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The Cohen-Macaulay property of invariant rings over the integers. (English summary)

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Classifications

- **13A50 - Actions of groups on commutative rings; invariant theory**
- 13F20 - Polynomial rings and ideals; rings of integer-valued polynomials
- 13H10 - Special types (Cohen-Macaulay, Gorenstein, Buchsbaum, etc.)

Citations

From References: 1
From Reviews: 0

Review

Let G be a finite group acting on $\mathbf{k}[x_1,\ldots,x_n]$ by sending each x_i to a $\mathbf{k}$-linear combination of the x_i, where $\mathbf{k}$ is a commutative ring. The invariant ring $\mathbf{k}[x_1,\ldots,x_n]^G$ turns out to be finitely generated if G is finite and $\mathbf{k}$ is a Noetherian ring. The author's attention is focused on the action of a finite group G on the polynomial ring over a principal ideal domain (PID), in particular the ring of integers. The first section is devoted to proving the existence of a homogeneous system of parameters, consisting of n homogeneous polynomials, for the invariant ring $\mathbf{Z}[x_1,\ldots,x_n]^G$. In the second section the author studies the Cohen-Macaulay property for the invariant ring over a PID. The results are that $\mathbf{k}[x_1,x_2]^G$ is Cohen-Macaulay and $\mathbf{k}[x_1,x_2,x_3]^G$ is Cohen-Macaulay in all cases except for the groups conjugate to a cyclic group of order 4 generated by a particular matrix.

Reviewer: Ciampella, Adriana

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This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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