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Brief paper

Global stability of multi-agent systems with heterogeneous transmission and perception functions ☆

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ABSTRACT

In this brief, we consider an extended opinion dynamic framework by including both transmitting and perceiving behaviors in the agents' interactions: the first represents how the agent transmits his own opinion to the neighbors, while the latter models how personal features of the agent affect the final perception of external opinions. The agents' interactions are modeled by general piecewise linear functions that can be heterogeneous and not necessarily monotone, thus generalizing the analytical framework usually considered in the literature, in particular, the well-known interval consensus (Fontanet al., 2020). In the considered novel multi-agent scenario, we formulate sufficient operative LMI conditions to assess global network asymptotic stability for either a consensus or a cluster equilibrium, without necessarily requiring strong network connectivity. The proposed approach is validated through illustrative examples.

1. Introduction

During recent years agreement problems have attracted great attention from the scientific communities due to their impact in different fields spanning from sensor networks, formation control, and electric power grid (see e.g. Dörfler, Chertkov, & Bullo, 2013; Manfredi & Angeli, 2018 and references therein) to opinion and social dynamics (Angeli & Manfredi, 2019; Hegselmann & Krause, 2002). For the latter, the main aim is to formulate sufficiently representative mathematical models of social groups' interactions and analyze when the attained equilibrium results in consensus or clustering of opinions. One important example is the well-known Hegselmann–Krause (HK) model, where the agents interact only with those ones with sufficiently close opinions and simultaneously update their opinions by averaging all the opinions of their neighbors (Hegselmann & Krause, 2002). HK models can be classified as agent-based, bounded confidence models, and have been presented in numerous variations in the literature, concerning the possible agents' homophilous, heterophilous, homogeneous, heterogeneous behaviors, see among others (Iervolino, Vasca, & Tangredi, 2018). Recently in Fontan, Shi, Hu, and Altafini (2020) an interesting and novel class of multi-agent networks has been considered, where each agent can limit the interval of values in which a consensus can be accepted. It is still a constrained consensus problem, but the bounds can be violated during the transient regime. More recently Altafini's model has been extended in Su, Wang, and Gao (2023) considering interval coordination problems for multi-agent systems with antagonistic interactions. In such models, each agent is an influencer in the sense that he transmits to his neighbors an opinion value according to a transmission function. Such a function represents how the influencer shows his opinion to the network, and it is assumed to be a saturation-like (and, hence, monotone) function, whose parameters are link-dependent, rather than node-dependent as in Fontan et al. (2020).

On the other hand, a growing body of literature has demonstrated the conditional influence of *issue frames* on self-reported opinion. The capacity *to frame issues* defines how an issue comes to be understood, and represents a key factor in communication strategies available to political elites and the media, strongly affecting the opinion dynamics in social networks (Joslyn & Haider-Markel, 2002). The effects of frames influence on respondents' perceptions of public opinion and are conditioned by message content, the medium of communication, and the predispositions of respondents (Jones, 1994; Rochefort & Cobb, 1994; Stone, 1997). Therefore, the issue frames, or the perception agent's behavior, are crucial as well to determine the overall opinion dynamics.

Herein we extend the above-mentioned multi-agent models (Fontan et al., 2020; Su et al., 2023) by considering a combined behavior derived from the composition of both transmission and

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perception functions. The resulting behavior is modeled by a more general piecewise linear function to take into account more complex types of interaction. A sufficient condition is then formulated to assess the global stability of both consensus and clustering final equilibria of the multi-agent system.

More specifically, the paper's contributions are:

- we extend the models presented in the literature (i.e. Fontan et al., 2020) by considering the frame issue effect. Specifically, the agent interacts with his neighbors through both active (or transmitting) and passive (or perceiving) behavior. The first represents how the agent transmits his own opinion to the neighbors, while the latter models how the personal features of the agent affect the final perception of external opinions. Therefore the same opinion of a node may be perceived in different ways by distinct agents. This involves the model including link-dependent interactions rather than node-dependent ones, differently from the model in Fontan et al. (2020);
- the agents' link interactions are modeled by general piecewise linear functions that can be heterogeneous and not necessarily monotone. In addition, the nature of the agent's interaction, being defined by the sign of the piecewise linear function, can exhibit a "piecewise" cooperative/antagonistic behavior. In this respect, we enlarge the analytical framework of interval consensus (Fontan et al., 2020; Su et al., 2023) where the single agent's interaction is modeled through a saturation-like function and can be merely cooperative or antagonistic.
- we formulate sufficient operative LMI conditions to assess global network asymptotic stability of the equilibrium point. Differently from similar works in the literature (Fontan et al., 2020; Su et al., 2023), herein it is not required the strong network connectivity assumption to formulate the stability conditions, which are valid for either a consensus or a cluster equilibrium.
- the proposed stability analysis approach can be extended to deal with networks of agents with more general nonlinear transmitting and perceiving interactions by approximating them with piecewise linear functions, i.e., splitting the function interval of definition into a number of subintervals and using a linear function approximation on each of them. Indeed, piecewise linear functions have universal approximation properties in the sense that they can approximate a generic nonlinear function with adequate accuracy, provided that the function domain is partitioned into a large enough number of subdomains where the linear function approximation is performed (see, e.g., Bemporad, Heemels, & Lazar, 2010). This extension will be the subject of a future paper.

2. Preliminaries

Given two positive integers p, q, we denote by $\mathbf{0}_{p\times q}$ the zero matrix of pq components. In the case of square matrices, we use the simplified notation $\mathbf{0}_p$. Analogously, $\mathbf{I}_p$ is the $p \times p$ identity matrix.

Let us consider a network of n agents, represented by a graph $\mathcal{G}(\mathcal{I}, \mathcal{E})$, where $\mathcal{I} = \{1, \ldots, n\}$ is the set of nodes, and $\mathcal{E} \subseteq \mathcal{I} \times \mathcal{I}$ the set of ordered and connected pairs of nodes. A weight $a_{ij} > 0$ is assigned to every edge $(j, i) \in \mathcal{E}$. We propose the following model for each agent's opinion dynamics:

$$\dot{x}_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}\left(\phi_{ij}(\theta_j(x_j(t))) - x_i(t)\right),\ \ i \in \mathcal{I}, \tag{1}$$

where $x_i \in \mathbb{R}$, and $\mathcal{N}_i \triangleq \{j \in \mathcal{I} : (j, i) \in \mathcal{E}, a_{ij} > 0\}$ is the neighbors set of a node i. It is assumed the network does not contain self-loops, i.e. $a_{ii} = 0$, $\forall i$. We denote by $A = [a_{ij}]$ the network adjacency matrix, and, by setting $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, the input degree matrix as $D = \mathrm{diag}([d_1, \ldots, d_n]^T)$. The vector $x = [x_1, \ldots, x_n]^T$ collects all the agents' opinions.

The model (1) includes the presence of stubborn agents, with a constant opinion (i.e., for a given i, $a_{ij} = 0$, $\forall j$ and there exists at least an agent j such that $a_{ji} > 0$), and we denote by $\mathcal{S} \subseteq \mathcal{I}$ their index set. Notice that isolated agents (i.e., for a given i, $a_{ij} = a_{ji} = 0$, $\forall j$) can be considered in this framework, too, however, being their evolution uncoupled to the dynamics of all the other agents, they are not considered in this study, for the sake of simplicity.

The multi-behavior characteristic of (1) is represented by the influence functions[1] θ_j and the perception functions ϕ_{ij}. Specifically, θ_j represents how the agent j transmits his own opinion to the network , while ϕ_{ij} represents how personal features of agent i affect the final perception of external opinions (defined for all the agents' pairs with $i \in \mathcal{I} \setminus \mathcal{S}$ and $j \in \mathcal{N}_i$).

The model (1) can be recast in the more compact form:

$$\dot{x}_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}\left(\psi_{ij}(x_j(t)) - x_i(t)\right),\ \ i \in \mathcal{I} \setminus \mathcal{S}, \tag{2a}$$

$$\dot{x}_i(t) = 0,\ \ i \in \mathcal{S}, \tag{2b}$$

where $\psi_{ij}(\cdot) \triangleq (\phi_{ij} \circ \theta_j)(\cdot)$, $i \in \mathcal{I} \setminus \mathcal{S}$, $j \in \mathcal{N}_i$, is the composite function of the influence function θ_j of the agent j on i, and the perception function ϕ_{ij} of the agent i with respect to j. It represents how the opinion of an agent $i \in \mathcal{I} \setminus \mathcal{S}$ depends on the combination of both individual and neighbor features. In the following, for the sake of presentation, we refer to $\psi_{ij}(x_j)$ as the *resulting belief* of agent $i \in \mathcal{I} \setminus \mathcal{S}$ with respect to agent $j \in \mathcal{N}_i$. In order to avoid trivial results, we assume that ψ_{ij} is not identically zero, whenever $a_{ij} \neq 0$. Of course, if $a_{ij} = 0$ this function can be arbitrary in the mathematical representation (2a).

For the analysis performed in this paper, we make the following assumptions, for all the pairs i, j of interacting agents, i.e., $i \in \mathcal{I} \setminus \mathcal{S}$, $j \in \mathcal{N}_i$.

Assumption 1. The perception functions ϕ_{ij} are bounded and the transmission functions θ_j are with an unbounded domain.

Assumption 2. The composition of perception and transmission functions generates a continuous resulting belief ψ_{ij}.

Assumption 3. The belief function ψ_{ij} is piecewise linear with a finite number of breaking points.

Assumptions 1 and 2 are usually present in the literature on interval consensus problem (Fontan et al., 2020), which this paper generalizes. Assumption 3 allows extending the current approach adopted for the case of limited transmitted/perceived opinions from saturation to piecewise linear functions. Such an assumption holds, for instance, when the internal functions θ_j are monotone and the external ones ϕ_{ij} are piecewise linear and bounded with a finite number of breaking points (see Chua & Kang, 1977, p. 923).

Note that model (2) extends (and contains) the framework proposed in Fontan et al. (2020), Su et al. (2023), as the piecewise linear functions ψ_{ij} have not to be necessarily all monotone and saturation like.

The problem of the existence of the equilibrium for the system (2) is related to the problem of the existence of the solution

[1] In the literature, they are also called *transmission functions* (Fontan et al., 2020).

for the system under analysis. The existence and uniqueness of classical solutions for system (2) for any initial condition $x(0) = x_0$ are guaranteed as the characteristics are locally Lipschitz from the Assumptions made (Agarwal & Lakshmikantham, 1993; Coddington & Levinson, 1955). Therefore, a solution always exists and it is unique in any compact set, for any fixed initial condition. Being ψ_{ij} bounded for all $i \in \mathcal{I} \setminus \mathcal{S}, j \in \mathcal{N}_i$ from Assumption 1, let $\overline{\psi} \triangleq \max_{i,j,i\neq j}\{\max_{x_j} \psi_{ij}(x_j)\}$, $\underline{\psi} \triangleq \min_{i,j,i\neq j}\{\min_{x_j} \psi_{ij}(x_j)\}$; it is easy to verify the polytope $\mathcal{P} \triangleq [\underline{\psi}, \overline{\psi}]^n$ is an invariant set under the ensemble dynamics (2). As a consequence, from Brouwer's fixed-point theorem (Basener, Brooks, & Ross, 2006), it is guaranteed the existence of at least one equilibrium point in $\mathcal{P}$. This result can be generalized, as shown by the following Lemma related to a polytope that takes into account the initial conditions of the system (2).

Lemma 4. *Consider the system (2), under the Assumptions 1 and 2. For any initial condition $x(0) = x_0$, the trajectory $x(t; x_0)$ is contained in the set $\mathcal{P}_0 \triangleq \left[\min\{x_0^m, \underline{\psi}\}, \max\{x_0^M, \overline{\psi}\}\right]^n$, where $x_0^M \triangleq \max_{i\in\mathcal{I}}(x_i(0))$ and $x_0^m \triangleq \min_{i\in\mathcal{I}}(x_i(0))$, i.e. the set $\mathcal{P}_0$ is positively invariant under the ensemble dynamics (2), and it includes at least one equilibrium point.*

Proof. If $x_0 \in \mathcal{P}$ the proof is obvious, being $\mathcal{P} \subseteq \mathcal{P}_0$ an invariant set. Suppose then that $x_0 \notin \mathcal{P}$, which means, for instance, and with no loss of generality, there exists at least an agent i such that $x_i(0) > \overline{\psi}$. At any instant of time $t \geq 0$, let $\mathcal{I}_M(t) \triangleq \{j \in \mathcal{I} | x_j(t) = \max_{i\in\mathcal{I}} x_i(t)\}$ be the set of the indices of the agents with the maximum opinion. Since $\psi_{ij}(x_j(t)) \leq \overline{\psi}$, $\forall i, j \in \mathcal{I}$, it is:

$$\max_{i\in\mathcal{I}_M} \dot{x}_i \leq \max_{i\in\mathcal{I}_M} \sum_{j\in\mathcal{N}_i} a_{ij}(\overline{\psi} - x_i(t)) \leq 0. \tag{3}$$

The second inequality in (3) is strict for all $t \geq 0$ such that $x_i(t) > \overline{\psi}$, with $i \in \mathcal{I}_M$, and there exists at least one j with $a_{ij} > 0$ (i.e. $\mathcal{N}_i \neq \emptyset$); it is zero if the agent i is a stubborn agent, i.e. if $a_{ij} = 0$ for all j (or, equivalently, $\mathcal{N}_i = \emptyset$), which implies $x_i(t) = x_i(0)$. As a result, the maximum opinion cannot increase. Analogously, we can conclude that the minimum opinion cannot decrease. Indeed, let $\mathcal{I}_m(t) \triangleq \{j \in \mathcal{I} | x_j(t) = \min_{i\in\mathcal{I}} x_i(t)\}$, the opinion $x_i(t)$ is not decreasing in time for all $t \geq 0$ such that $x_i(t) < \underline{\psi}$, with $i \in \mathcal{I}_m$. Thus, the convex hull of the opinions is nonincreasing in $\mathcal{P}_0 \setminus \mathcal{P}$, which means that the system trajectory $x(t; x_0)$ cannot escape from the set $\mathcal{P}_0$, for any $x_0 \in \mathcal{P}_0$. Finally, by applying Brouwer's theorem to $\mathcal{P}_0$ the proof is complete. □

The conditions to determine the equilibrium point, say $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$, are obtained by setting all the time derivatives of the state-variables in (2) equal to zero, i.e.:

$$\sum_{j\in\mathcal{N}_i} a_{ij}\left(\psi_{ij}(x_j^*) - x_i^*\right) = 0, \forall i \in \mathcal{I} \setminus \mathcal{S}, \tag{4a}$$

$$\dot{x}_i = 0, \quad \forall i \in \mathcal{S}. \tag{4b}$$

Since for all agents $i \in \mathcal{I} \setminus \mathcal{S}$ there exists at least one agent $j \in \mathcal{I}$ such that $a_{ij} > 0$, and by fixing the initial opinion values, conditions (4) can be rewritten as:

$$x_i^* = \frac{\sum_{j\in\mathcal{N}_i} a_{ij}\psi_{ij}(x_j^*)}{\sum_{j\in\mathcal{N}_i} a_{ij}}, \quad \forall i \in \mathcal{I} \setminus \mathcal{S}, \tag{5a}$$

$$x_i^* = x_i(0), \quad \forall i \in \mathcal{S}. \tag{5b}$$

In what follows we refer to the solution x^* as a *cluster equilibrium* if there exists at least a pair (i, j) such that $x_i^* \neq x_j^*$. Another interesting case corresponds to a *consensus equilibrium*, where $x_i^* = \bar{x}, \forall i$. From (5) it follows that a sufficient condition for the equilibrium being a consensus state is the existence of a common opinion value $\bar{x}$ such that $\psi_{ij}(\bar{x}) = \bar{x} = x_s(0)$, for all $i \in \mathcal{I}\setminus\mathcal{S}, j \in \mathcal{N}_i$, $s \in \mathcal{S}$.

For a given equilibrium point, let $e \triangleq x - x^*$ be the *disagreement* variable, and, for each pair of agents $i \in \mathcal{I} \setminus \mathcal{S}, j \in \mathcal{N}_i$, let $\psi'_{ij}(e_j) \triangleq \psi_{ij}(x_j) - \psi_{ij}(x_j^*)$ be the shifted characteristic function, then we can define the error model as:

$$\begin{aligned}\dot{e}_i &= \sum_{j\in\mathcal{N}_i} a_{ij}\left(\psi'_{ij}(e_j) + \psi_{ij}(x_j^*) - e_i - x_i^*\right)\\ &= \sum_{j\in\mathcal{N}_i} a_{ij}\left(\psi'_{ij}(e_j) - e_i\right) + \sum_{j\in\mathcal{N}_i} a_{ij}\left(\psi_{ij}(x_j^*) - x_i^*\right)\\ &= \sum_{j\in\mathcal{N}_i} a_{ij}\left(\psi'_{ij}(e_j) - e_i\right), \quad i \in \mathcal{I} \setminus \mathcal{S},\end{aligned} \tag{6}$$

where $\sum_{j\in\mathcal{N}_i} a_{ij}\left(\psi_{ij}(x_j^*) - x_i^*\right) = 0$ from (4a). If $i \in \mathcal{S}$, it is

$$\dot{e}_i = 0, \tag{7}$$

that, with $e_i(0) = 0$, means $e_i(t) = 0$, $\forall t \geq 0$, i.e. the stubborn agents are always at the equilibrium, by definition. It is easy to verify that the origin is an equilibrium point for the error dynamics, being $\psi'_{ij}(0) = 0$, for all i, j. Moreover, (6) and (7) are decoupled, being $\psi'_{ij}(e_j) = 0$ for all pairs (i, j) with $i \in \mathcal{I} \setminus \mathcal{S}$, $j \in \mathcal{N}_i \cap \mathcal{S}$, and the stubborn agents dynamics not influenced by other agent's opinions. As a result: (i) in the stability analysis of the error dynamics, we can refer to non-stubborn agents only, i.e., in order to avoid repetitions, it is implicitly assumed that $i \in \mathcal{I} \setminus \mathcal{S}$, and (ii) being $\sum_{j\in\mathcal{N}_i} a_{ij}\psi'_{ij}(e_j) = \sum_{j\in\mathcal{N}_i\setminus\mathcal{S}} a_{ij}\psi'_{ij}(e_j)$ for any (i, j) in (6), we also implicitly assume that $\mathcal{N}_i = \mathcal{N}_i \setminus \mathcal{S}$. For the sake of notation simplicity, the index set can be conveniently ordered such that the first $n' \leq n$ and the last $n - n'$ agents are non-stubborn and stubborn ones, respectively.

By assembling Eqs. (6), the dynamics of the disagreement variables for the first n' agents can be written in the matrix form:

$$\dot{e} = -D'e + \Psi'(e), \tag{8}$$

where, with some abuse of notation, $e = [e_1, e_2, \ldots, e_{n'}]^T$, D' is the input degree matrix related to the non-stubborn agents,[2] and:

$$\Psi'(e) = \begin{bmatrix} \sum_{j\in\mathcal{N}_1} a_{1j}\psi'_{1j}(e_j) \\ \vdots \\ \sum_{j\in\mathcal{N}_{n'}} a_{n'j}\psi'_{n'j}(e_j) \end{bmatrix}. \tag{9}$$

Notice that Ψ' defines the node interactions according to the network topology (i.e., the adjacency matrix A) and the agents' behavior (perception/transmission functions composition).

In the following section, we will provide a sufficient condition to assess the global asymptotic stability of the error dynamics. Some considerations on the formulation of a local stability result are also introduced, based on the following remark.

Remark 5. Referring to the error model (6), (7) it is easy to demonstrate that the polytope

$$\mathcal{P}'_0 \triangleq \left[\min\{e_0^m, \underline{\psi'}\}, \max\{e_0^M, \overline{\psi'}\}\right]^n, \tag{10}$$

where $e_0^M \triangleq \max_{i\in\mathcal{I}}(e_i(0))$, $e_0^m \triangleq \min_{i\in\mathcal{I}}(e_i(0))$, $\overline{\psi'} \triangleq \max_{i,j,i\neq j}\{\max_{e_j} \psi'_{ij}(e_j)\}$, and $\underline{\psi'} \triangleq \min_{i,j,i\neq j}\{\min_{e_j} \psi_{ij}(e_j)\}$, is a positively invariant set in the error space.

[2] Note that, even though the stubborn agents do not contribute to the overall agent's characteristic in the error model, their presence is anyway taken into account in the input degree computation.

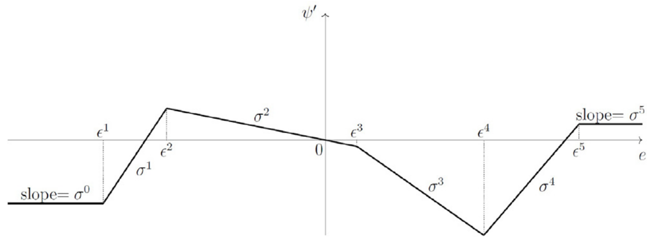

Fig. 1. An example of a piecewise linear (shifted) characteristic with $m = 5$ breaking points.

3. Global stability result

In this section, we present a global stability result based on using a quadratic Lyapunov function. In order to reduce the conservativism of the proposed approach, we take explicitly into account an analytical representation of the (shifted) characteristic functions of the agents. Under Assumptions 1, 2, 3, the resulting translated belief ψ'_{ij} is still a bounded, continuous, and piecewise linear function with a finite number of breaking points, for all $i, j \in \mathcal{I}$. More specifically, each composite function ψ'_{ij}, associated to an i, j pair, $i \in \mathcal{I} \setminus \mathcal{S}$, $j \in \mathcal{N}_i \setminus \mathcal{S}$, is composed of $m+1$ segments,[3] or, equivalently, defined by m distinct breaking points, say $\{\epsilon^1_{ij}, \ldots, \epsilon^m_{ij}\}$, and $m+1$ slopes, say $\{\sigma^0_{ij}, \sigma^1_{ij}, \ldots, \sigma^m_{ij}\}$ (see, e.g., Fig. 1 for a case with $m = 5$). Let $\mathcal{M} = \{1, \ldots, m\}$ be the index set related to the characteristic breaking points. Without loss of generality, it can be assumed that $\epsilon^1_{ij} < \cdots < \epsilon^m_{ij}$, $\forall(i, j)$, $i \neq j$, and the slopes can be numbered from left to right.

Any (continuous) piecewise linear function can be represented according to the following canonical form (Chua & Kang, 1977):

$$\psi'_{ij}(e_j) = \alpha^0_{ij} + \alpha^1_{ij} e_j + \sum_{h=1}^{m} \beta^h_{ij} |e_j - \epsilon^h_{ij}|, \tag{11}$$

where:

(a) $\alpha^0_{ij} = \psi'_{ij}(0) - \sum_{h=1}^{m} \beta^h_{ij} |\epsilon^h_{ij}|$;
(b) $\alpha^1_{ij} = \frac{1}{2}(\sigma^0_{ij} - \sigma^m_{ij}) = 0$[4];
(c) $\beta^h_{ij} = \frac{1}{2}(\sigma^h_{ij} - \sigma^{h-1}_{ij})$, $h \in \mathcal{M}$.

From (11) and definitions (a)–(c), the ith row of (9) is:

$$\Psi'_i(e) = \sum_{j \in \mathcal{N}_i} a_{ij} \psi'_{ij}(e_j) = \sum_{j \in \mathcal{N}_i} \left[\hat{\alpha}^0_{ij} + \sum_{h=1}^{m} \hat{\beta}^h_{ij} \left| e_j - \epsilon^h_{ij} \right| \right], \tag{12}$$

where $\hat{\alpha}^0_{ij} = a_{ij}\alpha^0_{ij}$, $\hat{\beta}^h_{ij} = a_{ij}\beta^h_{ij}$.

Before introducing the stability results, let us consider the following Lemma that provides some useful properties of the components of the vector $\Psi'(e)$.

Lemma 6. *Consider the vector $\Psi'(e)$ in (9) and the expression (12) of its ith row. For all (i, j) pairs, the following properties hold:*

(i) $\sum_{h=1}^{m} \hat{\beta}^h_{ij} = 0$.
(ii) $-\sum_{j \in \mathcal{N}_i} \hat{\alpha}^0_{ij} = \sum_{j \in \mathcal{N}_i} \sum_{h=1}^{m} \hat{\beta}^h_{ij} |\epsilon^h_{ij}| \leq 0$.
(iii) Let $\mathcal{M}^+_{ij} \triangleq \{h \in \mathcal{M} | \hat{\beta}^h_{ij} > 0, j \in \mathcal{N}_i\}$ *and* $\mathcal{M}^-_{ij} \triangleq \{h \in \mathcal{M} | \hat{\beta}^h_{ij} < 0, j \in \mathcal{N}_i\}$, $m^+_{ij} = |\mathcal{M}^+_{ij}|$ *and* $m^-_{ij} = |\mathcal{M}^-_{ij}|$, $\tilde{\beta}^+_i = \max_{j \in \mathcal{N}_i, h \in \mathcal{M}} \hat{\beta}^h_{ij} m^+_{ij} d_i$ *and* $\tilde{\beta}^-_i = \min_{j \in \mathcal{N}_i, h \in \mathcal{M}} \hat{\beta}^h_{ij} m^-_{ij} d_i$, *then it results:* $\tilde{\beta}^-_i \sum_{j \in \mathcal{N}_i} |e_j| \leq \Psi'_i(e) \leq \tilde{\beta}^+_i \sum_{j \in \mathcal{N}_i} |e_j|$.

Proof. The property (i) can be easily verified by using (12) and the assumption about the continuity and boundedness of (the sum of) all the characteristic functions for (all) $e_j \to \infty$. Indeed, if $e_j \leq \epsilon^1_{ij}$ or $e_j \geq \epsilon^m_{ij}$, $\forall j \in \mathcal{N}_i$ then it must be:

$$\Psi'_i(e) = \sum_{j \in \mathcal{N}_i} \hat{\alpha}^0_{ij} + \sum_{j \in \mathcal{N}_i} \sum_{h=1}^{m} \hat{\beta}^h_{ij} \epsilon^h_{ij}, \text{ if } e_j \leq \epsilon^1_{ij}, \tag{13a}$$

$$\Psi'_i(e) = \sum_{j \in \mathcal{N}_i} \hat{\alpha}^0_{ij} - \sum_{j \in \mathcal{N}_i} \sum_{h=1}^{m} \hat{\beta}^h_{ij} \epsilon^h_{ij}, \text{ if } e_j \geq \epsilon^m_{ij}. \tag{13b}$$

From (13a), (13b), it follows property (i), which implies that, except for the trivial case of all $\hat{\beta}^h_{ij} = 0$, for each agent i there must be at least two indices, say $h_{i,1}, h_{i,2} \in \mathcal{M}$, $h_{i,1} \neq h_{i,2}$, such that $\hat{\beta}^{h_{i,1}}_{ij} \hat{\beta}^{h_{i,2}}_{ij} < 0$, $\forall j \in \mathcal{N}_i \setminus \mathcal{S}$.

For $e = 0$ (equilibrium point) it is $\Psi'_i(0) = 0$ and hence from (12) the equality condition in property (ii) holds. Moreover, it is easy to verify that it is also $\sum_{j \in \mathcal{N}_i} \sum_{h=1}^{m} \hat{\beta}^h_{ij} |\epsilon^h_{ij}| \leq \epsilon^m_{ij} m(n-1) \sum_{h=1}^{m} \hat{\beta}^h_{ij} = 0$ from property (i).

Finally, $\forall j \in \mathcal{N}_i$, it is $|e_j - \epsilon^h_{ij}| \leq |e_j| + |\epsilon^h_{ij}|$, and then $\Psi'_i(e) \leq \sum_{j \in \mathcal{N}_i} \hat{\alpha}^0_{ij} + \sum_{j \in \mathcal{N}_i} \sum_{h \in \mathcal{M}} \hat{\beta}^h_{ij} |e_j| + \sum_{j \in \mathcal{N}_i} \sum_{h=1}^{m} \hat{\beta}^h_{ij} |\epsilon^h_{ij}|$. From property (ii) and simple algebra, it follows $\Psi'_i(e) \leq \sum_{j \in \mathcal{N}_i} \sum_{h \in \mathcal{M}} \hat{\beta}^h_{ij} |e_j| \leq \sum_{j \in \mathcal{N}_i} \sum_{h \in \mathcal{M}^+_{ij}} \hat{\beta}^h_{ij} |e_j| \leq \tilde{\beta}^+_i \sum_{j \in \mathcal{N}_i} |e_j|$. Analogously, it is $|e_j - \epsilon^h_{ij}| \geq |e_j| - |\epsilon^h_{ij}|$, and hence $\Psi'_i(e) \geq \sum_{j \in \mathcal{N}_i} \hat{\alpha}^0_{ij} + \sum_{j \in \mathcal{N}_i} \sum_{h \in \mathcal{M}} \hat{\beta}^h_{ij} |e_j| - \sum_{j=1}^{n} \sum_{h=1}^{m} \hat{\beta}^h_{ij} |\epsilon^h_{ij}|$. From property (ii) and simple algebra, it also follows $\Psi'_i(e) \geq \sum_{j=1}^{n} \sum_{h \in \mathcal{M}} \hat{\beta}^h_{ij} |e_j| + 2 \sum_{j \in \mathcal{N}_i} \hat{\alpha}^0_{ij} \geq \sum_{j \in \mathcal{N}_i} \sum_{h \in \mathcal{M}^-_{ij}} \hat{\beta}^h_{ij} |e_j| \geq \tilde{\beta}^-_i \sum_{j \in \mathcal{N}_i} |e_j|$. □

The expression (12) represents the cumulative effect of the perception/transmission function of agent i from his neighbors $j \in \mathcal{N}_i$. The boundedness assumption of all the resulting beliefs (and hence on their sum (12), made explicit by property (iii)) implies that each neighbor j has a limited effect on i, constrained by the following sufficient conditions for property (iii):

$$(\Psi'_i(e) - \tilde{\beta}^+_i |e_j|) \cdot (\Psi'_i(e) - \tilde{\beta}^-_i |e_j|) \leq 0, \ j \in \mathcal{N}_i. \tag{14}$$

In order to tackle (14) by more amenable conditions, we consider that, from property (i) in Lemma 6, it is $\tilde{\beta}^+_i \geq 0$ and $\tilde{\beta}^-_i \leq 0$. Therefore, by letting $\tilde{\beta}_i = \max\{\tilde{\beta}^+_i, |\tilde{\beta}^-_i|\}$,[5] (14) are implied by the conic sector conditions:

$$(\Psi'_i(e) - \tilde{\beta}_i e_j) \cdot (\Psi'_i(e) + \tilde{\beta}_i e_j) \leq 0, \ j \in \mathcal{N}_i. \tag{15}$$

In a more compact matrix form, conditions (15) become:

$$\begin{bmatrix} e_j \\ \Psi'_i(e) \end{bmatrix}^T M_i \begin{bmatrix} e_j \\ \Psi'_i(e) \end{bmatrix} \leq 0, \ j \in \mathcal{N}_i, \tag{16}$$

where

$$M_i = \begin{bmatrix} -\tilde{\beta}^2_i & 0 \\ 0 & 1 \end{bmatrix}.$$

For each element of the vectors e and $\Psi'(e)$ we can write

$$e_j = \nu^T_j e, \tag{17}$$

$$\Psi'_i(e) = \nu^T_i \Psi'(e), \tag{18}$$

[3] For ease of notation, we assume the same number of segments for all the pairs (i, j). Indeed, it is always possible to fictitiously decompose a characteristic into $m+1$ segments with m being the largest number of distinct breaking points $\forall(i, j)$, $i \neq j$.

[4] Note that $\alpha^1_{ij} = 0$, $\forall i, j$, being $\sigma^0_{ij} = \sigma^m_{ij} = 0$, for the boundedness assumption.

[5] Except the trivial case of all $\beta^h_{ij} = 0$, it is always $\tilde{\beta}_i > 0$.

where ν_i, ν_j is the ith and jth canonical vector in $\mathbb{R}^{n'}$, respectively. From (17)–(18) it is

$$\begin{bmatrix} e_j \\ \Psi_i'(e) \end{bmatrix} = T_{ij} \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix}, \tag{19}$$

with

$$T_{ij} = \begin{bmatrix} \nu_j^T & \mathbf{0}_{n'\times 1}^T \\ \mathbf{0}_{n'\times 1}^T & \nu_i^T \end{bmatrix}, \tag{20}$$

for which condition (16) becomes

$$\begin{bmatrix} e \\ \Psi'(e) \end{bmatrix}^T T_{ij}^T M_i T_{ij} \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix} \leq 0. \tag{21}$$

Now we are ready to state the following stability result.

Theorem 7. *Given the error model (8), under Assumptions 1–3, the origin is globally asymptotically stable if there exist a symmetric positive definite matrix P, and positive scalars τ_{ij}, $i \in \mathcal{I}\setminus\mathcal{S}$, $j \in \mathcal{N}_i\setminus\mathcal{S}$ such that the following LMI has a solution:*

$$\begin{bmatrix} -D'P - PD' + \sum_{i\in\mathcal{I}\setminus\mathcal{S}} \sum_{j\in\mathcal{N}_i\setminus\mathcal{S}} \tau_{ij}\tilde{\beta}_i^2 \nu_j\nu_j^T & P \\ P & -\sum_{i\in\mathcal{I}\setminus\mathcal{S}} \sum_{j\in\mathcal{N}_i\setminus\mathcal{S}} \tau_{ij}\nu_i\nu_i^T \end{bmatrix} < 0. \tag{22}$$

Proof. Let us consider as a candidate Lyapunov function for the error model (8) the quadratic function defined as follows:

$$V(e) = e^T P e \tag{23}$$

with P a symmetric positive definite matrix of appropriate dimensions. The derivative of the Lyapunov function along the trajectories of the system (8) is:

$$\dot{V}(e) = -e^T(D'P + PD')e + 2e^T P\Psi'(e), \tag{24}$$

or, in a more compact matrix form:

$$\dot{V}(e) = \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix}^T \begin{bmatrix} -D'P - PD' & P \\ P & \mathbf{0}_{n'} \end{bmatrix} \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix}. \tag{25}$$

It is possible to include the quadratic constraints (21) into the decreasing condition along the system trajectories of the Lyapunov function (23), i.e. $\dot{V}(e) < 0$, by applying the S-procedure. The latter is a Lagrange relaxation technique to express through LMIs a sign condition on a quadratic form (e.g., the Lie derivative of the Lyapunov function (25)) subject to some quadratic inequality constraints (e.g., the conic sector conditions (21)) (Fradkov & Yakubovich, 1979). Then we have:

$$\begin{bmatrix} e \\ \Psi'(e) \end{bmatrix}^T \begin{bmatrix} -D'P - PD' & P \\ P & \mathbf{0}_{n'} \end{bmatrix} \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix} - \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix}^T \Big(\sum_{i\in\mathcal{I}\setminus\mathcal{S}} \sum_{j\in\mathcal{N}_i\setminus\mathcal{S}} \tau_{ij} T_{ij}^T M_i T_{ij} \Big) \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix} < 0. \tag{26}$$

Indeed, it is easy to verify that condition (26) implies the decreasing condition $\dot{V}(e) < 0$ for all pairs $e, \Psi'(e)$ which satisfy the sector constraints. By considering that

$$T_{ij}^T M_i T_{ij} = \begin{bmatrix} -\tilde{\beta}_i^2 \nu_j\nu_j^T & \mathbf{0}_{n'} \\ \mathbf{0}_{n'} & \nu_i\nu_i^T \end{bmatrix}, \tag{27}$$

condition (26) becomes

$$\begin{bmatrix} e \\ \Psi'(e) \end{bmatrix}^T \begin{bmatrix} -D'P - PD' + \sum_{i\in\mathcal{I}\setminus\mathcal{S}} \sum_{j\in\mathcal{N}_i\setminus\mathcal{S}} \tau_{ij}\tilde{\beta}_i^2 \nu_j\nu_j^T & P \\ P & -\sum_{i\in\mathcal{I}\setminus\mathcal{S}} \sum_{j\in\mathcal{N}_i\setminus\mathcal{S}} \tau_{ij}\nu_i\nu_i^T \end{bmatrix} \begin{bmatrix} e \\ \Psi'(e) \end{bmatrix} < 0. \tag{28}$$

If the LMI condition (22) holds then (28) is satisfied. By virtue of Lyapunov's direct method (Khalil, 1992) (being $V(e)$ positive definite, radially unbounded, and its derivative along the system trajectories negative definite), the global asymptotic stability of the origin of the error system (6), under the sector conditions (21), can be concluded, which implies the global asymptotic stability of the equilibrium point of the original model (2). □

Theorem 7 allows evaluating if an equilibrium point of system (2) is asymptotically stable, regardless if it is a consensus or a clustering, and, being a global stability result, requires as a necessary condition, that the original model (1) admits a unique equilibrium point. Such a stability result is global also because the constraints (15) are all globally fulfilled, being the characteristics assumed to be bounded. The following considerations can be made on how to extend the proposed approach.

Remark 8. If the origin of error space belongs to the interior of the invariant set $\mathcal{P}_0'$, then the stability analysis through a candidate quadratic Lyapunov function, as performed in Section 3, necessarily leads to a global stability result. Indeed, the sign condition of a quadratic function (as the function (23) and (25)) on any set is equivalent to its sign condition on the conical hull of that set, which is the entire state space when it has the origin as an interior point. However, if the origin of system (8) is a boundary point of $\mathcal{P}_0'$, i.e. if $\underline{\psi}' = 0$ with $e_0^m \geq 0$, which implies that $\overline{\psi}' > 0$, or, equivalently, $\overline{\psi}' = 0$ with $e_0^M \leq 0$, which implies that $\underline{\psi}' < 0$,[6] by adopting cone-copositivity arguments (Iervolino, Tangredi, & Vasca, 2017; Iervolino, Trenn, & Vasca, 2018), it is possible to get local stability results even though a quadratic candidate Lyapunov function is employed. Due to the page limit, this study has not been reported and is the topic of an ongoing research project.

Remark 9. The LMI (22) becomes infeasible when the input degree matrix D' is null (i.e. the network is composed of all isolated agents), as it appears from the (1, 1) block element of the matrix. Such a circumstance is ruled out since we have considered the scenario where the network nodes are not isolated.

4. Illustrative examples

In what follows we validate the proposed LMI-based stability condition considering the network graph of Fig. 2, composed of 7 followers (from 1 to 7) and two stubborn agents (8 and 9). The edge weights are assumed to be unitary. The line style of the links (solid or dashed) refers to the considered piecewise linear characteristic $\tilde{\psi}$ and $\hat{\psi}$, reported in Fig. 3, whose canonical representations are:

$$\tilde{\psi}(u) = \frac{7}{4} - \frac{1}{2}|u| + \frac{3}{4}|u-1| - \frac{1}{4}|u-2|,$$
$$\hat{\psi}(u) = \frac{5}{4} - \frac{1}{2}|u| + \frac{1}{4}|u-1| + \frac{1}{4}|u-2|.$$

In particular, it is $\psi_{12} = \psi_{28} = \psi_{34} = \psi_{54} = \tilde{\psi}$, $\psi_{41} = \psi_{49} = \psi_{65} = \psi_{76} = \hat{\psi}$.

In the first scenario, consider the presence of the stubborn agent 8 only, as the root of the spanning tree corresponding to the network graph, with $x_8(0) = 1$. Since the belief functions are such that $\psi_{ij}(x_j) = x_j$ when $x_j = 1$, we get a consensus equilibrium

[6] It is easy to verify that, thanks to the continuity assumption of the characteristic functions, it is $\underline{\psi}' \leq 0$ and $\overline{\psi}' \geq 0$.

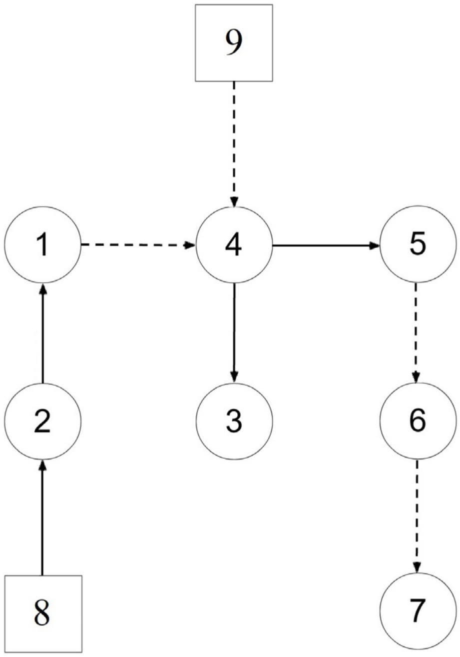

Fig. 2. Network graph with two stubborn agents.

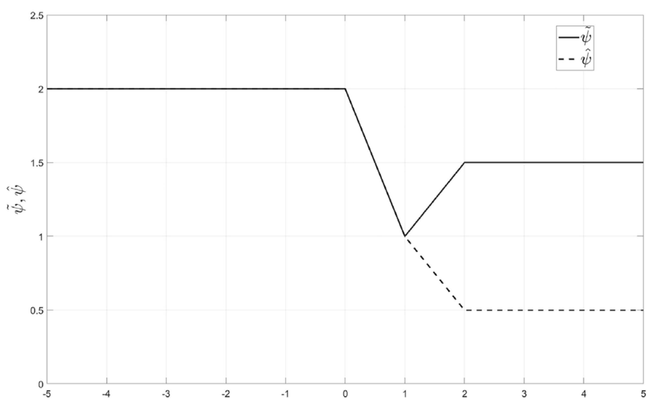

Fig. 3. Two piecewise linear characteristics representing two different agents' interaction behaviors.

value $x^* = \begin{bmatrix}1, & 1, & 1, & 1, & 1, & 1, & 1, & 1\end{bmatrix}^T$. The input degree matrix of the follower agents is $D = \text{diag}(1, 1, 1, 1, 1, 1, 1)$, while the characteristic parameters of the conic sectors are $\tilde{\beta}_1 = \tilde{\beta}_3 = \tilde{\beta}_5 = 1$, $\tilde{\beta}_4 = \tilde{\beta}_6 = \tilde{\beta}_7 = \frac{1}{2}$. By applying Theorem 7, the solutions of the LMIs are $P = \text{diag}(0.221, 0.081, 1.327, 1.196, 0.060, 0.618, 2.873)$ and $\tau_{12} = 0.130$, $\tau_{34} = 0.913$, $\tau_{41} = 0.986$, $\tau_{54} = 0.022$, $\tau_{65} = 0.487$, $\tau_{76} = 2.057$. The application of Theorem 7 allows assessing that the correspondent consensus equilibrium point is globally asymptotically stable. Fig. 4 shows the opinions dynamic evolutions of the agents from the initial condition $x(0) = \begin{bmatrix}0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 1\end{bmatrix}^T$, confirming the final equilibrium is a consensus.

As a second simulation experiment, we modify the previous scenario by reversing the link between agents 4 and 5. Starting from the same initial condition, the final equilibrium is a clustering, as shown in Fig. 5. Indeed, the stubborn agent 8 is no more a root of a spanning tree covering the overall network graph and $\psi_{54}(x_5(0)) \neq 1$ (and $\psi_{56}(x_5(0)) \neq 1$), resulting in a final clustering equilibrium.

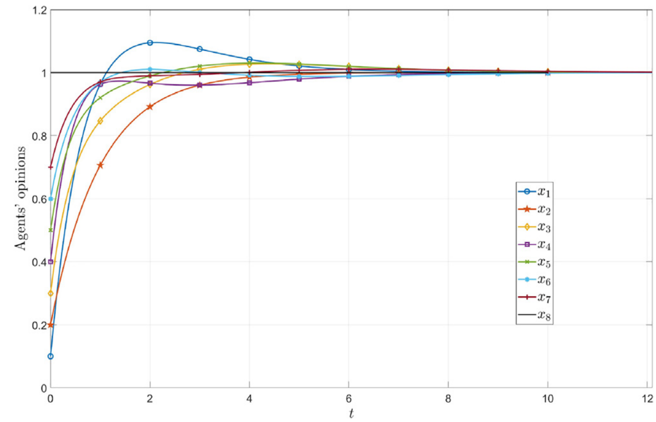

Fig. 4. Consensus scenario with one stubborn 8.

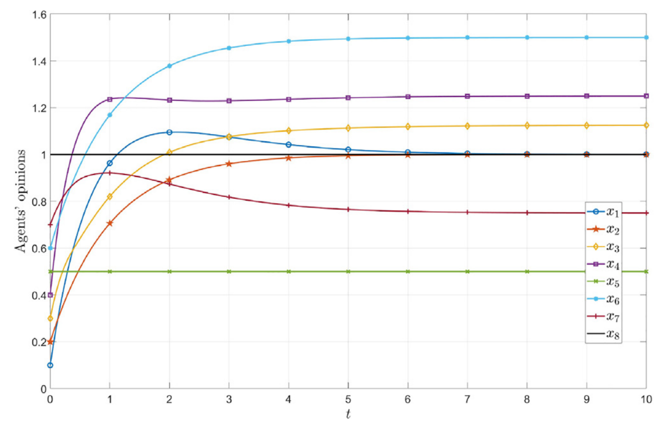

Fig. 5. Clustering scenario with one stubborn 8.

Finally, consider the complete scenario with the two stubborn agents 8 and 9 in Fig. 2. If the stubborn agents 8 and 9 are in agreement (e.g. $\psi_{82}(x_8(0)) = x_8(0) = 1$ and $\psi_{94}(x_9(0)) = x_9(0) = 1$), then the network presents a consensus equilibrium $x^* = \begin{bmatrix}1, & 1, & 1, & 1, & 1, & 1, & 1, & 1, & 1\end{bmatrix}^T$. By using Theorem 7 we can conclude the global asymptotical stability of such an equilibrium, see Fig. 6. When the two stubborn are not in agreement, the equilibrium is a clustering. For $x_8(0) = 1$ and $x_9(0) = 2$, it is $x^* = \begin{bmatrix}1, 1, 1.25, 0.75, 1.25, 0.875, 1.125, 1, 2\end{bmatrix}^T$, which is globally asymptotically stable as can be proved by applying Theorem 7.

Finally, the simulation from the initial condition $x(0) = \begin{bmatrix}0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 1, 2\end{bmatrix}^T$ is reported in Fig. 7, showing how the final followers' states are affected by both stubborn 8 and 9 opinions and their propagation through the network belief functions $\psi_{ij}(\cdot)$.

Remark 10. Notice that the stability analysis of the network equilibrium (either consensus or clustering) in the example considered cannot be carried out for instance by using the approaches proposed in Fontan et al. (2020), Su et al. (2023) since herein we require weaker assumptions and consider more general interactions. Specifically, the graph is not strongly connected, the agents' characteristics are not saturation-like ones, and the monotonicity assumption is not needed since our aim is to provide global stability conditions for a given equilibrium, not necessarily a consensus one.

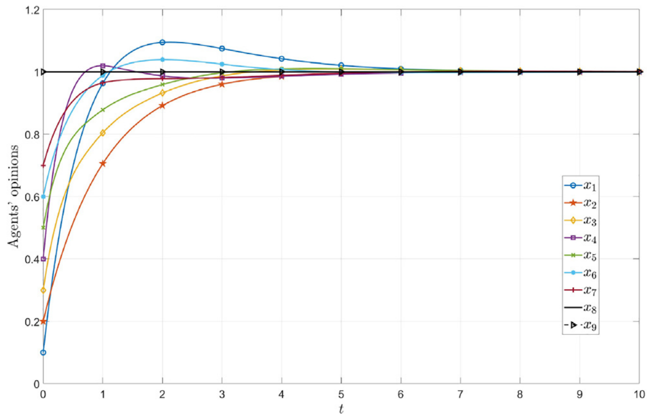

Fig. 6. Consensus scenario with two stubborn 8, 9.

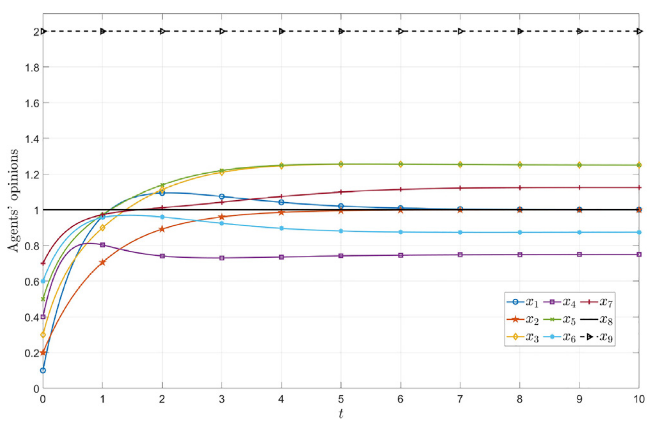

Fig. 7. Clustering scenario with two stubborn 8, 9.

5. Conclusions and future work

In this paper, the stability analysis of the equilibrium point of a multi-behavioral agent system has been analyzed by considering both transmitting and perceiving agents' interactions. The resulting beliefs of the agent are modeled by heterogeneous piecewise linear functions. Sufficient operative LMI condition has been derived to evaluate the global asymptotic stability of the origin of the error space.

A potential extension of the proposed approach may include the stability analysis of more general multi-agent systems with continuous nonlinear interactions approximated by piecewise linear functions. A further development of this study could be the local stability analysis of the error dynamics performed with respect to a positively invariant set having the origin on its boundary and exploiting some results on cone-copositivity of quadratic forms, or to a more general agreement subspace. Moreover, a vectorial opinion framework with possibly discontinuous characteristic interactions could be a natural future extension of the present work. Finally, the formulation of sufficient conditions for the LMIs' feasibility is an interesting topic of ongoing research.

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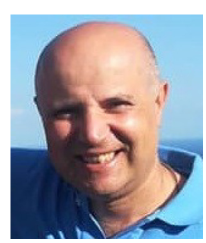

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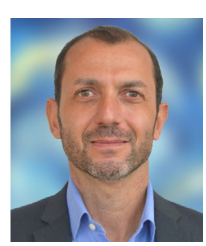

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