamma)^2\left(\alpha^2\beta^2\tau_S+\tau_0\right)^2}\end{aligned} \tag{18}$$

Now consider part 1 of the result. In the limit for $\theta \to \infty$ we have that $\frac{\mathrm{d}\alpha}{\mathrm{d}\delta}$ diverges negatively, while in $\frac{\partial WL^\delta}{\partial\delta}$ the leading term is:

$$-\frac{2\alpha^4\beta^4\theta(\theta+1)}{\alpha^4\beta^4\tau_S^4} \to -\infty$$

Moreover, we have seen in Proposition 3 that $\frac{\partial WL^\delta}{\partial\alpha} > 0$ for θ small enough: so we get that $\frac{\partial WL^\delta}{\partial\delta} < 0$ for θ large enough.

Consider part 2. The total derivative goes to zero as $\theta \to -1$ (and $\alpha \to 0$). We can observe that both $\frac{\partial WL}{\partial \delta}$ and $\frac{d\alpha}{d\delta}$ have a factor of α. So, we collect α, and calculating we get:

$$\lim_{\theta\to-1} \frac{1}{\alpha}\frac{dWL^\delta}{d\delta} = \lim_{\theta\to-1} \frac{1}{\alpha}\left(\frac{\partial WL^\delta}{\partial\delta} + \frac{\partial WL^\delta}{\partial\alpha}\frac{d\alpha}{d\delta}\right) = \frac{4\gamma}{(\beta+\gamma)^2\tau_0} > 0$$

Now consider part 3. If $\theta = 0$ expression (18) shows that $\frac{\partial WL^\delta}{\partial\delta} = \frac{a^*(1-\gamma a^*)(a^*\beta\tau_S(a^*(\beta+\gamma)-1)+\tau_0)}{(\beta+\gamma)((a^*)^2\beta^2\tau_S+\tau_0)^2}$. If $a^* = a^T$, by definition, $\frac{\partial WL^\delta}{\partial\alpha} = 0$, and $\frac{d\alpha}{d\delta}$ remains finite. So the total derivative is positive if and only if $\alpha\beta\tau_S(\alpha(\beta+\gamma)-1)+\tau_0 > 0$. □

C.8. Proof of Proposition 6

All the equilibrium expressions are analogous to what derived in Proposition 1, with $\gamma+\delta$ in place of γ, but where, crucially, β is *not* substituted by $\beta+\delta$ as in the proof of Proposition 5. The level of trade for agent i is:

$$D_i = (\theta+1)as_i + \frac{(1-(\gamma+\delta)a)(\theta+1)\mathbb{E}(V \mid p) - p}{\gamma+\delta}$$

where a solves:

$$(\gamma+\delta)a = \frac{\tau_\epsilon}{\tau_\epsilon + \tau(a)}$$

We now use these expressions to compute the expression of the new Welfare loss, and compute its derivatives.

From Lemma 1.1, we know that the expression for the welfare loss is:

$$\frac{1}{2}\left((\beta+\gamma)\mathbb{E}(D^o - \overline{D})^2 + \gamma\mathbb{E}Var(D_i)\right)$$

The first best solution D^o is of course not affected by the tax. We have to compute the two terms using the individual demands under a tax δ. The dispersion term has the same form as a function of a as would without the tax.

Instead, for the volatility term:

$$\overline{D} = \frac{1}{\beta+\gamma+\delta}\left(S + \mu_S + (\gamma+\delta)a(\theta+1)V + (1-(\gamma+\delta)a)(\theta+1)\mathbb{E}(V \mid p)\right)$$

$$\begin{aligned}
&\overline{D}^o - \overline{D}\\
&= \frac{\mu_S + S + V}{\beta+\gamma} - \frac{1}{\beta+\gamma+\delta}\left(S + \mu_S + (\gamma+\delta)a(\theta+1)V + (1-(\gamma+\delta))(\theta+1)\mathbb{E}(V \mid p)\right)\\
&= \frac{(\beta+\gamma)((1-(\gamma+\delta)a(\theta+1)V) + (1-(\gamma+\delta)a)(\theta+1)\mathbb{E}(V \mid p)) + \delta(\mu_S + S + V)}{(\beta+\gamma)(\beta+\gamma+\delta)}\\
&= \frac{1}{(\beta+\gamma)(\beta+\gamma+\delta)}\left((\beta+\gamma)(1-(\gamma+\delta)\alpha)(V - \mathbb{E}(V \mid p)) + \right.\\
&\left. + (\beta+\gamma)\theta\mathbb{E}(V \mid p) + \delta(\mu_S + V + S)\right)
\end{aligned}$$

Taking the square and the expectation we get:

$$\begin{aligned}
\mathbb{E}(D^o - \overline{D})^2 = &\frac{(1-(\gamma+\delta)\alpha)}{(\beta+\gamma+\delta)^2\tau}\left((1-(\gamma+\delta)\alpha) + \frac{2\delta}{\beta+\gamma}(1+\beta\alpha)\right)\\
&+ \frac{\delta^2(\mu_S^2 + \tau_S^{-1} + \tau_0^{-1})}{(\beta+\gamma)^2(\beta+\gamma+\delta)^2}\\
&+ \left(\frac{1}{\beta+\gamma+\delta}\right)^2\left(\theta^2 - \frac{2\delta\theta}{\beta+\gamma}\left(1 - \frac{\tau_0}{\beta\alpha\tau_S}\right)\right)\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right)
\end{aligned}$$

So the total welfare loss is:

$$\begin{aligned}
WL^\delta = &\frac{1}{2}\left(\frac{(1-(\gamma+\delta)\alpha)}{(\beta+\gamma+\delta)^2\tau}((1-(\gamma+\delta)\alpha)(\beta+\gamma) + 2\delta(1+\beta\alpha))\right.\\
&+ \frac{\delta^2(\mu_S^2 + \tau_S^{-1} + \tau_0^{-1})}{(\beta+\gamma)(\beta+\gamma+\delta)^2}\\
&\left.+ \left(\frac{1}{\beta+\gamma+\delta}\right)^2\left(\theta^2(\beta+\gamma) - 2\delta\theta\left(1 - \frac{\tau_0}{\beta\alpha\tau_S}\right)\right)\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right) + \frac{\gamma\alpha^2}{\tau_\epsilon}\right)
\end{aligned} \tag{19}$$

Using the implicit function theorem, the effect of δ on the loadings is:

$$\frac{d\alpha}{d\delta} = -\frac{\alpha}{(\gamma+\delta)\left(\frac{2\alpha^2\beta^2(\theta+1)^2\tau_S}{\alpha^2\beta^2(\theta+1)^2\tau_S+\tau_0+\tau_\epsilon} + 1\right)} < 0$$

$$\begin{aligned}
\frac{d\eta}{d\delta} &= -\frac{\theta+1}{(\gamma+\delta)^2} + \frac{\alpha}{(\gamma+\delta)\left(\frac{2\alpha^2\beta^2(\theta+1)^2\tau_S}{\alpha^2\beta^2(\theta+1)^2\tau_S+\tau_0+\tau_\epsilon} + 1\right)}\\
&= -\frac{\theta+1}{(\gamma+\delta)^2}\left(1 - \frac{\tau_\epsilon}{\tau+\tau_\epsilon}\frac{1}{\left(\frac{2\alpha^2\beta^2(\theta+1)^2\tau_S}{\alpha^2\beta^2(\theta+1)^2\tau_S+\tau_0+\tau_\epsilon} + 1\right)}\right) < 0
\end{aligned}$$

For $\theta \to \infty$ we can see that since $\alpha \to \infty$ we have $\frac{d\alpha}{d\delta} \to -\infty$. For $\theta \to -1$ since $\alpha \to 0$ we have $\frac{d\alpha}{d\delta} \to 0$.

Calculating the derivatives in $\delta = 0$ we get:

$$\frac{\partial WL^\delta}{\partial\alpha}\Big|_{\delta=0} = -\frac{(1-\alpha\gamma)\left(\gamma\tau_0 + \alpha\beta^2\tau_S\right) + \theta^2\alpha\beta^2\tau_S}{(\beta+\gamma)\left(\alpha^2\beta^2\tau_S + \tau_0\right)^2} + \frac{\alpha\gamma}{\tau_\epsilon} \tag{20}$$

$$\frac{\partial WL}{\partial\delta}\Big|_{\delta=0} = -\frac{\alpha\beta\theta\left(\alpha\beta(\theta+1)\tau_S - \tau_0\right)}{\tau_0(\beta+\gamma)^2\left(\alpha^2\beta^2\tau_S + \tau_0\right)} \tag{21}$$

Now consider part 1 of the result. The total derivative is:

$$\frac{dWL^\delta}{d\delta}\Big|_{\delta=0} = \frac{\partial WL^\delta}{\partial\delta}\Big|_{\delta=0} + \frac{\partial WL^\delta}{\partial\alpha}\frac{d\alpha}{d\delta}\Big|_{\delta=0}$$

We have $\frac{\partial WL^\delta}{\partial\alpha}\Big|_{\delta=0} > 0$, $\frac{d\alpha}{d\delta}\Big|_{\delta=0}$ *to* $-\infty$, and $\frac{\partial WL^\delta}{\partial\delta}\Big|_{\delta=0}$ also goes to $-\infty$. So, the welfare loss is decreasing for θ high enough.

Consider part 2. The total derivative goes to zero as $\theta \to -1$ (and $\alpha \to 0$). We can observe that both $\frac{\partial WL^\delta}{\partial\delta}\Big|_{\delta=0}$ and $\frac{d\alpha}{d\delta}\Big|_{\delta=0}$ have a factor of α. So, we collect α, and calculating we get:

$$\lim_{\theta\to-1}\frac{1}{\alpha}\frac{dWL^\delta}{d\delta}\Big|_{\delta=0} = \lim_{\theta\to-1}\frac{1}{\alpha}\left(\frac{\partial WL^\delta}{\partial\delta}\Big|_{\delta=0} + \frac{\partial WL^\delta}{\partial\alpha}\frac{d\alpha}{d\delta}\Big|_{\delta=0}\right) = \frac{2\gamma}{\tau_0^2(\beta+\gamma)^2} > 0$$

Now consider part 3. If $\theta = 0$ expression (21) shows that $\frac{\partial WL^\delta}{\partial\delta}\Big|_{\theta=\delta=0} = 0$. Moreover, for $\theta = 0$ the welfare loss is the same function of α as the Bayesian, and we know from Vives (2017) that it is convex, with a minimum in $a^* = a^T$. □

Data availability

No data was used for the research described in the article.

References

Adam, K., Marcet, A., Beutel, J., 2017. Stock price booms and expected capital gains. Amer. Econ. Rev. 107 (8), 2352–2408.

Afrouzi, H., Kwon, S.Y., Landier, A., Ma, Y., Thesmar, D., 2023. Overreaction in expectations: Evidence and theory. Q. J. Econ. qjad009.

Angeletos, G.-M., Pavan, A., 2007. Efficient use of information and social value of information. Econometrica 75 (4), 1103–1142.

Angeletos, G.-M., Pavan, A., 2009. Policy with dispersed information. J. Eur. Econom. Assoc. 7 (1), 11–60.

Arnold, D., Dobbie, W., Yang, C.S., 2018. Racial bias in bail decisions. Q. J. Econ. 133 (4), 1885–1932.

Auerbach, A.J., Bradford, D.F., 2004. Generalized cash-flow taxation. J. Public Econ. 88 (5), 957–980.

Ba, C., Bohren, J.A., Imas, A., 2022. Over-and underreaction to information. Available at SSRN 4274617.

Bastianello, F., Fontanier, P., 2023. Partial equilibrium thinking, extrapolation, and bubbles. Extrapolation, and Bubbles (December 15, 2023).

Bayona, A., 2018. The social value of information with an endogenous public signal. Econom. Theory 66, 1059–1087.

Bayona, A., Manzano, C., 2022. Competition in schedules with traders that neglect the informational content of the price. Available at SSRN.

Bernardo, A.E., Welch, I., 2001. On the evolution of overconfidence and entrepreneurs. J. Econ. Manag. Strategy 10 (3), 301–330.

Bordalo, P., Coffman, K., Gennaioli, N., Shleifer, A., 2016. Stereotypes. Q. J. Econ. 131 (4), 1753–1794.

Bordalo, P., Conlon, J.J., Gennaioli, N., Kwon, S.Y., Shleifer, A., 2023a. Memory and probability. Q. J. Econ. 138 (1), 265–311.

Bordalo, P., Gennaioli, N., Kwon, S.Y., Shleifer, A., 2020a. Diagnostic bubbles. J. Financ. Econ.
Bordalo, P., Gennaioli, N., La Porta, R., O'Brien, M., Shleifer, A., 2023b. Long term expectations and aggregate fluctuations. NBER Chapters.
Bordalo, P., Gennaioli, N., Ma, Y., Shleifer, A., 2020b. Overreaction in macroeconomic expectations. Amer. Econ. Rev. 110 (9), 2748–2782.
Bordalo, P., Gennaioli, N., Shleifer, A., 2018. Diagnostic expectations and credit cycles. J. Finance 73 (1), 199–227.
Bordalo, P., Gennaioli, N., Shleifer, A., 2022. Overreaction and diagnostic expectations in macroeconomics. J. Econ. Perspect. 36 (3), 223–244.
Bouchaud, J.-P., Krueger, P., Landier, A., Thesmar, D., 2019. Sticky expectations and the profitability anomaly. J. Finance 74 (2), 639–674.
Buss, A., Dumas, B., Uppal, R., Vilkov, G., 2016. The intended and unintended consequences of financial-market regulations: A general-equilibrium analysis. J. Monetary Econ. 81, 25–43.
Colombo, L., Femminis, G., Pavan, A., 2014. Information acquisition and welfare. Rev. Econ. Stud. 81 (4), 1438–1483.
Daniel, K.D., Hirshleifer, D., Subrahmanyam, A., 2001. Overconfidence, arbitrage, and equilibrium asset pricing. J. Finance 56 (3), 921–965.
d'Arienzo, D., 2020. Maturity increasing overreaction and bond market puzzles. Available at SSRN 3733056.
Dávila, E., 2023. Optimal financial transaction taxes. J. Finance 78 (1), 5–61.
Dávila, E., Walther, A., 2023. Prudential policy with distorted beliefs. Amer. Econ. Rev. 113 (7), 1967–2006.
Engen, E.M., Gale, W., 2000. The Effects of 401 (K) Plans on Household Wealth: Differences Across Earnings Groups. National Bureau of Economic Research Cambridge, Mass., USA.
Eyster, E., Rabin, M., 2005. Cursed equilibrium. Econometrica 73 (5), 1623–1672.
Eyster, E., Rabin, M., Vayanos, D., 2019. Financial markets where traders neglect the informational content of prices. J. Finance 74 (1), 371–399.
Farhi, E., Gabaix, X., 2020. Optimal taxation with behavioral agents. Amer. Econ. Rev. 110 (1), 298–336.
Gabaix, X., 2019. Behavioral inattention. In: Handbook of Behavioral Economics: Applications and Foundations 1, vol. 2, Elsevier, pp. 261–343.
Gennaioli, N., Shleifer, A., 2018. A Crisis of Beliefs. Princeton University Press.
Goldin, J., 2015. Optimal tax salience. J. Public Econ. 131, 115–123.
Grossman, S.J., Stiglitz, J.E., 1980. On the impossibility of informationally efficient markets. Amer. Econ. Rev. 70 (3), 393–408.
Hayek, F.A., 1945. The use of knowledge in society. Amer. Econ. Rev. 35, 519–530.
Kahneman, D., Tversky, A., 1972. Subjective probability: A judgment of representativeness. Cognit. Psychol. 3 (3), 430–454.
Kyle, A.S., Wang, F.A., 1997. Speculation duopoly with agreement to disagree: Can overconfidence survive the market test? J. Finance 52 (5), 2073–2090.
Maxted, P., 2024. A macro-finance model with sentiment. Rev. Econ. Stud. 91 (1), 438–475.
Mondria, J., Vives, X., Yang, L., 2022. Costly interpretation of asset prices. Manage. Sci. 68 (1), 52–74.
Moore, D.T., Slemrod, J., 2021. Optimal tax systems with endogenous behavioral biases. J. Public Econ. 197, 104384.
Mullainathan, S., Schwartzstein, J., Congdon, W.J., 2012. A reduced-form approach to behavioral public finance. Annu. Rev. Econ. 4 (1), 511–540.
O'Donoghue, T., Rabin, M., 2006. Optimal sin taxes. J. Public Econ. 90 (10–11), 1825–1849.
Ostrizek, F., Sartori, E., 2021. Acquisition,(mis) use and dissemination of information: The blessing of cursedness and transparency. Available at SSRN 3954947.
Rochet, J.-C., Biais, B., 2023. Taxing financial transactions: A mirrleesian approach. TSE Working Paper.
Sandroni, A., Squintani, F., 2007. Overconfidence, insurance, and paternalism. Amer. Econ. Rev. 97 (5), 1994–2004.
Spinnewijn, J., 2015. Unemployed but optimistic: Optimal insurance design with biased beliefs. J. Eur. Econom. Assoc. 13 (1), 130–167.
Stiglitz, J.E., 1989. Using tax policy to curb speculative short-term trading. Regul. Reform Stock Futures Markets: A Spec. Issue J. Financ. Serv. Res. 3–17.
Summers, L.H., Summers, V.P., 1989. When financial markets work too well: A cautious case for a securities transactions tax. J. Financ. Serv. Res. 3, 261–286.
Tobin, J., 1978. A proposal for international monetary reform. East. Econ. J. 4 (3/4), 153–159.
Uhlig, H., 1996. A law of large numbers for large economies. Econom. Theory 8, 41–50.
Vives, X., 2010. Information and Learning in Markets: The Impact of Market Microstructure. Princeton University Press.
Vives, X., 2014. On the possibility of informationally efficient markets. J. Eur. Econom. Assoc. 12 (5), 1200–1239.
Vives, X., 2017. Endogenous public information and welfare in market games. Rev. Econ. Stud. 84 (2), 935–963.
Whitehouse, E., 2005. Taxation: The Tax Treatment of Funded Pensions. World Bank, Washington, DC.