Axionless strong *CP* problem solution: The spontaneous *CP*-violation case

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We propose an alternative to the axion mechanism for addressing the charge parity (CP) problem in QCD. Our approach involves imposing CP as an inherent symmetry of the Lagrangian, which is then spontaneously broken. To generate the correct texture for the Yukawa matrices, we introduce a discrete $\mathbb{Z}_2$ symmetry that is softly broken by the scalar potential. By identifying a benchmark point for the Yukawa couplings that aligns with the measured quark masses, the Cabibbo-Kobayashi-Maskawa matrix, and low-energy flavor-changing constraints, our findings suggest that this model offers a viable solution to the CP problem.

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I. INTRODUCTION

The nonobservation of the neutron's dipole moment provides a strong limit on the CP violating parameter

$$\bar{\theta} = \theta + \mathrm{Arg}\,\mathrm{Det}(M_u M_d), \tag{1}$$

where θ is the coefficient of the QCD topological term

$$\mathcal{L}_{\mathrm{QCD}} \supset \frac{\theta}{32\pi^2} G^{\mu\nu}\tilde{G}_{\mu\nu}, \tag{2}$$

G is the gluon field strength, and $M_{u,d}$ are the quark mass matrices. The strong CP problem consists of explaining the extreme smallness of the parameter $\bar{\theta} \lesssim 10^{-10}$ (see Ref. [1] for a recent review). The strategy to solve this problem involves implementing symmetries (continuous or discrete) that constrain $\bar{\theta}$.

Peccei and Quinn proposed a solution based on a continuous global symmetry $U(1)_{PQ}$ [2] implying the existence of a pseudo-Goldstone boson (the axion) that is also a viable dark matter candidate [3–5]. Even though the Peccei-Quinn mechanism is elegant and simple, it suffers from the so-called quality problem. Furthermore, global symmetries are broken by gravity, meaning that higher-dimensional operators, suppressed by powers of the Planck scale [6–8], result in a minimum of the potential at $\bar{\theta} \neq 0$, unless the coefficients of these operators are exponentially suppressed ($\lesssim 10^{-55}$).

On the other hand, in the so-called *axionless* models relying on discrete global symmetries (see Refs. [9–15] for pioneer works), the quality problem is absent. These models rely on imposing CP or P besides extending either the gauge symmetry or the quark sector to make both terms in Eq. (1) vanish. The topological term proportional to θ is absent due to CP invariance, and $\mathrm{Det}(M_u M_d)$ is a real quantity (at tree level) even if $M_{u,d}$ are complex, allowing one to obtain the sizable phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [16] (a recent advance in this direction is given in Ref. [17]). Nevertheless, once the symmetry is broken either spontaneously or softly, contributions to $\bar{\theta}$ will typically come from one-loop diagrams. Recently, axionless models have been revamped [18–26], also in connection with the flavor problem [27–30] and in particular by means of the modular invariance [31–36]. Different axionless solutions have also been proposed based on supersymmetry [37,38], vacuum accumulation mechanism [39], extra dimensions [40], anomalous $U(1)$ [41], and long-distance vacuum effects [42].

This work considers an axionless solution to the strong CP problem based on Georgi's mechanism [11]. Georgi's original work focused on two families of quarks. Here, we extend the analysis to the realistic case with three families. We compare our results with current experimental constraints, including quark masses and mixings, low-energy flavor-changing neutral currents, and bounds on $\bar{\theta}$. By assuming that the heaviest scalar has a mass of $\lesssim 1$ TeV, the strong constraints on the scalar sector from perturbative unitarity, perturbativity, and boundedness from below are included (see Ref. [43]). We conclude that this model remains a viable solution to the CP problem.

II. MODEL

The model contains two Higgs fields, ϕ_1 and ϕ_2, that are, respectively, even and odd under a $\mathbb{Z}_2$ symmetry. The third left-handed quark family is odd under the discrete $\mathbb{Z}_2$ symmetry and couples only with the second Higgs, ϕ_2. The rest of the Standard Model (SM) fermions are even under $\mathbb{Z}_2$ and therefore couple only with the first Higgs, ϕ_1.

If the scalar potential is completely invariant under $CP \times \mathbb{Z}_2$, the two Higgs fields take real vacuum expectation values (VEVs). However, if the scalar potential contains a $\mathbb{Z}_2$ soft-breaking term, ϕ_1 and ϕ_2 develop complex VEVs after CP is spontaneously broken, as shown in Ref. [11].

The minimization of the two Higgs potential with a softly broken $\mathbb{Z}_2$ symmetry has been recently studied, for instance, in Ref. [44] (more references can be found in the review given in Ref. [45]). Complex VEVs are required to fit the CP-violating phase of the CKM matrix.

The key idea is that, since every row in the Yukawa coupling matrix for the up-quarks and down-quarks comes with ϕ_i and $\tilde{\phi}_i = -i\sigma_2\phi_i^*$ respectively, the phases entering each mass matrix are opposite, and therefore the argument of $\mathrm{Det}(M_u)$ exactly cancels the argument of $\mathrm{Det}(M_d)$. Moreover, if ϕ_2 couples only to the third left-handed quark family, the flavor-changing neutral currents (FCNCs) are under control [46].

As mentioned before, in the three-family realization of Georgi's mechanism considered here, all the right-handed quark fields, U_{iR} and D_{iR}, as well as the left-handed quark doublets,

$$Q_{1L} = \begin{pmatrix} U_{1L} \\ D_{1L} \end{pmatrix}, \qquad Q_{2L} = \begin{pmatrix} U_{2L} \\ D_{2L} \end{pmatrix}, \tag{3}$$

are even under $\mathbb{Z}_2$, while the left-handed quarks Q_{3L} are odd under $\mathbb{Z}_2$. Therefore, the Yukawa couplings become

$$\begin{aligned}\mathcal{L}_Y = &\sum_{i=1}^{2}\sum_{k=1}^{3} Y^u_{ik}\overline{Q_{iL}}U_{kR}\phi_1 + \sum_{i=1}^{3} Y^u_{3i}\overline{Q_{3L}}U_{iR}\phi_2 \\ &+ \sum_{i=1}^{2}\sum_{k=1}^{3} Y^d_{ik}\overline{Q_{iL}}D_{kR}\tilde{\phi}_1 + \sum_{i=1}^{3} Y^d_{3i}\overline{Q_{3L}}D_{iR}\tilde{\phi}_2 + \mathrm{H.c.}\end{aligned} \tag{4}$$

The two Higgs fields take VEVs $\langle\phi_i\rangle = (0, v_i e^{i\alpha_i})^T$, where v_i are real and α_i are phases. Because the model imposes CP invariance, the 18 Yukawa couplings Y^u_{ij} and Y^d_{ij} are real; however, spontaneous CP breaking implies that $\alpha_i \neq 0$. It is possible to show that one of the phases α_i (we choose α_1) can be reabsorbed, leaving only one phase $\alpha_2 \equiv \alpha$.

The two Higgs model gives (from the diagonalization of the most generic potential involving ϕ_1 and ϕ_2) two scalar (CP even) neutral states h and H, one pseudoscalar (CP odd) state A, and two charged ones $H^\pm$.

III. QUARK MASS TEXTURES AND RADIATIVE CORRECTIONS

After electroweak symmetry breaking, from (4), we obtain

$$M_u^0 = \frac{v_1}{\sqrt{2}}\begin{pmatrix} Y^u_{11} & Y^u_{12} & Y^u_{13} \\ Y^u_{21} & Y^u_{22} & Y^u_{23} \\ Y^u_{31}t_\beta e^{i\alpha} & Y^u_{32}t_\beta e^{i\alpha} & Y^u_{33}t_\beta e^{i\alpha} \end{pmatrix},$$

$$M_d^0 = \frac{v_1}{\sqrt{2}}\begin{pmatrix} Y^d_{11} & Y^d_{12} & Y^d_{13} \\ Y^d_{21} & Y^d_{22} & Y^d_{23} \\ Y^d_{31}t_\beta e^{-i\alpha} & Y^d_{32}t_\beta e^{-i\alpha} & Y^d_{33}t_\beta e^{-i\alpha} \end{pmatrix}, \tag{5}$$

where $t_\beta \equiv \tan\beta = v_2/v_1$ and the 0-apex refers to tree-level mass matrices. Loop contributions are expected to modify the $M^0_{u,d}$ pattern. The quark mass matrices, including one-loop corrections (see the diagram of Fig. 1), take the form

$$M_{u,d} = M^0_{u,d} + M^{(1)}_{u,d},$$

where the (1) apex indicates the one-loop contribution that is

$$M_u^{(1)} \approx \begin{pmatrix} \delta^u_{11}e^{+2i\alpha} & \delta^u_{12}e^{+2i\alpha} & \delta^u_{13}e^{+2i\alpha} \\ \delta^u_{21}e^{+2i\alpha} & \delta^u_{22}e^{+2i\alpha} & \delta^u_{23}e^{+2i\alpha} \\ \delta^u_{31}t_\beta^{-1}e^{-i\alpha} & \delta^u_{32}t_\beta^{-1}e^{-i\alpha} & \delta^u_{33}t_\beta^{-1}e^{-i\alpha} \end{pmatrix},$$

$$M_d^{(1)} \approx \begin{pmatrix} \delta^d_{11}e^{-2i\alpha} & \delta^d_{12}e^{-2i\alpha} & \delta^d_{13}e^{-2i\alpha} \\ \delta^d_{21}e^{-2i\alpha} & \delta^d_{22}e^{-2i\alpha} & \delta^d_{23}e^{-2i\alpha} \\ \delta^d_{31}t_\beta^{-1}e^{+i\alpha} & \delta^d_{32}t_\beta^{-1}e^{+i\alpha} & \delta^d_{33}t_\beta^{-1}e^{+i\alpha} \end{pmatrix}, \tag{6}$$

where (for $i = 1$, 2 and $j = 1$, 2, 3)

$$\delta^{u,d}_{ij} = \frac{\lambda_5 v_1^3 t_\beta^2}{16\pi^2 M^2}\sum_{k=1}^{3}\sum_{m=1}^{2} Y^{u,d}_{ik}Y^{u,d*}_{mk}Y^{u,d}_{mj}, \tag{7}$$

and for $i = 3$

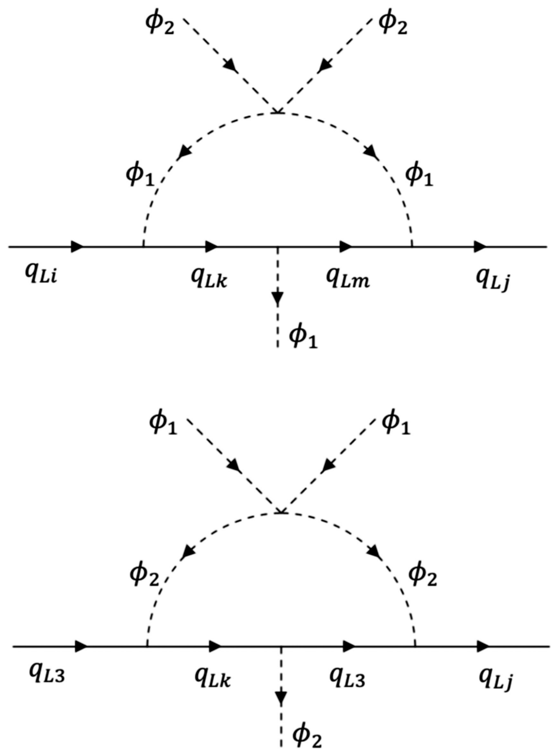

FIG. 1. One-loop diagrams correcting $\bar{\theta}$.

$$\delta^{u,d}_{3j} = \frac{\lambda_5 v_1^3 t_\beta^2}{16\pi^2 M^2} \sum_{k=1}^{3} Y^{u,d}{}_{3k} Y^{u,d*}{}_{3k} Y^{u,d}_{3j}, \tag{8}$$

where $M \sim 1$ TeV is the scale of heaviest Higgs in the loop and λ_5 is the coupling of $((\phi_1^\dagger \phi_2)^2 + (\phi_2^\dagger \phi_1)^2)$ term in the potential (see Ref. [44]).

Including one-loop contribution, a simple estimation gives

$$\mathrm{Arg\,Det}(M_u\, M_d) \sim \sin\alpha \lambda_5 f_Y \frac{v_1^2 \tan\beta}{16\pi^2 M^2} \sim \lambda_5 f_Y 10^{-5}, \tag{9}$$

where f_Y is a function of the δ's and therefore of the Yukawa couplings. This estimation agrees with other estimations, for instance, Refs. [11,12,47]. The parameter λ_5 cannot be too small because of the collider limit on the CP-odd state's mass A. Indeed, $m_A^2 = 2\lambda_5(v_1^2 + v_2^2)$ from which it follows $\lambda_5 \gtrsim 0.02$ from $m_A \gtrsim 50$ GeV.

IV. EXPERIMENTAL LIMITS ON STRONG *CP* PARAMETER

To test this expression, it is necessary to compare it with experimental measurements. To obtain the relation between the measured neutron electric dipole moment (EDM) [48] and $\bar{\theta}$, one must compute the corresponding contribution from the expectation value $\langle n|J_\mu|n\rangle$, which requires an estimation of nonperturbative effects. The first computation of this relation was performed in Ref. [49] using chiral perturbation theory. In this approach, one-loop pion penguin diagrams with the emission of photons between neutron states were calculated. In contrast, Ref. [50] used the operator product expansion and sum rules. More recently, *ab initio* calculations from lattice QCD have also been used to compute the relation [51]. However, despite significant effort, the relation is still known with considerable uncertainty, with at least a ~20% error. Furthermore, Ref. [52] pointed out that the form factors used in Ref. [51] were incorrect and provided the necessary corrections. Recently, using the gradient flow in the lattice the relation was recalculated [53,54], finding agreement with the sum rule method. Additionally, a recent recalculation of the sum-rule approach was conducted in Ref. [55]. In that work, the relation between $\bar{\theta}$ and the neutron EDM was obtained using a different linear combination of the two independent nucleon currents contributing to the neutron EDM. The conclusion was that extracting numerically reliable results with this new linear combination is challenging. Consequently, there is significant uncertainty regarding the relation, which weakens the present constraints on new physics models derived from neutron EDM measurements. In this work, we assume that the experimental constraints correspond to $\bar{\theta}$ centered at zero with an error of 5×10^{-10} at 1σ. It is worth noting the importance of the experimental efforts to measure a signal on the strong CP problem not only of the neutron EDM but also in time-reversal invariance violation neutron-nucleus interaction experiments that will test θ in the same order of magnitude as the neutron EDM experiments [56–58].

V. RESULTS

As in the SM, the mass matrices $M^0_{u,d}$ are diagonalized using a biunitary transformation $V_L^{u,d\dagger} M^0_{u,d} V_R^{u,d}$, with the implication that the charged current interactions are not diagonal in the quark mass basis. The mixing is described by the CKM matrix $V_{\mathrm{CKM}} \equiv V_L^{u\dagger} V_L^d$. In our model (similar to observations found by Georgi in Ref. [11] for the two-flavor case), the mass matrices satisfy $\mathrm{Arg\,Det}(M_u^0 M_d^0) = 0$, but $\mathrm{Det}[M_u^0 M_u^{0\dagger}, M_d^0 M_d^{0\dagger}] \neq 0$, which leaves room to agree with the Jarlskog determinant.

Generally, two Higgs doublet models yield FCNC at the tree level. This can readily be seen in the Higgs basis [59–61], where one rotates ϕ_1 and ϕ_2 to H_1 and H_2 in such a way that only H_1 acquires a VEV. The quark masses are therefore given by H_1, and the interaction between this Higgs and the quarks is diagonal in the quark mass basis. In contrast, the Yukawa structure that couples H_2 to the quarks is not necessarily diagonal in the quark mass basis. This would lead to flavor-changing neutral interactions mediated by H_2. Typically, a final rotation is needed to rotate the scalar fields containing H_1 and H_2 to the scalar mass basis, yielding flavor-changing couplings between all the neutral scalars and the quarks. Here, we assume that the lightest scalar is the Higgs field and that it makes the strongest contribution to a flavor-changing process.

In the quark flavor basis, the Yukawa interaction with H_2 is

$$N_u^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} -s_\beta Y_{11}^u & -s_\beta Y_{12}^u & -s_\beta Y_{13}^u \\ -s_\beta Y_{21}^u & -s_\beta Y_{22}^u & -s_\beta Y_{23}^u \\ Y_{31}^u c_\beta e^{i\alpha} & Y_{32}^u c_\beta e^{i\alpha} & Y_{33}^u c_\beta e^{i\alpha} \end{pmatrix},$$

$$N_d^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} -s_\beta Y_{11}^d & -s_\beta Y_{12}^d & -s_\beta Y_{13}^d \\ -s_\beta Y_{21}^d & -s_\beta Y_{22}^d & -s_\beta Y_{23}^d \\ Y_{31}^d c_\beta e^{-i\alpha} & Y_{32}^d c_\beta e^{-i\alpha} & Y_{33}^d c_\beta e^{-i\alpha} \end{pmatrix}. \quad (10)$$

By applying the same biunitary transformation that leads to the diagonalization of the mass matrices, one can then obtain the FCNC. In Ref. [62], it was shown that matrices with the structure of Eq. (10) lead to controlled flavor-changing contributions. Their analysis was based on the results of Ref. [63], which computed the bounds on the quark's effective flavor-changing couplings to a scalar of mass 125 GeV from low-energy flavor-changing constraints, like meson mixing. We use their notation and write the interaction in the mass basis as

$$\mathcal{L}_{\text{eff}} = \sum_{i,j=d,s,b i\neq j} C_{ij}^d \bar{d}_L^i d_R^j h + \sum_{i,j=u,c,t i\neq j} C_{ij}^u \bar{u}_L^i u_R^j h, \quad (11)$$

where $C_{ij}^{d,u} = V_L^{d,u\dagger} N_{d,u}^0 V_L^{d,u}$ for $i, j = d, s, b$ and $i, j = u, c, t$, respectively.

To determine whether the model is viable, we constructed a χ^2, which includes the quark masses, the CKM elements [64] shown in Table I, and the FCNC constraints from Ref. [63] shown in Table II. Therefore, the total number of constraints is 35: six quark masses, three mixing angles, the Jarlskog determinant, 24 low-energy flavor-changing constraints from Ref. [63], and $\bar{\theta}$, while the number of free parameters is 20 (18 real Yukawas, the CP breaking phase, and t_β). We note that we fix the value of the parameter λ_5 that enters in Eq. (9) to be $\lambda_5 = 1$. The χ^2 is a complicated function of the Yukawas with many local minima. Nevertheless, finding the global minima is unnecessary since we must show that a neighborhood exists where the model is compatible with the experiments.

Just as in the case of the Standard Model, constraints from perturbative unitarity can impose strong bounds on the scalar masses of the theory. In our calculations, we assumed that the mass of the heaviest scalar is around 1 TeV. We take this value since it is the largest that can be achieved without fine tuning the $\mathbb{Z}_2$ soft-breaking term, as shown in Ref. [43]. In addition, we comply with the constraints from the Peskin–Takeuchi (S, T, U) parameters,

TABLE I. Quark masses and CKM elements. For asymmetric errors given in Ref. [64], we take as reference the largest one to compute the pulls [the pull is defined as $(\mathcal{O}_{\text{exp}} - \mathcal{O}_{\text{model}})/\delta O_{\text{exp}}$]. Given the good agreement, taking one or the other gives irrelevant changes in the overall agreement. We do for show the experimental value used as can trivially be deduced from the pull.

Observable	Optimal point	Experimental error	Pull
$\hat{m}_u(M_W)$	1.3 MeV	0.4 MeV	0.08
$\hat{m}_d(M_W)$	2.7 MeV	0.4 MeV	0.04
$\hat{m}_s(M_W)$	54 MeV	10 MeV	0.57
$\hat{m}_c(M_W)$	0.631 GeV	0.009 GeV	0.05
$\hat{m}_b(M_W)$	2.883 GeV	0.008 GeV	0.04
$\hat{m}_t(M_W)$	173.1 GeV	0.6 GeV	0.04
$\lvert V_{ud}\rvert$	0.97435	0.00016	0.01
$\lvert V_{us}\rvert$	0.22501	0.00068	0.00
$\lvert V_{ub}\rvert$	0.003732	0.000090	0.19
$\lvert V_{cd}\rvert$	0.22487	0.00068	−0.02
$\lvert V_{cs}\rvert$	0.97349	0.00016	0.02
$\lvert V_{cb}\rvert$	0.04183	0.0008	−0.02
$\lvert V_{td}\rvert$	0.00858	0.00019	0.05
$\lvert V_{ts}\rvert$	0.041110	0.00077	−0.03
$\lvert V_{tb}\rvert$	0.999118	0.000034	0.02
J	0.000031	0.000001	0.17
$\bar{\theta}$	-8.3×10^{-10}	5×10^{-10}	−1.7

TABLE II. Comparison of FCNC 95% confidence range to the benchmark point prediction. If the ratio is bigger than 1, there is tension of 2σ.

Observable	Bound 95%	Model/bound
$\lvert C_{sd}C_{ds}^*\rvert$	1.1×10^{-10}	8×10^{-8}
$\lvert C_{ds}^2\rvert$	2.2×10^{-10}	8×10^{-7}
$\lvert C_{sd}^2\rvert$	2.2×10^{-10}	2×10^{-9}
$\text{Im}(C_{sd}C_{ds}^*)$	4.1×10^{-13}	3×10^{-14}
$\text{Im}(C_{ds}^2)$	8×10^{-13}	-1×10^{-14}
$\text{Im}(C_{sd}^2)$	8×10^{-13}	-2×10^{-15}
$\lvert C_{cu}C_{uc}^*\rvert$	9×10^{-10}	2×10^{-7}
$\lvert C_{uc}^2\rvert$	1.4×10^{-9}	8×10^{-5}
$\lvert C_{cu}^2\rvert$	1.4×10^{-9}	3×10^{-10}
$\text{Im}(C_{cu}C_{uc}^*)$	1.7×10^{-10}	2×10^{-11}
$\text{Im}(C_{uc}^2)$	2.5×10^{-10}	4×10^{-12}
$\text{Im}(C_{cu}^2)$	2.5×10^{-10}	-6×10^{-14}
$\lvert C_{bd}C_{db}^*\rvert$	9×10^{-9}	9×10^{-4}
$\lvert C_{db}^2\rvert$	1×10^{-8}	0.8
$\lvert C_{bd}^2\rvert$	1×10^{-8}	7×10^{-7}
$\text{Im}(C_{bd}C_{db}^*)$	2.7×10^{-9}	6×10^{-16}
$\text{Im}(C_{db}^2)$	3×10^{-9}	2×10^{-14}
$\text{Im}(C_{bd}^2)$	3×10^{-9}	1×10^{-18}
$\lvert C_{bs}C_{sb}^*\rvert$	2×10^{-7}	0.02
$\lvert C_{sb}^2\rvert$	2.2×10^{-7}	0.8
$\lvert C_{bs}^2\rvert$	2.2×10^{-7}	2×10^{-4}
$\text{Im}(C_{bs}C_{sb}^*)$	2×10^{-7}	-2×10^{-17}
$\text{Im}(C_{sb}^2)$	2.2×10^{-7}	1×10^{-15}
$\text{Im}(C_{bs}^2)$	2.2×10^{-7}	1×10^{-18}

TABLE III. Benchmark point used to obtain the results of Tables I and II.

Parameter	Value
$\tan\beta$	4.14177
Y^u_{11}	$+6.99737 \times 10^{-2}$
Y^u_{12}	-7.33074×10^{-2}
Y^u_{13}	-7.11714×10^{-2}
Y^u_{21}	-6.87086×10^{-2}
Y^u_{22}	$+7.22767 \times 10^{-2}$
Y^u_{23}	$+6.96334 \times 10^{-2}$
Y^u_{31}	$+5.63088 \times 10^{-1}$
Y^u_{32}	-6.72942×10^{-1}
Y^u_{33}	-5.25847×10^{-1}
Y^d_{11}	$+8.51142 \times 10^{-6}$
Y^d_{12}	-6.09872×10^{-4}
Y^d_{13}	$+3.37296 \times 10^{-4}$
Y^d_{21}	$+1.81040 \times 10^{-4}$
Y^d_{22}	$+1.05304 \times 10^{-3}$
Y^d_{23}	-4.03567×10^{-4}
Y^d_{31}	$+1.58555 \times 10^{-2}$
Y^d_{32}	$+6.25660 \times 10^{-3}$
Y^d_{33}	-2.00748×10^{-4}
α	-2.37994

perturbativity, and the boundedness from below of the potential, using a value of $t_\beta < 5$, as shown in Ref. [44].

It is indeed possible to find values of the Yukawas that agree with the experimental constraints. The benchmark point is given in Table III, while the tensions with the experimental constraints can be seen in Tables I and II. One must be aware that there is fine-tuning since relatively small variations ∼1% yield a drastic increase of the tension. For example, changing the first Y^u_{11} by this amount changes the χ^2 by 4 orders of magnitude. This is a typical situation in two Higgs doublet models.

Other constraints, like $\Gamma(t \to hq)$ measured at ATLAS [65–67] and CMS [68,69] also impose constraints on the flavor violating couplings, although looser than the ones from indirect low-energy flavor-changing process [43].

VI. CONCLUSIONS

In this paper, we presented a model that addresses the strong *CP* problem through spontaneous symmetry breaking, drawing inspiration from the model proposed by Georgi [11]. By treating *CP* as a symmetry of the Lagrangian, the model naturally sets the topological term θ to zero. After symmetry breaking, corrections to this term make it nonzero, but its value remains small. This minimal model requires one additional Higgs doublet with a $\mathbb{Z}_2$ symmetry which is softly broken. The $\mathbb{Z}_2$ symmetry plays a crucial role in ensuring that the texture of the fermion matrices leads to controlled flavor-changing neutral currents. Despite fine-tuning required in the Yukawa sector, the model remains viable for solving the *CP* problem, successfully satisfying current experimental constraints while maintaining theoretical consistency. A more detailed analysis of constraints from colliders and a complete global fit, including the scalar sector, is left for future work.

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