Energy Balance During Disruptions

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1. Introduction

One of the major threats for the integrity of future fusion devices are disruptions [1], i.e. events during which the plasma energy content is lost on a very fast time scale and can be released under various forms to the structures circumventing the plasma. The typical chain of events during a disruption is the following. When a given operational limit is exceeded on at least one of the main plasma parameters (e.g. density, pressure, safety factor etc.), a fast instability occurs, leading to a very fast plasma cool down (Thermal Quench, TQ). As a consequence, plasma resistivity increases dramatically, giving rise to a rapid current decay (Current Quench, CQ), leading to the termination of the experiment. During such phases, control of the vertical position is often lost, causing a Vertical Displacement Event (VDE).

During a disruption, significant heat, particle and electromagnetic loads may arise on the surrounding structures, which may cause non negligible damage. It is hence fundamental to study and quantify the energy exchange between the plasma and the structures. Being a fusion plasma an intrinsically multiphysics system, particular care must be taken when deriving an overall energy balance. This problem has been also tackled in [2-5].

The analysis of disruptions is usually carried out by coupling in cascade several different models: detailed plasma models [6,7], accounting for the plasma behavior during transient phases (e.g. TQ and CQ); evolutionary equilibrium models [8,9] to describe the electromagnetic interaction of plasma with surrounding structures during the event and predict the electromagnetic loads; thermal models [10] to evaluate the thermal loads. This approach does not allow easily a comprehensive understanding of the actual energy exchange mechanism during such events. This paper instead provides a framework in which electromagnetics, mechanics and thermodynamics are simultaneously taken into account in order to pursue a complete energy balance. The paper is organized as follows. Section 2 is devoted to the derivation of the energy balance, while Section 3 applies the general framework to a specific case study, using the CarMa0NL [9] code to compute the various quantities in the energy balance.

2. Energy balance

Supposing that the one-fluid MHD equations describe the system [11], the following equations must be considered.

- Poynting theorem. This is a direct consequence of Maxwell's equations only and can be interpreted in terms of electromagnetic power balance, involving the variation of toroidal and poloidal magnetic energy $W_{mag,pol} + W_{mag,tor}$, the work done on charge careers and the flux Φ_S of Poynting vector.

- Kinetic energy balance. This is derived from the momentum balance equation and states that kinetic energy K varies due to work done both by pressure force and by Lorentz force.

- Internal energy balance. From thermodynamics, we know that plasma internal energy U may vary due to deformation work, heat flux dQ (bremmstrahlung, radiation losses, external heating, etc.) and Joule losses.

None of these equations can be considered alone, since each is coupled to the others by one or more terms. Combining all these relations and integrating in time, we obtain the energy balance over a fixed volume:

$$\Delta K + \Delta U + \Delta W_{mag,pol} + \Delta W_{mag,tor} = -\Delta Q - \int_{t_1}^{t_2} \Phi_S \, dt \qquad (1)$$

where the symbol Δ indicates difference in time of the various quantities at instant t_2 and t_1 and $\Delta Q = \int_{t_1}^{t_2} dQ$. As spatial integration domain, we consider the fixed region V_{fw} delimited by the inner side of the first wall ∂V_{fw}. We call S_{fw} its poloidal cross section and ∂S_{fw} its contour (a line in the poloidal plane). We assume axisymmetry inside V_{fw} and no halo currents. The quantity K is the (macroscopic) plasma kinetic energy. It is usually assumed that the plasma inertia can be safely neglected during disruptions [12], which is certainly true on a time scale much longer than Alfvén time. In this hypothesis, this term will be neglected.

The internal energy at any time instant can be written as:

$$U(t) = \iiint_{V_{fw}} u(t) \, dV = \iiint_{V_p(t)} u(t) \, dV$$

since the internal energy density $u(t)$ vanishes outside the plasma volume $V_p(t)$, whose poloidal cross section is $S_p(t)$. For a classical ideal gas with three degrees of freedom, the plasma pressure is equal to $p(t) = (\Gamma - 1)u(t)$, where $\Gamma = \frac{5}{3}$, so that the internal energy $U(t)$ is proportional to "pressure energy" $W_{press}(t)$:

$$U(t) = \frac{1}{\Gamma - 1} \iiint_{V_p(t)} p(t) \, dV = \frac{2\pi}{\Gamma - 1} \iint_{S_p(t)} p(t) \, r \, dS = \frac{1}{\Gamma - 1} W_{press}(t)$$

The energy of the toroidal magnetic field $B_{tor} = \frac{f}{r}$ can be expressed as:

$$W_{mag,tor}(t) = \iiint_{V_{fw}} \frac{B_{tor}^2}{2\mu_0} \, dV = \frac{1}{2\mu_0} \iint_{S_{fw}} \frac{f^2}{r^2} 2\pi r \, dS = \frac{\pi}{\mu_0} \iint_{S_{fw}} \frac{f^2}{r} \, dS$$

The poloidal magnetic energy can be written as:

$$W_{mag,pol}(t) = \iiint_{V_{fw}} \frac{B_{pol}^2}{2\mu_0} \, dV = \iiint_{V_{fw}} \frac{|\nabla \psi|^2}{2\mu_0 r^2} \, dV = \frac{\pi}{\mu_0} \iint_{S_{fw}} \frac{|\nabla \psi|^2}{r} \, dS$$

where ψ is the poloidal magnetic flux per radian.

The flux of the Poynting vector is:

$$\Phi_S = \iint_{\partial V_{fw}} (\bar{E} \times \bar{H}) \cdot \hat{n} \, dS = \iint_{\partial V_{fw}} (\bar{E}_{tor} \times \bar{H}_{pol} + \bar{E}_{pol} \times \bar{H}_{tor}) \cdot \hat{n} \, dS$$

$$\Phi_{S1} = \iint_{\partial V_{fw}} (\bar{E}_{tor} \times \bar{H}_{pol}) \cdot \hat{n} \, dS = -\frac{2\pi}{\mu_0} \int_{\partial S_{fw}} \frac{d\psi}{dt} \frac{1}{r} \frac{\partial \psi}{\partial n} \, dl$$

$$\Phi_{S2} = \iint_{\partial V_{fw}} (\bar{E}_{pol} \times \bar{H}_{tor}) \cdot \hat{n} \, dS = -\frac{2\pi}{\mu_0} f_0 \frac{d\phi_{tor}}{dt}$$

where f_0 is the value of f due to TF coils current and to the poloidal current in the vacuum vessel, and ϕ_{tor} is the flux of the toroidal magnetic field thorugh S_{fw}.

Combining some of the terms of (1) we get:

$$\Delta W_{mag,tor} + \int_{t_1}^{t_2} \Phi_{S2} \, dt = \frac{\pi}{\mu_0} \iint_{S_{fw}} \frac{1}{r} \left(f^2(t_2) - f^2(t_1) - \int_{t_1}^{t_2} 2f_0 \frac{df}{dt} dt \right) dS$$

If the toroidal field due to external poloidal currents overwhelms that produced by the plasma ($f_0 \gg f - f_0$), it results $\frac{df^2}{dt} = 2f \frac{df}{dt} \approx 2f_0 \frac{df}{dt}$, so that the variation of the toroidal magnetic energy is almost compensated by the flux of one of the components of the Poynting vector [2].

3. Case study

The energy balance equation (1) is applied to a disruptive plasma; the various quantities are computed with the CarMa0NL code [8]. We analyze a fictitious circular tokamak (Fig. 1); the reference plasma configuration has major radius 1.00 m, minor radius 0.20 m, internal inductance 1.125, poloidal beta 0.586, plasma current 1.5 MA.

The disruption is simulated as a Thermal Quench, bringing to zero the poloidal beta in 0.1 ms, followed by a Current Quench, with a linear decay of the toroidal current of 1 MA/ms. Assuming as initial time instant t_1 the start of the TQ, Fig. 1 reports the various quantities defined in (1). We notice that in fact, as expected, the variation of the toroidal magnetic energy

is almost perfectly compensated by the time integral of the portion Φ_{S2} of the Poynting vector flux. By the way, in the case under analysis, the contribution to the toroidal magnetic flux of the poloidal current induced in the vessel is not negligible, as compared to the toroidal flux due to plasma poloidal current. Conversely, no compensation occurs between the variation of the poloidal magnetic energy and the time integral of the portion Φ_{S1} of the Poynting vector flux; this is coherent with experimental findings [2-4]. Consequently, in the specific case under consideration, the heat flux is almost equal to the time variation of internal energy only during the TQ.

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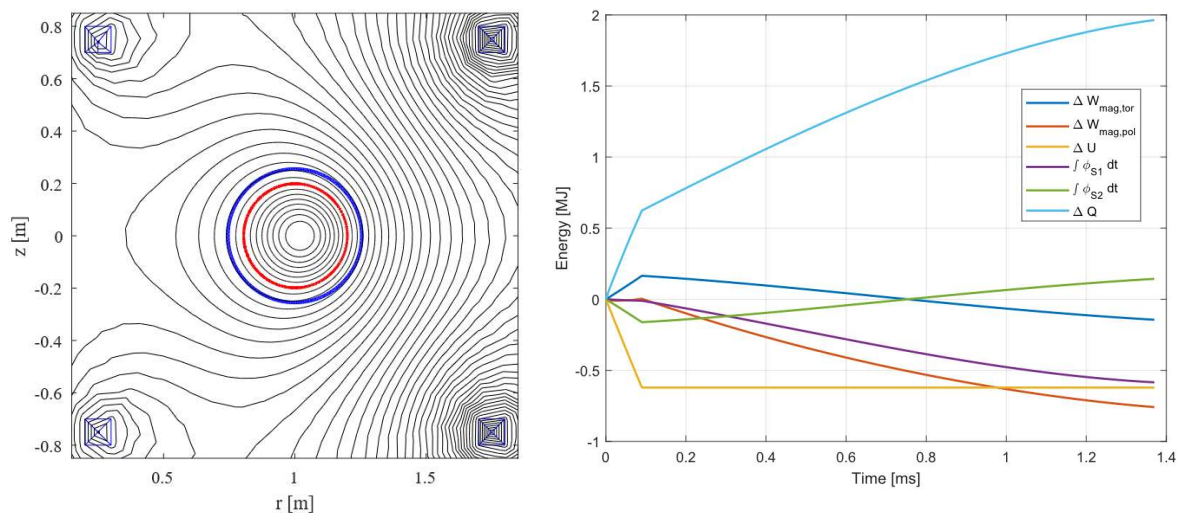[12] Pustovitov V. et al, *Nucl. Fusion* **57** (2017) 126038

Figure 1. Reference equilibrium configuration and quantities defined in (1) for the case study under analysis