METRON
https://doi.org/10.1007/s40300-023-00251-6

Dynamic modelling of price expectations and judgments

Rosaria Simone[1] · **Marcella Corduas**[1] · **Domenico Piccolo**[1]

Received: 27 October 2022 / Accepted: 27 June 2023

Abstract

Official data about consumers' qualitative expectation and perception of inflation are derived from repeated surveys conducted by national statistical institutes. In EU, these data are published in aggregate form, and cannot be described by means of classical methods based on cumulative models for ordinal data. This article illustrates an integrated approach that locates CUB mixture models for ratings in a time series perspective in order to investigate the joint evolution of inflation judgments and expectations in Italy. In order to measure the common sentiment of interviewees through the feeling component of the model, net of possible uncertainty and nuisance effects, its estimation is pursued through profile likelihood methods given the empirical frequency distributions of consumers' opinions observed over time. Then, the relationship between the time series of the estimated feeling parameters is modelled using a dynamic regression model and the results are compared in three periods marked by different economic conditions. Results indicate that each series has a substantial inertial component, and thus it is characterized by a slow variation over time, and that both judgments about past price levels and previous expectations affect current expectations about the future in fairly different ways for the three time periods.

Keywords Time series models · CUB models · Profile likelihood · Official surveys · Perceived Inflation

Mathematics Subject Classification 62-F99 · 62-P20

R. Simone and M. Corduas have contributed equally to this work.

✉ Rosaria Simone
rosaria.simone@unina.it

Marcella Corduas
marcella.corduas@unina.it

Domenico Piccolo
domenico.piccolo@unina.it

[1] Department of Political Sciences, University of Naples Federico II, Via Leopoldo Rodinó, Naples I-80128, Italy

Published online: 26 July 2023

1 Introduction

Consumer opinion surveys are regularly promoted by the European Commission in order to collect qualitative and quantitative data concerning the past and future development of the main economic indicators [4]. In particular, the investigation of consumers' inflation perceptions and expectations represents an important issue for the analysis of consumers' behaviour in terms of purchase or savings decisions [28, 43]. Experimental studies have shown that inflation judgments and expectations are formed using various sources of information [38]. Judgments are affected by the personal experience of price changes, the shopping frequency, the product accessibility, the attitudes towards inflation [17, 20]. Expectations, instead, can be influenced by the model of economy, the perceived level of inflation, the knowledge of official data and expert forecasts [3, 8, 14, 38]. Moreover, consumers' attitude about price changes varies according to the socio-demographic profile of respondents (such as personal income, age, gender, level of economic literacy) and the responsiveness to positive/negative news [42].

The present study examines inflation opinions from monthly surveys conducted by the Italian National Statistical Institute [23] from January 1994 to January 2019 within the harmonized European programme of business and consumer surveys. In particular, the analysis refers to the qualitative assessments of inflation that seem more informative on consumers' perceptions and expectations than the quantitative one. These are in fact available only from 2004 and are affected by several shortcomings as discussed by Biau et al. [6]; Arioli et al. [2]; Meyler and Reiche [26, 27].

Qualitative data on inflation beliefs are published in aggregate form as time series of frequency distributions of rating variables and can be downloaded from: https://ec.europa.eu/info/business-economy-euro/indicators-statistics/economic-databases/business-and-consumer-surveys. Then, the subjective factors driving the opinions formation cannot be explicitly taken into account because individual responses and features are not available: therefore, the classical approach based on cumulative models for ordinal data is not applicable [1, 44]. This article aims at studying the evolution of qualitative judgments and expectations about price levels during three periods marked by special economic conditions. This is achieved by means of a dynamic approach to CUB mixture models [16, 34] that exploits the analytic advantages of ordinal data and time series models, as firstly discussed in Proietti [36]; Piccolo and Simone [35]; Simone et al. [41]. The approach has been further extended and applied to explore gender differences in inflation perception [12, 13]. At a given instant, the chosen modelling framework allows to parametrize the rating choices by means of the underlying feeling, the uncertainty and to include, possibly, the presence of inflated frequencies. In this way, observed time series are considered as realizations of a data generating process by which respondents react to simple questions by expressing their perception on an ordinal rating scale. Thus, data are the registration and aggregation of a complex psychological process which should be analyzed with proper methodological methods and models able to detect the main components of such a procedure.

The article proposes a comprehensive procedure to assess the feeling component net of uncertainty. The problem at hand lends itself to introduce the profile likelihood estimation [31] of the feeling measure across time to describe how consumers' latent attitude underlying their opinion has changed over time, net of nuisance effects induced by heterogeneity in the distribution and possible inflated frequencies.

Further, the analysis examines the relationship between the estimated feeling associated to the distribution of inflation judgments and expectations using a dynamic linear models

[19, 30] with the purpose of detecting the extent by which this relationship modifies across time. This bond has been previously reported by various authors using consumers' quantitative assessment of inflation from experimental data or ad hoc survey data (see the earliest contribution by Jonung [24]; Armantier et al. [3]; Axelrod et al. [5]). Inflation perceptions have been suggested as one of the elements that may justify the deviations of subjective expectations of individuals from the assumptions of full-information rational expectations [10].

The paper is organized as follows: Sect. 2 introduces the data motivating the study and Sect. 3 briefly presents the modelling approach to ordinal data. Section 4 discusses univariate time series models for the proposed feeling series, whereas Sect. 5 investigates the dynamic relationship between the feeling characterizing judgments and expectation of inflation in Italy. Some concluding remarks end the paper. Finally, Appendix A illustrates the procedure for the profile likelihood estimation of the feeling parameter with the support of some simulation experiments, whereas B supplements the discussion with some comparative comments with the well-known Balance Statistic, an indicator whose usage is popular within official statistical analysis of price expectations and perception [26].

2 Motivating data

The dataset is part of the consumer confidence survey carried out monthly by ISTAT. Observations range from January 1994 to January 2019. Respondents have been asked the following questions:

- **Judgments**: How do you think the price level changed over the previous 12 months?
- **Expectations**: How do you think the price level will change over the next 12 months?

Possible responses are selected over a 5-point scale with categories ordered according to beliefs of increasing inflation, and are labelled as follows: Those categories do not have a

'*fall*'	'*stay about the same*'	'*rise slightly*'	'*rise moderately*'	'*rise a lot*'

central reference point, but three out of the five categories suggest a rising level of prices. Besides, individual responses are not available and only the ordinal frequency distribution is published monthly.

Hereafter, three different time windows have been considered on the basis of 107 macro-events which had high impact on the economic situation: 1) from January 1994 up to the end of 2001; 2) between January 2002 and the end of 2007; 3) from January 2008 up to January 2019. The first period was characterized by the implementation of economic policies in order to reduce the inflation rate and satisfy the convergence goals established by the Maastricht Treaty for the creation of the Euro area. The second period started with the introduction of the Euro as official currency. At that time, European consumers perceived a much larger rise in prices than it was recorded in official statistics [15, 18]. In Italy, this gap reached an unprecedented size and persisted for a long period, and a similar divergent pattern characterized also expected and perceived inflation. The third period includes some years of financial turbulence that severely affected the economic growth in Italy. In January 2008 the spread between the Italian and German sovereign bond started to increase achieving a very high volatility between 2009

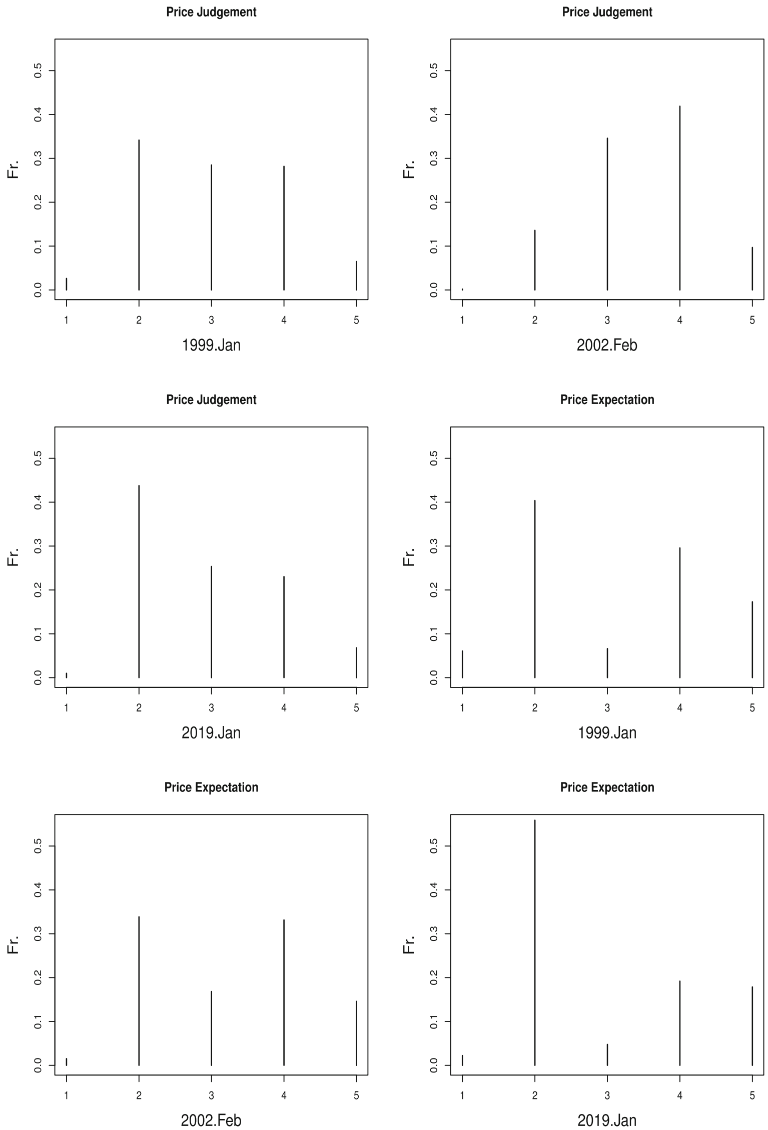

Fig. 1 Observed frequency distributions of judgments (top) and expectation(down) of price levels at selected time periods

and 2013 due to the increased risk about the sustainability of the Italian sovereign debt. This negative outlook was amplified by the worldwide financial crises originated in the United States in 2008 and by the national debt crises 123 that affected several European countries and requested the intervention of the European Central Bank. Of course, all these events were widely covered by media and greatly influenced the formation of consumers' opinions about inflation. The time span 2008–2019 was also characterized by a deflationary period between 2014 and 2017 when very low values of inflation, sometimes closely oscillating about zero, were observed.

For selected time points, one for each period, Fig. 1 displays the observed frequency distribution of judgments and expectations about prices level. The shape of the distributions shows the greater heterogeneity of the future prices expectations as compared to the judgments over the past year. Sometimes, the distribution of price expectations is bimodal. For this reason, checking the evolution of responses in terms of the average rate may yield an important lack of information [37], although the use of central tendency measures as median or mode are possible even in case of ordinal scales. As it will be shown in Sect. 3, some indexes currently used to summarize price expectations and judgments are indeed functions of the average response. Furthermore, we advocate a parameterization of the latent sentiment that is possibly covariate-free to make comparisons possible over time and also to compare the latent sentiment of perceptions and expectations: indeed, no individual data is available, as the series consist of the aggregated response frequency distribution for each time point. For the sake of parsimony, we resort to a class of statistical model that uses only one parameter to shape the underlying feeling, as opposed to the set of four logits required under classical ordinal models. In this regard, it is worth of noticing that empirical evidence often is against standard assumptions of classical models [35].

3 A class of mixture models with uncertainty

The special nature of the responses suggests to investigate the ordinal structure of the categories to summarize essential features of the observed distributions derived from interviewees' opinions. For this purpose, a modelling framework that describes a simplified process ruling the respondent's rating choice is introduced. This relies on CUB models [16, 32] and their many variants introduced for finer specifications [33–35]. CUB is an acronym that stands for *C*ombination of *U*niform and *B*inomial to indicate the two-component mixture distributions of uncertainty and feeling.

CUB models provide an effective and parsimonious parameterization of the distributions of rating data characterized by different shapes without requiring necessarily the specification of covariates. In fact, the class of CUB models interprets the selection of a category out of m ordered options as a procedure mixing discrete distributions ruled by the feeling, the overall uncertainty *generating heterogeneity*, and by a possible over-dispersion or inflated frequencies.

Assuming the Binomial distribution as a suitable model for feeling, the basic CUB model for a rating response R is defined by:

$$Pr\,(R = r \mid \boldsymbol{\theta}) = \pi\, g_r(\xi) \;+\; (1-\pi)\,\frac{1}{m}, \qquad r = 1, \ldots, m, \tag{1}$$

with $\boldsymbol{\theta} = (\pi, \xi)'$ where $\pi \in (0, 1]$, $\xi \in [0, 1]$, $g_r(\xi) = \binom{m-1}{r-1}\xi^{m-r}(1-\xi)^{r-1}$ being the shifted Binomial distribution (for short, $R \sim \text{CUB}(\pi, \xi)$), and $m > 3$ (identifiability constraint). Both $1-\xi$ (related to the feeling) and $1-\pi$ (related to the heterogeneity of the distribution) range in the unit interval. Thus, the position of the point $(1-\pi,\ 1-\xi)$ over the unit square summarizes the whole distribution, and provides an effective representation of the intrinsic features of the responses that can be used for comparative analysis. The greater ξ is, the stronger is the skewness to the right of the distribution. Then, for the data under consideration, $1-\xi$ is a direct indicator of beliefs about the increasing direction of price changes as perceived in the recent past or expected in the next future. Notice that when $\pi = 1$ the model simply describes a shifted Binomial distribution whereas $\pi = 0$ is not admissible since model (1) becomes unidentifiable.

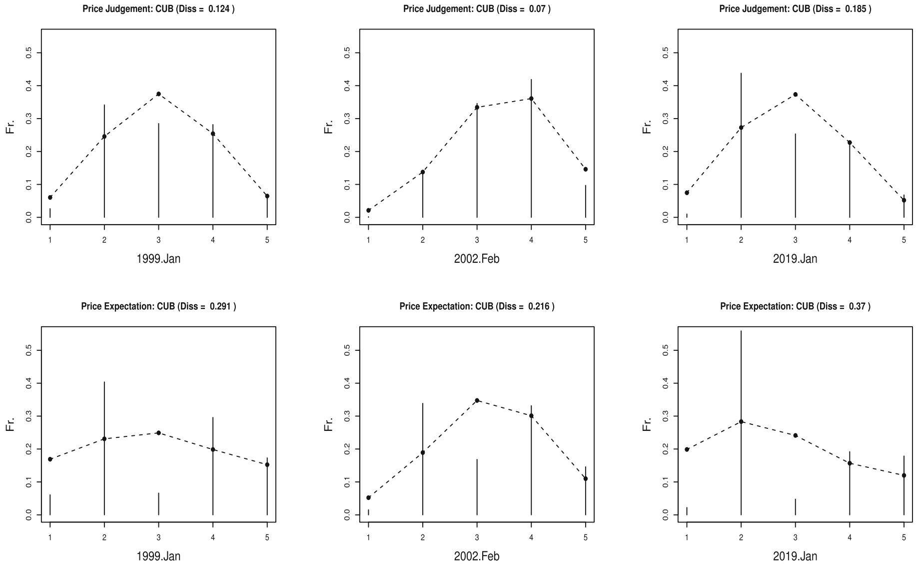

Fig. 2 CUB models fitted on rating distributions of judgments and expectation about price levels

At each time $t = 1, \ldots, T$, a CUB model can be estimated in order to describe the response distribution of the price judgments $R_t^{(j)}$ and the price expectations $R_t^{(e)}$ [36, 41]. For illustrative purposes, Fig. 2 displays the estimated CUB probability masses superimposed to the observed frequency distributions for the time points chosen in Fig. 1. Overall, the response patterns of judgments on past price trends are rather regular, with some exceptions in the last period. Conversely, ratings on price expectations show that the second category (‘*stay about the same*’) is selected by respondents to a larger extent than expected according to the CUB model. For this reason, the fitting is poor as indicated by the reported values of the normalized dissimilarity index[1]. Plots suggest that the second category is often used by individuals as a safe and quick refuge against an aware answer.

Thus, to take this *shelter* effect into account, the basic model is modified and a degenerate distribution is introduced to account for the category attracting an excess in frequency [11, 21]. Explicitly, a CUB model with a shelter at the category c for the response R (for short, $R \sim CUB_{\text{she}}(\pi^{(1)}, \pi^{(2)}, \xi)$) is specified by:

$$Pr\,(R = r \mid \boldsymbol{\theta}) = \pi^{(1)} g_r(\xi) + \pi^{(2)} \frac{1}{m} + \left(1 - \pi^{(1)} - \pi^{(2)}\right) D_r^{(c)}, \quad r = 1, 2, \ldots, m. \tag{2}$$

Here, $\boldsymbol{\theta} = \left(\pi^{(1)}, \pi^{(2)}, \xi\right)'$, and $D_r^{(c)}$ is the degenerate distribution with mass concentrated at the category $c \in \{1, 2, \ldots, m\}$. The parameter $\delta = 1 - \pi^{(1)} - \pi^{(2)}$ measures the overall shelter effect, whereas $\pi^{(2)}$ is the uncertainty measure, meant as importance of the residual heterogeneity in the distribution.

[1] The normalized dissimilarity index between observed frequencies, f_r, and fitted probabilities, $p_r(\widehat{\boldsymbol{\theta}}) = Pr\left(R = r \mid \widehat{\boldsymbol{\theta}}\right)$, is defined by: $Diss = 0.5 \sum_{r=1}^{m} \mid f_r - p_r(\hat{\boldsymbol{\theta}}) \mid$, as discussed by [34, pp.408–409], among others.

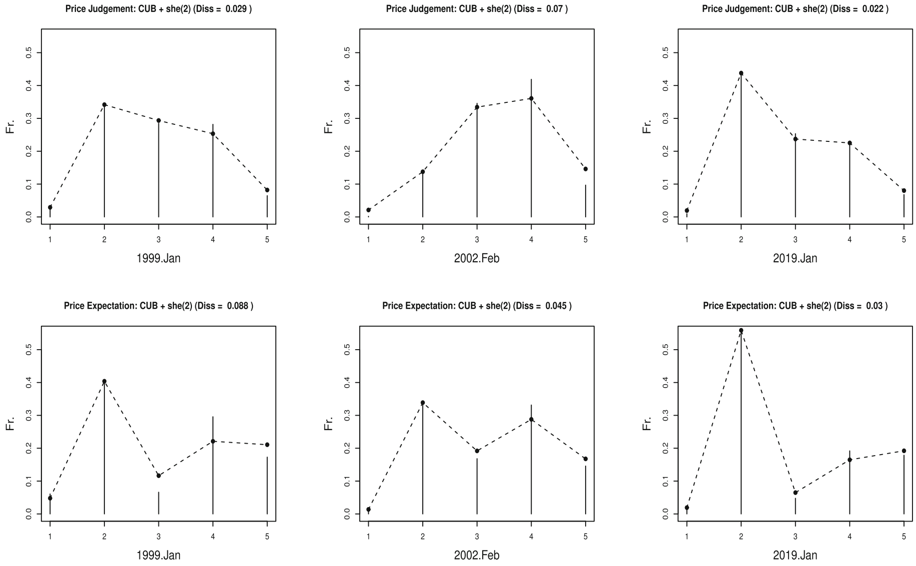

Fig. 3 CUB model fitting on ratings for judgments and expectation of price levels after adjusting for shelter

Going back to the previous illustrative example, Fig. 3 shows the CUB models with the shelter component at $c = 2$ fitted to the selected distributions. A part from the distributions of judgements in the first two instants, the goodness of fit substantially improves as confirmed by the reduction of the dissimilarity index.

The circumstance that a shelter specification at $c = 2$ improves goodness of fit applies to several time points, for each period, either to the baseline (shifted) Binomial or to the general CUB. Notice that the finer model specification achieved while accounting for shelter effect does not alter the meaning of the feeling component in the chosen series of models: (shifted) Binomial without or with shelter, CUB without or with shelter. In general, parameters of CUB or CUB with shelter are estimated by maximum likelihood method via an EM algorithm for mixtures, as implemented in `R` [22, 39, 40] and STATA packages [9].

Since policy analysis is concerned with people sentiment (feeling), an accurate estimation of the feeling component should be pursued net of nuisance effects surrounding the rating choice. In the present study, regardless the specified model, we focus solely on feeling and thus resort to profile likelihood estimation of ξ to assess this measure net of possible uncertainty contamination and without the need of pursuing model selection to find the best fitting model among alternative ones. This methodological approach represents a novelty within the literature on mixture models with uncertainty: Appendix 1 provides some details and simulation results that support this choice.

In the next section, the dynamic of respondents' beliefs about current and future inflation as expressed by the monthly frequency distributions of opinions is investigated using the time series obtained by collecting the corresponding estimated feeling parameters over time. In this way it is possible to study the evolution of the latent response signal changes over time. In order to investigate how the relative importance of feeling component within mixture specification varies over time, likelihood of the binomial mixing weight should be investigated instead.

4 Time series modelling of price level evaluations

Consider the rating responses about price judgement and expectation, $R_t^{(j)}$ and $R_t^{(e)}$ for $t = 1, 2, \ldots, T$. At each instant, Binomial, Binomial with shelter (at $c = 2$), CUB and CUB with shelter (at $c = 2$) models are candidate to fit the ordinal distributions of $R_t^{(j)}$ and $R_t^{(e)}$. Instead of performing a point-wise model-selection, a profile likelihood estimation of the underlying feeling parameters $\xi_t^{(j)}$, $\xi_t^{(e)}$ is proposed as a straightforward procedure to assess latent sentiment, net of possible nuisance effects[2]. Since the feeling parameter estimates are constrained within the unit range, to facilitate the interpretation of results and the implementation of adequate models, the logit transformation is considered:

$$y_t^{(e)} = \log\left(\frac{1-\xi_t^{(e)}}{\xi_t^{(e)}}\right); \qquad y_t^{(j)} = \log\left(\frac{1-\xi_t^{(j)}}{\xi_t^{(j)}}\right), \quad t = 1, \ldots, T, \tag{3}$$

so that, when low ratings (corresponding to a decreasing or stable price level) are selected with higher probability than high ratings, the logit series of feeling tend to assume low values. Vice versa, when high ratings (corresponding to increasing prices) are selected with higher probability than low ratings, the logit series of feeling tend to assume high values. Thus, the time series defined in (3) can be interpreted as point-wise belief that prices have increased rather than decreased (regarding judgements) or that prices will increase rather than decrease (regarding expectations), net of uncertainty effects. In the rest of the article, for brevity, these time series will be simply referred to as 'feeling series' omitting the explicit reference to the logit transform.

Figure 4 shows the time series $y_t^{(e)}$ and $y_t^{(j)}$ in the three time windows defined in Sect. 2.

The inspection of the plots highlights how the pattern of the series $y_t^{(j)}$, related to judgments, and $y_t^{(e)}$, corresponding to expectations, reflects the impact of economic events. In particular, the divergent behaviour of judgments and expectations occurring after the introduction of Euro (with reference to the second period) is evident and quantifiable. The feeling series of judgments is much higher than expectations. The upward movement of $y_t^{(j)}$, starting in 2002, is due to the increasing probabilities attached to the categories describing high inflation. As mentioned previously, in those years consumers judgments remarkably overstated the price changes [15]. In the third period, from 2008 to January 2019, it is possible to recognize various events that characterize those years. The 2008 crisis corresponds to a rapid decline in the feeling series of judgments indicating that consumers correctly perceived the declining price level due to the contraction of the economy. In the deflationary period, between 2014 and 2017, the series $y_t^{(j)}$ assumes low values and shows a very stable pattern. In those years, the probability of selecting ratings denoting decreasing or stable levels of prices is high, whereas feeling about expectations increases.

The dynamics of the feeling series is investigated by fitting a linear model to data. The kernel histograms of the series $y_t^{(e)}$ and $y_t^{(j)}$ are sufficiently smooth and symmetric to assume that the generating process is Gaussian. The plots of the series, as well as the estimated autocorrelation functions (here, omitted for brevity), show the presence of strong inertial components that can be taken into account using either a difference operator or an ARMA operator with balanced orders.

The selection has been performed by comparing the explained variance of the competing models. Recalling the basic notation of the Box-Jenkins methodology [7], let $\varphi(B)$ be a

[2] The standard *hat* notation for estimates is omitted for notation simplicity.

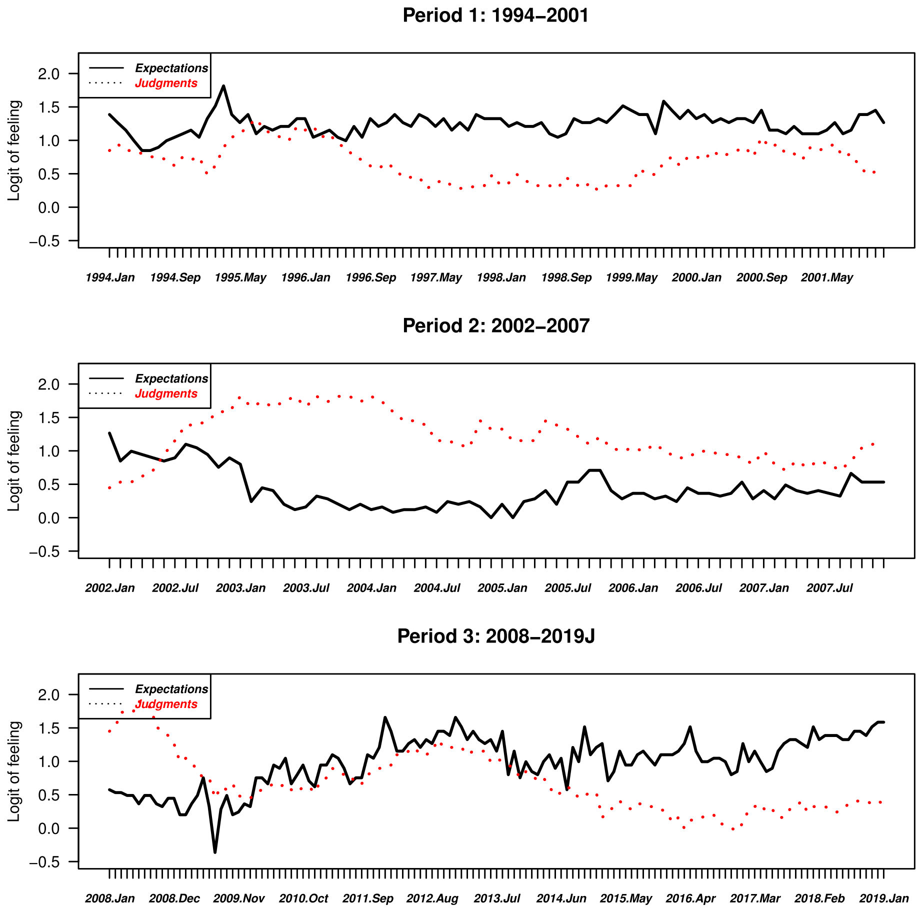

Fig. 4 Feeling series defined in (3) for expectations and judgments about price levels in Italy, in different periods

linear operator in B such that $B^k\, X_t = X_{t-k},\ k = 0, \pm 1, \pm 2, \ldots$, and assume that the time series x_t is the realization of the process $X_t = \varphi(B)\eta_t$. The index: $\nu = 1 - \dfrac{Var(\eta_t)}{Var(X_t)}$ is a measure of the explanatory ability of the $\varphi(B)$ operator to decrease the variance of X_t down to the variance of η_t. Assuming that the estimated parameters of $\varphi(B)$ are all significant, the ν measure allows for a direct comparison of models in terms of explanatory power of that operator (Table 1).

The inspection of the estimated autocorrelation functions of the feeling series suggests a model including a generalized ARMA operator (with a possible unit root) as the following ones:

- First difference model: ARIMA(0,1,0) $\Longrightarrow\ \varphi_1(B) = \nabla^{-1} = \dfrac{1}{1-B}$.
- Stationary ARMA(1,1) model: ARIMA(1,0,1) $\Longrightarrow\ \varphi_2(B) = \dfrac{1-\theta\, B}{1-\phi\, B}$.
- Non-stationary model: ARIMA(0,1,1) $\Longrightarrow\ \ \varphi_3(B) = \dfrac{1-\theta\, B}{1-\ B}$.

Table 1 Explanatory ability index ν for the selected estimated models (for each period, the highest value is highlighted in bold)

Models	Expectations			Judgments		
	Per. 1	Per. 2	Per. 3	Per. 1	Per. 2	Per. 3
ARIMA(0,1,0)	0.149	0.713	0.661	0.837	0.884	0.950
ARIMA(1,0,1)	**0.353**	**0.756**	**0.734**	**0.851**	**0.888**	**0.950**
ARIMA(0,1,1)	0.263	0.752	0.731	0.846	0.886	0.950

Table 2 Estimated ARMA(1,1) models for feeling series (standard errors are reported in parentheses)

Estimates	Expectations			Judgments		
	Per. 1	Per. 2	Per. 3	Per. 1	Per. 2	Per. 3
$\widehat{\phi}_0$	1.246	0.590	1.004	0.688	1.053	0.775
	(*0.035*)	(*0.237*)	(*0.220*)	(*0.130*)	(*0.261*)	(*0.323*)
$\widehat{\phi}$	0.708	0.961	0.969	0.941	0.956	0.981
	(*0.107*)	(*0.035*)	(*0.025*)	(*0.033*)	(*0.034*)	(*0.016*)
$\widehat{\theta}$	0.201	0.350	0.517	0.179	0.036	0.038
	(*0.141*)	(*0.111*)	(*0.092*)	(*0.095*)	(*0.105*)	(*0.082*)

Note that ARIMA(0,1,0) model specifies a *random walk* and the ARIMA(1,0,1) model reproduces the traditional *Exponential Smoothing* model when $\phi > \theta$. Thus, all the considered models are nested in a generalized ARMA(1,1,1) model.

Table 2 reports the maximum likelihood estimates of the parameters with asymptotic standard errors (in parentheses). In most cases the roots of the AR operator are close to unit circle. Apart from the moving average estimates $\hat{\theta}$ of the models for the judgment feeling in the last two periods, all the estimates are significantly different from zero and are coherent in terms of sign, dimension and relative ratios. In addition, the autocorrelations of residuals (Fig. 5) confirms that no further structure can be detected. The estimated relative reduction in variance, as measured by ν, is reported in Table 1. The best model for each period is denoted with bold font.

With exception of the feeling series of judgments in the third period (for which models seem equivalent), a stationary ARMA(1,1) model with a constant term seems to be preferable. For reader convenience, it is useful to recall that, for a time series X_t with expectation μ, this model is specified by:

$$X_t = \phi_0 + \phi\, X_{t-1} + a_t - \theta\, a_{t-1}, \qquad \text{with } \phi_0 = \mu\,(1-\phi)\,,$$

where a constant $\phi_0 = \mu\,(1-\phi)$ is necessary when $\phi \neq 1$.

The dynamic structure of the estimated models is depicted by the corresponding (normalized) parametric spectra (Fig. 6). Much of the spectral power is concentrated at low frequencies due to the strong inertia that characterizes the data and the presence of components near to non stationarity. Some differences can be detected in the first period (1994–2001), where the spectrum of judgments exhibits a greater weight in a wider frequency band about zero whereas the spectrum of expectations results more peaked than the others. This reflects the fact that the shape of the distributions of consumers' opinions (as measured by the feeling series) changes very slowly over time.

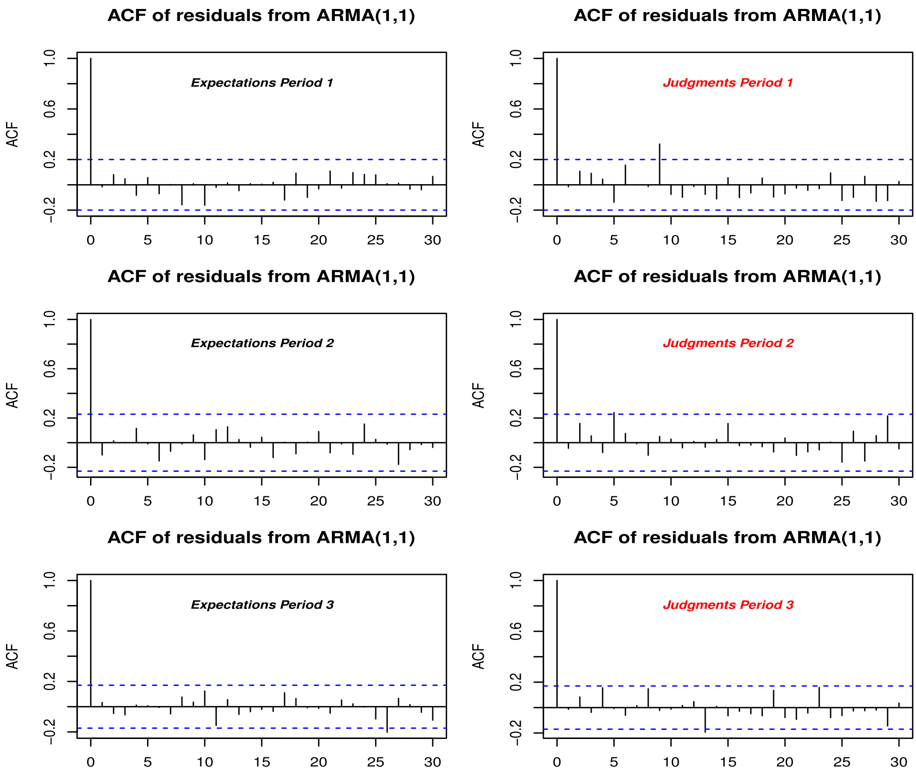

Fig. 5 Residual autocorrelation functions from ARMA(1,1) models fitted to feeling series

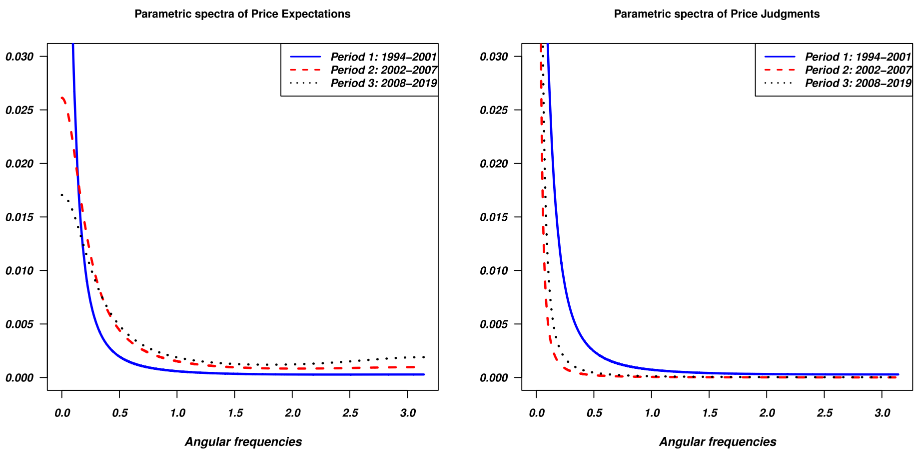

Fig. 6 Parametric spectra of estimated AR models, in the three periods

5 The relationship between judgments and expectations

As mentioned previously, various experimental studies have related the expectations about future inflation to a number of variables including the recent judgments about past price

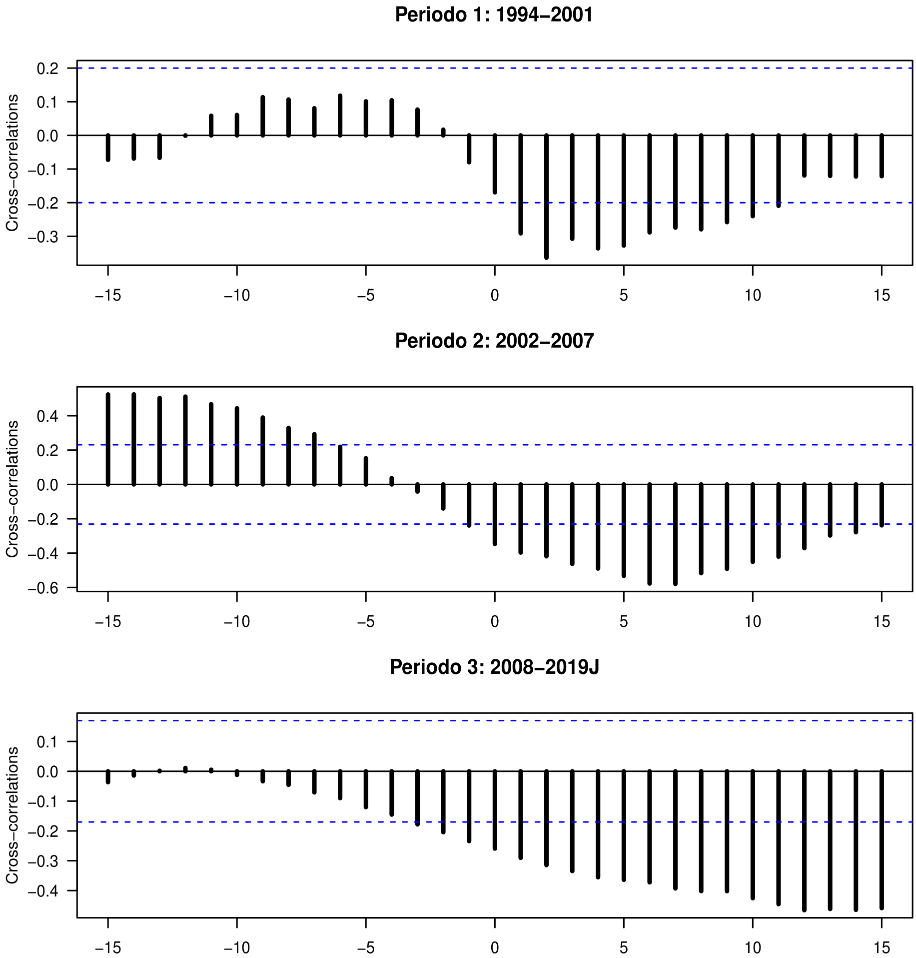

Fig. 7 Estimated cross-correlations between feeling series of judgments and expectations, in each period

changes [38]. Then, it is interesting to study whether this relationship holds at aggregate level when the distributions of beliefs concerning past and future inflation are considered, and more specifically, when the evolution of those distributions over time are described by the feeling series derived in the previous section.

Figure 7 illustrates the cross-correlation functions of the feeling series of expectations and judgments, that is $Corr\left(y^{(e)}_{t+k},\ y^{(j)}_t\right)$, for lags $k = 0, \pm 1, \pm 2, \ldots$. They show a clear relationship between lagged values of the feeling series of expectation and the corresponding judgment series. More precisely, a contemporaneous relationship is significant for the last two periods whereas a sharp correlation is evident between expectations and past judgments. This relationship has been confirmed to be unidirectional by VAR models preliminarily fitted to data and selected using the BIC criterion.

Then, we hypothesize the presence of a relationship acting from judgments to expectations: *Judgments* $\Longrightarrow$ *Expectations*. Thus a dynamic regression (with no-feedback) has been specified [30]:

Table 3 Estimation results (standard errors in parenthesis)

Period	$\widehat{\alpha}_0$	$\widehat{\alpha}_1$	$\widehat{\alpha}_2$	$\widehat{\beta}_0$	$\widehat{\beta}_1$	$\widehat{\beta}_2$	$\widehat{\beta}_3$	Adj-R^2
Period 1	0.448	0.445	0.238			−0.435	0.354	0.424
1994–2001	*(0.137)*	*(0.100)*	*(0.101)*			*(0.118)*	*(0.122)*	
Period 2	0.158	0.806		0.282	−0.622	0.269		0.760
2002–2007	*(0.072)*	*(0.071)*		*(0.135)*	*(0.168)*	*(0.142)*		
Period 3	0.159	0.502	0.378	0.301	−0.342			0.745
2008–2019J	*(0.061)*	*(0.082)*	*(0.081)*	*(0.171)*	*(0.168)*			

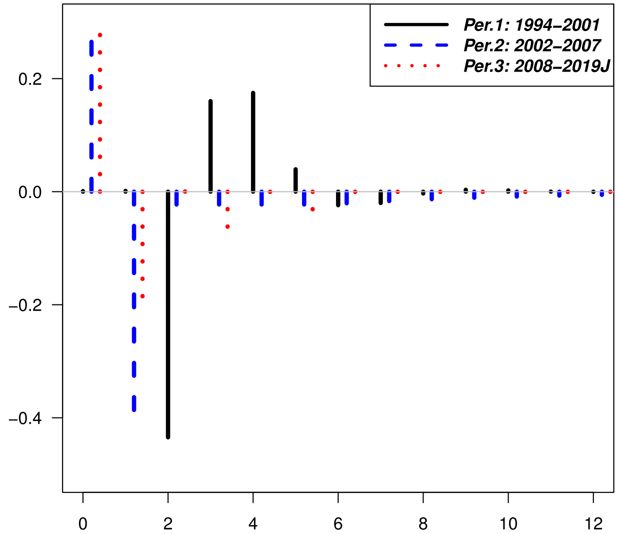

Fig. 8 Impulse function implied by the operator $\beta(B)\,\alpha^{-1}(B)$ for the estimated models in the three periods

$$y_t^{(e)} = \alpha_0 + \alpha_1 y_{t-1}^{(e)} + \cdots + \alpha_k y_{t-k}^{(e)} + \beta_0 y_t^{(j)} + \beta_1 y_{t-1}^{(j)} + \cdots + \beta_h y_{t-h}^{(j)} + \epsilon_t,$$

where the feeling series of expectations depends on its own past values and on the past and contemporaneous values of judgments.

Table 3 reports the estimation results. Although the relationship between the feeling series of judgments and expectations is confirmed, the models differ in the three periods. In the years 1994–2001, the contemporaneous relationship between the two series is not significant and the only link is with values at lag 2 and 3. The model fitting is lower than in the other two periods.

Conditionally to the past, the impact of judgments on expectations is illustrated by the impulse functions which are defined, for each period, as the coefficients in the expansion of the $\beta(B)\,\alpha^{-1}(B)$ operator, as shown in Fig. 8. Generally, the impact of judgments on expectations wears off within a few months but the reaction is different in the first period with respect to the last two. In the first period a modification is evident in months 2 to 4 with alternating signs (and this cause a fluctuating pattern in the series of feeling). In the last two periods, instead, the main effect is contemporaneous and positive. As a consequence, when at a certain month there is a shift from low to high ratings in the probability distribution of judgments, this has a positive effect on expectations whose probability distribution shows

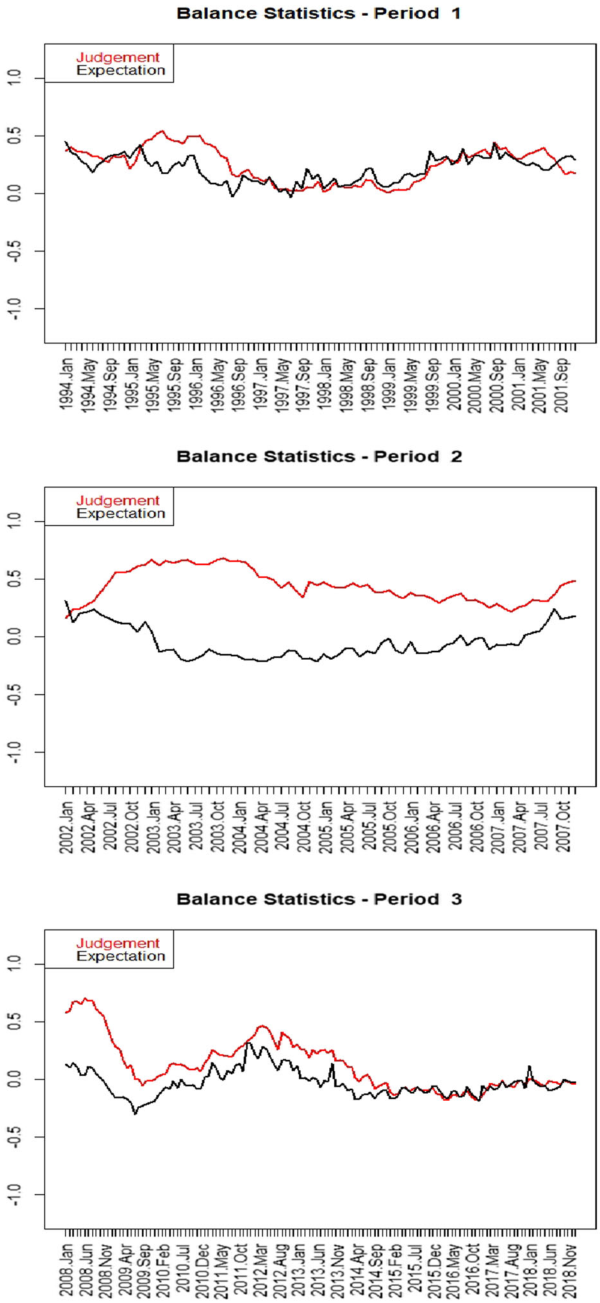

Fig. 9 Time series of Balance Statistic for qualitative price judgments and expectations over the three periods

Table 4 Different scenarios for the simulation plan

Scenario	Models	Parameters	m	n
1	Binomial	$\xi = 0.6$	5	2000
2	CUB	$\pi = 0.4;\ \xi = 0.6$	5	2000
3	CUB + shelter($c = 2$)	$\pi^{(1)} = 0.35;\ \pi^{(2)} = 0.50;\ \xi = 0.6$	5	2000
4	CUB + shelter($c = 5$)	$\pi^{(1)} = 0.6;\ \pi^{(2)} = 0.25;\ \xi = 0.3$	10	1000
5	Binomial	$\xi = 0.2$	7	1000
6	CUB	$\pi = 0.3;\ \xi = 0.1$	5	500
7	Binomial + shelter($c = 1$)	$\xi = 0.75; \delta = 0.2$	6	800
8	CUB	$\pi = 0.2; \xi = 0.5$	9	1500

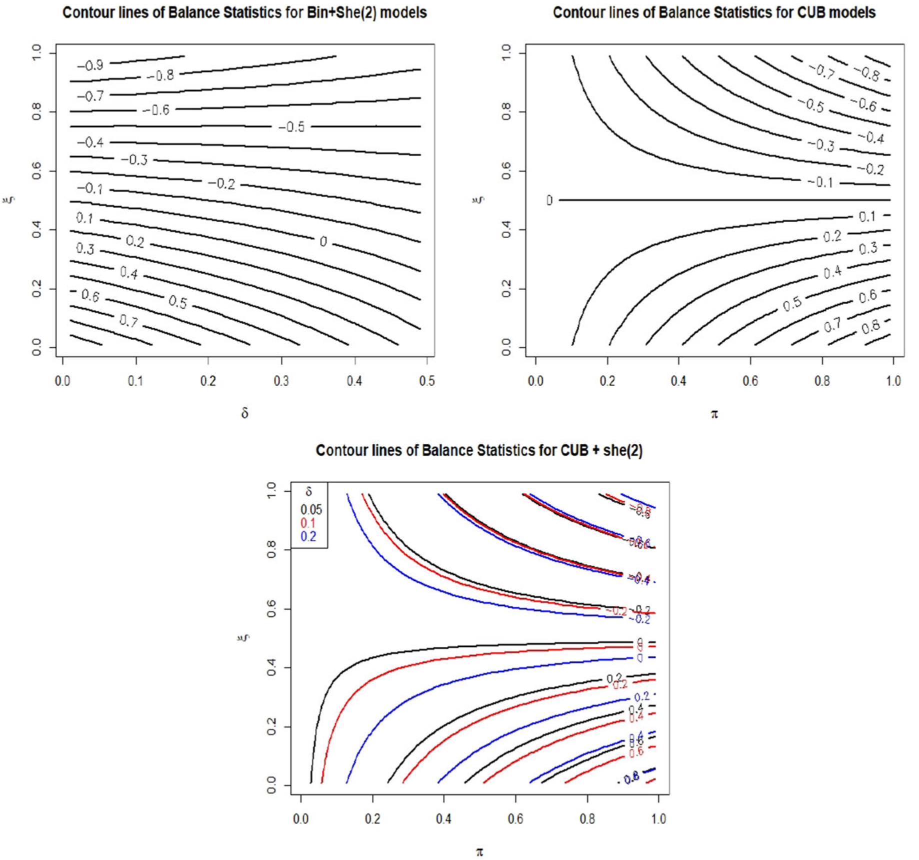

Fig. 10 Contour lines of (theoretical) Balance Statistic for modified Binomial models (with inflation, heterogeneity, and both, respectively)

an analogous change. However, the process has a very short memory, and a further effect of judgments is present after one period. The impulse coefficient at lag 1 is negative. This implies that the mentioned variation in the shape of the distribution of the expectations is negatively adjusted the subsequent month. The adjustment is more pronounced in the second period when the consequences of the Euro cash over had a major effect.

6 Conclusions

In this article we have discussed a dynamic approach to the class of CUB models for rating data to describe time patterns of consumers' beliefs about price levels in Italy. Specifically, our proposal is to resort to profile estimation of feeling parameters to investigate the evolution of the latent sentiment conveyed by qualitative judgments and expectations on price inflation, whose distribution is released monthly by ISTAT. Our analysis has covered a 24-year time span, from 1994 to January 2019: three different periods, characterized by different economic conditions, have been studied separately.

Given the feeling series for judgments and expectations, we have first run a search for the best fitting time series linear model, founding that an ARMA(1,1) can be safely assumed for their generating process. The inspection of their spectra induces to claim that consumers' latent sentiment on price inflation has a substantial inertial component, and thus it is characterized by a slow variation over time. Further, we have investigated how feeling on judgments and expectations interact over time by means of a dynamic regression: specifically, we have tested how judgments about past price levels affect future expectations, founding somewhat different results for the three time periods.

Summarizing, our proposal is an inferential procedure to extract the latent feeling about qualitative judgments and expectations of price levels, net of possible nuisance effects, suitable to investigate the dynamics of the underlying consumers' opinion. The approach is framed within a lively scientific literature on the topics, and it can be applied whenever a synthetic measure of feeling is needed to summarize ordinal distributions as they vary over time. Alternative methods to jointly model the dynamic of CUB parameters and their association have been preliminary investigated in Simone et al. [41]: in this setting, further options could involve non-linear non-Gaussian state space model for time-varying CUB parameters [25].

As a concluding remark, it is possible to advance a parallelism between the idea of profiling feeling out of a general model for qualitative ordered data and the approach advanced in Meyler and Reiche [26] for the analysis of quantitative survey measures on inflation perceptions and expectations, which foresees to focus separately on those 'certain' respondents that use figures with digits, assuming that rounded outcomes are deemed to be associated with uncertainty of the evaluation.

Funding No funding was received for conducting this study.

Declarations

Conflict of interest The authors declare that there are no conflict of interests.

Appendix A: Profile likelihood estimation of feeling

The feeling parameter in CUB mixture models conveys the underlying sentiment towards the trait under examine, whereas the uncertainty weight and possible shelter effect represent a discrete noise surrounding feeling with respect to heterogeneity and inflated frequencies, respectively. For the purpose of the paper, we resort to profile likelihood methods [29, 31, 45] to estimate feeling parameters (of both perceived and expected price inflation) by profiling the nuisance parameters out of the likelihood to be optimized.[3]

In the following, we will consider the profile likelihood estimator of ξ under the more general CUB with shelter specification. As a matter of fact, for $t = 1, \ldots, T$, the best model for $R_t^{(e)}$ and $R_t^{(j)}$ can be either Binomial, Binomial with shelter, CUB or CUB with shelter.[4] Thus, instead of performing a point-wise search for the best model, the feeling series analysed in Sects. 4–5 consider the profile estimation of ξ under CUB with shelter specification (at the second category) at each time point. More generally, the location of the shelter can be either fixed based on theoretical or graphical evidence, or parametrized in turn as a further nuisance parameter: the latter choice has been made for the forthcoming simulation study.

Hereafter, ξ is the parameter of interest whereas $\eta = (\pi^{(1)}, \pi^{(2)}, c)'$ is considered as the nuisance parameter vector (for CUB model, $\eta = \pi^{(2)} = \pi$, for instance). Let $L(\xi, \eta)$ be the log-likelihood function (dependence on the sample is omitted for notational convenience) of CUB with shelter, with unspecified shelter location. Profile likelihood estimation of ξ involves the following steps:

1. For each value of ξ in its parameter space $[0, 1]$, determine: $\hat{\eta}_\xi = \underset{\eta}{argmax}\ L(\xi, \eta)$, that is, the value of the nuisance parameter that maximises the likelihood $L(\xi, \eta)$ over η in its admissible range $\Omega(\eta)$.
2. The profile likelihood function $L_p(\xi)$ of ξ is defined as: $L_p(\xi) = L(\xi, \hat{\eta}_\xi)$.
3. Finally, the profile likelihood estimate $\hat{\xi}_p$ of ξ is obtained by maximizing $L_p(\xi)$ over the admissible range $[0, 1]$.

To show the usefulness of this approach, a small simulation experiment has been carried out. For 150 runs, a sample of n ratings over a scale with m ordered categories is generated according to each of the scenarios listed in Table 4.

Figure 11 shows the boxplot of the empirical distribution of ξ estimators corresponding to the fit of candidate models and profile methods. Results indicate that profile estimation of feeling parameter under the general CUB with shelter is a safe method to measure the latent *sentiment*, whatever the true generating model is, since it performs always equivalently to the best fitting model.

Thus, profile estimation of the feeling parameter can be successfully performed within the class of CUB models to assess the latent feeling while not disregarding possible nuisance effects and avoiding model-selection procedures in cases interpretation of the best model is not of primary importance for the analysis.

[3] For the sake of completeness, it is worth to emphasize that profile likelihood can be performed also to derive time series of uncertainty parameters in order to assess the dynamics of the propensity to uncertainty over time.

[4] For our empirical analysis, the shelter is considered at $c = 2$, but it could be itself parametrized as shown within the simulation experiment.

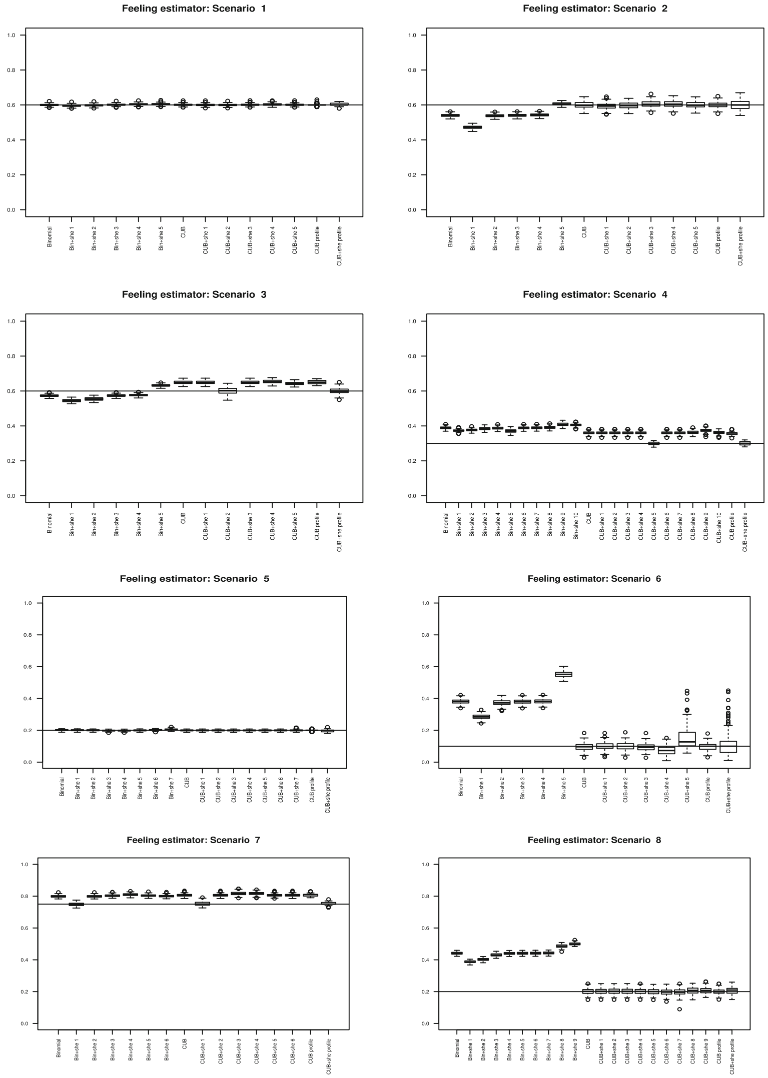

Fig. 11 Boxplots of the sampling distribution of ML estimators for ξ under candidate models and profile methods for CUB and CUB with shelter, for scenarios listed in Table 4

Appendix B: Balance statistic and feeling

In order to support the motivation of resorting to Binomial feeling parameter to assess the dynamics of latent sentiment expressed with qualitative surveys by consumers over time on price inflation judgments and expectations, the behavior of the Balance Statistic and feeling is briefly compared and discussed.

From a comparative perspective, it is worth to emphasize the link that the feeling parameter has with the balance statistic $\mathcal{B}$, which is formally defined as a weighted difference between the proportions of respondents who perceived (expected) a moderate or high increase in the price level and the proportions of respondents who perceived (expected) a decrease or stability. Given the observed relative frequencies $f_1, f_2, \ldots, f_5$, and according to our notation, the balance statistic is: $\mathcal{B} = f_5 + 0.5\, f_4 - 0.5\, f_2 - f_1$, and $\mathcal{B} \in [-1,\ 1]$. Simple algebra shows that $\mathcal{B} = \dfrac{\bar{R}_n - 3}{2}$, where $\bar{R}_n$ is the mean rating; as a consequence, *the balance statistics is implicitly assuming discrete equidistant numbers for the ordinal responses*.

If a shifted Binomial is assumed as the ratings distribution with $m = 5$ (that is a CUB model with $\pi = 1$), the maximum likelihood estimate of the parameter is: $\hat{\xi} = \dfrac{5 - \bar{R}_n}{4}$. As a consequence, $\mathcal{B} = 1 - \dfrac{\hat{\xi}}{2}$ under the Binomial model. In general, the Balance statistics will be a function of the model parameters, and not exclusively of the feeling (see Appendix B for further comparative discussion of the Balance Statistic and feeling parameter). Thus, feeling estimates obtained with profile likelihood method are better suited for the goal of the paper since they measure the latent sentiment net of possible nuisance parameters.

With reference to observed data, Fig. 9 shows the corresponding time series for the three time periods. It is seen that, except for Period 2, for Period 1 and, to a greater extent for Period 3, the Balance Statistic follows a different pattern compared to the feeling series displayed in Fig. 4.

We have already discussed that, under the Binomial model, the Balance Statistic is indeed linearly related to feeling ML estimates. However, when the data do not obey to a Binomial model, we can see from the contour curves of the Balance Statistic, displayed in Fig. 10, that it cannot be considered as a reliable indicator of the latent feeling only, as this relationship is modified by possible heterogeneity and inflated frequencies.

References

1. Agresti, A.: Analysis of Ordinal Categorical Data, 2nd edn. J. Wiley & Sons, Hoboken (2010)
2. Arioli, R., Bates, C., Dieden, H., Duca, I., Friz, R., Gayer, C., Geoff, K., Meyler, A., Pavlova, I.: EU consumers' quantitative inflation perceptions and expectations: an evaluation. European Central Bank Occasional Paper no. 186 (2017)
3. Armantier, O., Bruine de Bruin, W., Topa, G., van der Klaauw, W., Zafar, B.: Inflation expectations and behavior: do survey respondents act on their beliefs. Int. Econ. Rev. **56**, 505–36 (2015)
4. Arnold, I.J., Lemmen, J.J.: Inflation expectations and inflation uncertainty in the Eurozone: evidence from survey data. Rev. World. Econ. **144**, 325–346 (2008)
5. Axelrod, S., Lebow, D., Peneva, E.: Perceptions and Expectations of Inflation by U.S. Households. Finance and Economics Discussion Series 2018-073. Washington: Board of Governors of the Federal Reserve System (2018)
6. Biau, O., Dieden, H., Ferrucci, G., Fritz, R., Linden, S.: Consumers Quantitative Inflation Perceptions and Expectations in the Euro Area: an Evaluation. Mimeo, New York Federal Reserve Bank Conference (October 2010)
7. Box, G.E.P., Jenkins, G.M., Reinsel, G.C., Ljung, G.M.: Time Series Analysis: Forecasting and Control. Wiley Series in Probability and Statistics, 5th edn. Wiley, New York (2016)
8. Burke, M., Manz, M.: Economic literacy and inflation expectations: Evidence from a laboratory experiment. J. Money Credit Bank **46**, 1421–56 (2014)
9. Cerulli, C., Simone, R., Di Iorio, F., Piccolo, D., Baum, C.F.: The CUB STATA module: mixture models for feeling and uncertainty of rating data. STATA J **22**(1), 195–223 (2022)
10. Coibion, O., Gorodnichenko, Y., Kamdar, R.: The formation of expectations, inflation, and the Phillips curve. J. Econ. Lit. **56**(4), 1447–91 (2018)
11. Corduas, M., Iannario, M., Piccolo, D.: A class of statistical models for evaluating services and performances. In: Bini, M., et al. (eds.) Statistical methods for the evaluation of educational services and quality of products, Contribution to Statistics, pp. 99–117. Physica-Verlag, Springer, Berlin Heidelberg (2009)
12. Corduas, M.: A dynamic model for ordinal time series: an application to consumers' perceptions of inflation. In: Statistical Learning and Modeling in Data Analysis: Methods and Applications, Springer International Publishing, pp. 37-45 (2021)

13. Corduas, M.: Gender differences in the perception of inflation. J. Econ. Psychol. **90**(7), 102522 (2022)
14. de Bruin, W.B., Van der Klaauw, W., Topa, G.: Expectations of inflation: The biasing effect of thoughts about specific prices. J. Econ. Psychol. **32**(5), 834–845 (2011)
15. Del Giovane, P., Fabiani, S., Sabbatini, R.: Perceived and measured inflation after the launch of the Euro: explaining the gap in Italy. Giornale degli Economisti e Annali di Economia **65**, 155–192 (2006)
16. D'Elia, A., Piccolo, D.: A mixture model for preference data analysis. Comput. Stat. Data Anal. **49**, 917–934 (2005)
17. Del Missier, F., Ranyard, R., Bonini, N.: Perceived inflation: The role of product accessibility and attitudes towards inflation. J. Econ. Psychol. **56**, 97–106 (2016)
18. Dias, F., Duarte, C., Rua, A.: Inflation (mis) perceptions in the Euro area. Empir. Econ. **39**, 353–369 (2010)
19. Hamilton, J.D.: Time Series Analysis. Princeton University Press, Princeton (1994)
20. Huber, O.W.: Frequency of price increases and perceived inflation. An experimental investigation. J. Econ. Psychol. **32**, 651–661 (2011)
21. Iannario, M.: Modelling shelter choices in a class of mixture models for ordinal responses. Stat. Methods Appl. **21**, 1–22 (2012)
22. Iannario M., Piccolo D., Simone, R.: CUB: A class of mixture models for ordinal data (R package version 1.1.3). (2018) http://CRAN.R-project.org/package=CUB
23. ISTAT. I. stat data warehouse. (2019) http://dati.istat.it
24. Jonung, L.: Perceived and Expected Rates of Inflation in Sweden. Am. Econ. Rev. **71**, 961–968 (1981)
25. Koopman, D.: Time series analysis by state space methods, 2nd edn. Oxford Press, UK (2012)
26. Meyler, A., Reiche, L.: Making sense of consumers? Inflation perceptions and expectations - the role of (un)certainty. Econ. Bull. Articles **2**, 104–120 (2021)
27. Meyler, A., Reiche, L.: Making sense of consumer inflation expectations: the role of uncertainty, Working Paper Series 2642 (February 2022), European Central Bank (2022)
28. Monroe, K.B.: Buyers' subjective perception of price. J. Mark. Res. **10**, 70–80 (1973)
29. Murphy, S.A., Der Vaart, A.W.: On Profile Likelihood. J. Am. Stat. Assoc. **95**(450), 449–465 (2000)
30. Pankratz, A.: Forecasting with Dynamic Regression Models. John Wiley & Sons, Hoboken (2012)
31. Pawitan, J.: In All Likelihood. Oxford University Press, UK (2013)
32. Piccolo, D.: On the moments of a mixture of uniform and shifted binomial random variables. Quaderni di Statistica **5**, 85–104 (2003)
33. Piccolo, D.: Inferential issues on CUBE models with covariates. Communications in Statistics. Theo. Methods **44**, 5023–5036 (2015)
34. Piccolo, D., Simone, R.: The class of CUB models: statistical foundations, inferential issues and empirical evidence (with discussions and rejoinder). Stat. Methods Appl. **28**, 389–435 (2019)
35. Piccolo, D., Simone, R.: Rejoinder to the discussion of 'The class of CUB models: statistical foundations, inferential issues and empirical evidence'. Stat. Methods Appl. **28**, 477–493 (2019)
36. Proietti, T.: Discussion of 'The class of CUB models: statistical foundations, inferential issues and empirical evidence', by D. Piccolo and R. Simone. Stat. Methods Appl. **28**, 451–456 (2019)
37. Proietti, P., Frale, C.: New proposals for the quantification of qualitative survey data. J. Forecast. **30**, 393–408 (2011)
38. Ranyard, R., Del Missier, F., Bonini, N., Duxbury, D., Summers, B.: Perceptions and expectations of price changes and inflation: A review and conceptual framework. J. Econ. Psychol. **29**, 378–400 (2008)
39. Simone R.: FastCUB: Fast EM and Best-Subset Selection for CUB Models for Rating Data (R package version 0.0.2). (2020) https://CRAN.R-project.org/package=FastCUB
40. Simone, R.: An accelerated EM algorithm for mixture models with uncertainty for rating data. Comput. Stat. **36**, 691–714 (2021)
41. Simone R., Piccolo D., Corduas, M.: Dynamic modeling of price expectations, In: G.C. Porzio, F. Greselin, S. Balzano (eds.), CLADAG 2019 Book of Short Papers, pp.449–452. (2019) ISBN 978-88-8317-108-6
42. Soroka, S.N.: Good news and bad news: Asymmetric responses to economic information. J. Politics **68**, 372–385 (2006)
43. Sudipt, R., Chan, T., Cheen, A.: Price Expectation and Purchase Decisions: Evidence from an Online Store Experiment. Customer Needs Solut. **1**, 117–130 (2014)
44. Tutz, G.: Regression for Categorical Data. Cambridge University Press, Cambridge (2012)
45. Venzon, D.J., Moolgavkar, S.H.: A Method for Computing Profile-Likelihood Based Confidence Intervals. J. R. Stat. Soc. Series C. **37**(1), 87–94 (1988)