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Distributed Fixed-Time Leader-Tracking Control for Heterogeneous Uncertain Autonomous Connected Vehicles Platoons

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***Abstract*— Longitudinal control for platoons of connected autonomous vehicles is a hot research topic in the Cooperative Intelligent Transport Systems (C-ITSs) domain. Most of the existing results solve the platooning problem asymptotically, without ensuring that the consensus could be achieved in a finite settling time. To this aim, in this work we address the problem of guaranteeing the leader-tracking for heterogeneous vehicles platoons in a fixed time despite the presence of external disturbances. To solve this problem, by exploiting the integral sliding mode (ISM) approach and the Lyapunov theory, we propose a distributed control strategy able to ensure the leader-tracking in a finite settling time which is independent from any vehicles initial conditions. The simulation analysis, carried out in two different driving scenarios, confirms the effectiveness of the theoretical derivation.**

***Index Terms*— Connected Autonomous Vehicles Platoons; heterogeneous vehicle dynamics; uncertain vehicles dynamics; leader-tracking control problem; Distributed Fixed-time control strategy.**

I. INTRODUCTION

Over the past decades, the advances in information communication technology have attracted considerable attention in C-ITS domain due to the benefit they could lead in terms of road safety increasing and the decreasing of the environmental pollution [1]. Specifically, the deployment of connected autonomous vehicles platoons moving in formation with a common velocity while keeping a prefixed inter-vehicular distance may significantly improve different aspects of the vehicular traffic flow such as the traffic congestion [2]. In this driving scenario, all vehicles are connected through Vehicle-to-Vehicle (V2V) wireless communication paradigm and exchange information by exploiting the de facto IEEE 802.11p communication protocol [3]. Leveraging these information, the aim of the cooperative control strategies is to guarantee that each vehicle within the platoon tracks the reference behaviour as imposed by the leader, i.e. the first vehicle of the fleet, while achieving the desired formation w.r.t. the communicating vehicles [4].

Exploiting the Multi-Agent Systems (MASs) paradigm [5], in the wide technical literature, different cooperative driving controllers have been proposed both for homogeneous vehicles platoons (see for example [6], [7], [8] and the reference therein) and for heterogeneous ones (see for example [9], [10], [11]). Among them, robust protocols are suggested to counteract both the parameter uncertainties in [12] and the co-presence of parameter uncertainty and time-delays in [13]. Again, resilient strategies able to deal with cyber-attacks have been recently proposed in [4], [6], [14], while adaptive approaches have been designed in [15] and in [10].

However, all the aforementioned works focus on the design of an appropriate distributed control able to asymptotically achieve consensus while it is expected that the final prescribed inter-gap formation could be formed in a finite-time interval [16]. Moreover, it has been shown in [17], [18] that finite-time consensus controllers have a faster convergence rate and better disturbance rejection to system uncertainty and external disturbance w.r.t. the typical consensus-based strategies. Despite these crucial aspects, only few works have designed finite-time cooperative control for autonomous vehicles platoons. Along this line, nonlinear finite-time consensus protocols are suggested in [16] to solve the platooning problem over a fixed and switching communication topologies and in [19] to guarantee the string-stability of heterogeneous vehicles platoons in finite-time.

Although the consensus is pursued in a finite time, the settling time estimation explicitly relies on the initial conditions of each agent within the MAS [20], [21]. It implies that there might be an applicability limit of the finite-time consensus approaches in those cases when agents initial states are unknown or unavailable a-priori. This brought to the emerging of fixed-time consensus protocols whose stability, as well as the settling time estimation, is completely independent from agents initial conditions [22].

To this aim, in this work, to the best of authors knowledge, we address, for the first time, the fixed-time leader-tracking control problem for heterogeneous uncertain autonomous vehicles platoons. To solve this problem, we propose a fixed-time control protocol able to: *i*) counteract the vehicles heterogeneity and unknown external disturbances; *ii*) guarantee the leader-tracking in a fixed settling time whose estimation only depends on the proper choice of the control gains, and not on any vehicles initial conditions. The stability of the vehicular network under the action of the proposed distributed control protocol is analytically proved by exploiting the fixed-time stability tools and the Lypaunov theory and an estimation of the fixed settling time is provided. Numerical analysis, carried out considering two exemplar driving scenarios, confirms the theoretical derivation and discloses the effectiveness of the control strategy in ensuring the robust leader-tracking in a fixed settling time, despite the variation of vehicles initial conditions and the network communication topology. Finally, the rest of

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the paper is organized as follows. In Section II, all the mathematical background, notations and definitions are given while the problem statement is stated in Section III. Section IV describes the proposed distributed fixed-time platooning controller and analytically proves the stability of the closed-loop vehicular network. The numerical analysis, carried out for different driving scenarios, is presented in Section V. Conclusions are drawn in Section VI.

II. Mathematical Preliminaries

A. Graph Theory

A set of N connected vehicles can be modeled as a directed graph $\mathscr{G}_N = (\mathscr{V}_N, \mathscr{E}_N)$, where $\mathscr{V}_N$ is the set of vehicles while $\mathscr{E}_N \subseteq \mathscr{V}_N \times \mathscr{V}_N$ is the set of edges that defines the communication topology allowing the connection among them. The communication network can be hence described via the adjacency matrix $\mathscr{A}$ and the in-degree matrix $\mathscr{D}$. The adjacency matrix $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is such that the element $a_{ij} = 1$ if there exists a directed link from vehicle j to vehicle i, 0 otherwise. Conversely, the in-degree matrix is $\mathscr{D} = \text{diag}\{d_1, d_2, \cdots, d_N\} \in \mathbb{R}^{N\times N}$, where $d_i = \sum_{j=1}^{N} a_{ij}$ indicates the number of connection links with the vehicle i. Based on the above definition we can define the Laplacian matrix as $\mathscr{L} = \mathscr{D} - \mathscr{A} \in \mathbb{R}^{N\times N}$.

In the rest of the paper, we consider a platoon composed of N vehicles plus a leader, taken as an additional agent, labeled with index 0 and assumed to have no neighbors, i.e. it shares its state information without any edge that enters into it. To model the resulting vehicular network we use an augmented graph $\mathscr{G}_{N+1}$. To model the leader connections, we introduce the pinning matrix $\mathscr{B} = \text{diag}\{b_1, b_2, \cdots, b_N\} \in \mathbb{R}^{N\times N}$ with $b_i = 1$ if the leader information is directly available for the $i-$th vehicle, 0 otherwise. Defining a spanning tree as a union of communication links connecting all vehicles of the network, directly or indirectly, the following assumption holds.

Assumption 1: [23] The graph $\mathscr{G}_{N+1}$ contains a spanning tree with the leader as a root node. Hence, $\mathscr{L} + \mathscr{B}$ is a non-singular diagonally dominant M-matrix and all its eigenvalues have positive real parts.

B. Definition and Lemmas

In this section we introduce some definitions and lemmas useful for deriving the main result.

*Definition 1 (*Fixed-Time Stability*[24]):* Considering the system

$$\dot{x}(t) = f(x(t)),$$

if there exist a continuous, positive definite function $V(x(t)) : \mathbb{R}^n \to \mathbb{R}$, such that $\dot{V}(x(t)) \leq -a\, V^p(x(t)) - \, b\, V^q(x(t))$, $\forall x(t) \in \mathbb{R}^n$, $a, b > 0$, $p \in (0,1)$ and $q \in (1,\infty)$, then the system is fixed-time stable and the fixed settling time is bounded by $T \leq \frac{1}{a(1-p)} + \frac{1}{b(q-1)}$.

Lemma 1: [25] Consider the n ($n \geq 2$) order integrator system $\dot{x}_1(t) = x_2(t)$, $\dot{x}_2(t) = x_3(t)$, $\cdots$, $\dot{x}_n(t) = u(t)$, $x(0) = x_0$, with $x_k(t) \in \mathbb{R}(i = 1, \cdots, n)$ and $u(t) \in \mathbb{R}$. Let k_i ($\forall i = 1, \cdots, n$) positive constants such that the characteristic polynomial, defined with the Laplace operator, $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ and $s^n + 3k_n s^{n-1} + \cdots + 3k_2 s + 3k_1$ is Hurwitz. There exists a constant $\varepsilon \in \left(\frac{n-2}{n+2}, 1\right)$ such that, for every $\gamma \in (\varepsilon, 1)$, the integrator system is stabilized at the origin in fixed-time under the feedback control

$$u(t) = -\sum_{r=1}^{n} k_r(\lceil x_r(t)\rfloor^{\gamma_r} + \lceil x_r(t)\rfloor + \lceil x_r(t)\rfloor^{\gamma_r'}),$$

with parameters γ_r and γ_r' satisfying $\gamma_{n-g} = \frac{\gamma}{(g+1)-g\gamma}$, $\gamma_{n-g}' = \frac{2-\gamma}{g\gamma-(g-1)}$, $(g = 0, 1, \cdots, n-1)$.

Lemma 2: [26] Let $x_1, x_2, \cdots, x_N \geq 0$. If $p \in (0,1]$ we have

$$\sum_{i=1}^{N} x_i^p \geq \Big(\sum_{i=1}^{N} x_i\Big)^p. \tag{1}$$

Instead, if $p \in (1, +\infty)$, we have

$$\sum_{i}^{N} x_i^p \geq N^{1-p}\Big(\sum_{i=1}^{N} x_i\Big)^p. \tag{2}$$

Finally, throughout the paper, the following notation is adopted. Given a non-negative value of c, we indicate the sig function [27] as

$$\lceil x\rfloor^c = |x|^c \text{sign}(x),\ \forall x \in \mathbb{R}$$

being $sign(\cdot)$ is the signum function. According the above notation, we have $x\lceil x\rfloor^c = |x|^{c+1}$.

III. Problem Statement

Consider a platoon composed of N autonomous connected vehicles plus a leader sharing information about their position and velocity through a V2V wireless communication network. The behaviour of each vehicle i ($\forall i = 1, \cdots, N$) can be described by its linearized longitudinal dynamics as [28]

$$\begin{aligned} \dot{p}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t) + w_i(t), \end{aligned} \tag{3}$$

where $p_i(t)$ $[m]$ and $v_i(t)$ $[m/s]$ are the $i-$th vehicle absolute position (w.r.t. a given reference framework) and longitudinal velocity, respectively; $u_i(t)$ $[m/s^2]$ the desired longitudinal acceleration to be imposed to the vehicle, which can be computed as $\frac{1}{M_i}\tilde{u}_i(t)$, being M_i $[kg]$ the $i-$th vehicle mass and $\tilde{u}_i(t)$ $[kg\, m/s^2]$ the desired driving/brake force; $w_i(t)$ $[m/s^2]$ is the unknown external disturbance acting on the vehicle dynamics arising from environmental factors, such as variations in wind velocity and/or road slope.

Similarly, the leading vehicle, which imposes the reference behaviour for the whole vehicles platoon, is described by the following non-autonomous dynamical system [16]:

$$\begin{aligned} \dot{p}_0(t) &= v_0(t) \\ \dot{v}_0(t) &= u_0(t), \end{aligned} \tag{4}$$

where $p_0(t)$ $[m]$ and $v_0(t)$ $[m/s]$ are the leading vehicle absolute position (w.r.t. a given reference framework) and velocity, respectively; $u_0(t)$ $[m/s^2]$ the leader acceleration which can be computed as $\frac{1}{M_0}\tilde{u}_0(t)$, being M_0 $[kg]$ its mass

and $\tilde{u}_0(t)$ $[kg\,m/s^2]$ the leader driving/brake force.
Given the autonomous vehicles platoon dynamics as in (3) and (4), the following assumptions hold.

Assumption 2 ([28]): The unknown disturbances $w_i(t)$, $i=1,\cdots,N$ are bounded, i.e. $|w_i(t)| < \bar{w}_i < +\infty$, with $\bar{w}_i$ are finite constant known to all vehicles.

Assumption 3: The leader behavior is unknown, but bounded due to physical constraints [29], i.e. $|u_0(t)| < u_0^{max} < +\infty$, with the finite constant u_0^{max} known to all vehicles.

Now the platooning control problem can be stated as follows.

Problem 1 (Platooning Control in fixed-time): Given a platoon composed of N connected autonomous vehicles plus a leader imposing the reference behaviour for the whole vehicular network, find a distributed control input $u_i(t)$ $(\forall i=1,\cdots,N)$ such that each vehicle i, in a fixed-time $T^\star$, tracks the leader motion $v_0(t)$ while maintaining a desired inter-vehicular distance d_{ij} w.r.t. its neighbors j $(j=0,1,2,\cdots,N)$, i.e.

$$\begin{aligned}&\lim_{t\to T^\star}\|p_i(t)-p_j(t)-d_{ij}\|=0,\\&\lim_{t\to T^\star}\|v_i(t)-v_0(t)\|=0,\end{aligned} \tag{5}$$

where $T^\star < T_{max} < +\infty$, being T_{max} $[s]$ the settling time estimation, independent from the platoon initial conditions and computed according to Definition 1.

IV. Distributed Fixed-Time Platooning Control

In this section, firstly, we propose a novel distributed fixed-time control protocol able to solve Problem 1, and, then, we analytically prove the fixed-time stability of the vehicular network under the action of the proposed controller.

A. Control Design

Define for each vehicle i $(\forall i=1,\cdots,N)$ the position and velocity tracking errors vectors as

$$\delta_{pi}(t)=\sum_{j=1}^{N}a_{ij}(p_i(t)-p_j(t)-d_{ij})+b_i(p_i(t)-p_0(t)-d_{i0}) \tag{6}$$

$$\delta_{vi}(t)=\sum_{j=1}^{N}a_{ij}(v_i(t)-v_j(t))+b_i(v_i(t)-v_0(t)), \tag{7}$$

where a_{ij} and b_i model the network communication topology as defined in Section II-A.
By introducing the state error of the $i-$th vehicle w.r.t. the leader as

$$\begin{aligned}\tilde{p}_i(t)&=p_i(t)-p_0(t)-d_{i0},\\\tilde{v}_i(t)&=v_i(t)-v_0(t),\end{aligned} \tag{8}$$

the disagreement vectors in (6) and (7) can be recast as

$$\delta_{pi}(t)=\sum_{j=1}^{N}a_{ij}(\tilde{p}_i(t)-\tilde{p}_j(t))+b_i\tilde{p}_i(t) \tag{9}$$

$$\delta_{vi}(t)=\sum_{j=1}^{N}a_{ij}(\tilde{v}_i(t)-\tilde{v}_j(t))+b_i\tilde{v}_i(t). \tag{10}$$

Now, taking into account the definition of the disagreement vectors as in (9) and (10), from (3) and (4), the error tracking dynamics can be derived as

$$\begin{aligned}\dot{\delta}_{pi}(t)&=\delta_{vi}(t),\\\dot{\delta}_{vi}(t)&=\sum_{j=1}^{N}a_{ij}\Big(u_i(t)+w_i(t)-u_j(t)-w_j(t)\Big)\\&\quad+b_i\Big(u_i(t)+w_i(t)-u_0(t)\Big).\end{aligned} \tag{11}$$

Now, to solve the fixed-time platooning control as stated in Problem 1, we introduce the following integral sliding surface for each vehicle i:

$$\begin{aligned}\sigma_i(t)=\delta_{iv}(t)+\int_0^t &k_{i1}(\lceil\delta_{pi}(s)\rfloor^{\gamma_1}+\lceil\delta_{pi}(s)\rfloor+\lceil\delta_{pi}(s)\rfloor^{\gamma_1'})\\&+k_{i2}(\lceil\delta_{vi}(s)\rfloor^{\gamma_2}+\lceil\delta_{vi}(s)\rfloor+\lceil\delta_{vi}(s)\rfloor^{\gamma_2'})ds\end{aligned} \tag{12}$$

where k_{i1}, k_{i2}, $\gamma_{1,2}$ and $\gamma_{1,2}'$ have to be chosen according to Lemma 1. Specifically, regarding the choice of k_{i1}, k_{i2}, since the error dynamics as in (11) is of order $n=2$, it is sufficient selecting k_{i1} and $k_{i2}\in\mathbb{R}_+$. In view of (12) we propose for each vehicle i the following distributed fixed-time control protocol:

$$\begin{aligned}u_i(t)=&-\Big(\sum_{j=1}^{N}a_{ij}+b_i\Big)^{-1}(k_{i1}(\lceil\delta_{pi}(t)\rfloor^{\gamma_1}+\lceil\delta_{pi}(t)\rfloor\\&+\lceil\delta_{pi}(t)\rfloor^{\gamma_1'})+k_{i2}(\lceil\delta_{vi}(t)\rfloor^{\gamma_2}+\lceil\delta_{vi}(t)\rfloor+\lceil\delta_{vi}(t)\rfloor^{\gamma_2'}\\&-\sum_{j=1}^{N}a_{ij}u_j(t)+\lceil\sigma_i(t)\rfloor^{p}+\lceil\sigma_i(t)\rfloor+\lceil\sigma_i(t)\rfloor^{q}\\&+\kappa_i\mathrm{sign}(\sigma_i(t)))\end{aligned} \tag{13}$$

being $p\in(0,1)$, $q\in(1,\infty)$ and κ_i a control gain to be properly tuned.

B. Stability Analysis

The fixed-time stability of the heterogeneous autonomous vehicles platoon under the action of the proposed controller in (13) is guaranteed by the following theorem.

Theorem 1: Consider a platoon composed of N heterogeneous vehicles plus a leader imposing the reference behaviour, whose dynamics are as in (3) and (4), respectively. Let Assumptions 2 and 3 hold. The Problem 1 is solved in a fixed-time $T^\star$ by the distributed control input (13) if we select the control κ_i, $\forall i$ as:

$$\kappa_i\geq\sum_{j=1}^{N}a_{ij}(\bar{w}_i+\bar{w}_j)+b_i(\bar{w}_i+u_0^{max}), \tag{14}$$

being $\bar{w}_i$ and $\bar{w}_j$ the known upper bound of the external disturbances acting on the vehicles i and j, respectively, and u_0^{max} the maximum known value of the leader acceleration.

Proof: Consider the following candidate Lyapunov function:

$$V=\frac{1}{2}\sum_{i=1}^{N}\sigma_i^2(t). \tag{15}$$

Differentiating (15), by taking into account the definition of the sliding surface as in (12) as well as the error tracking dynamics as in (11), we obtain:

$$\begin{aligned}\dot{V} = &\sum_{i=1}^{N}\sigma_i(t)\Big((\sum_{j=1}^{N}a_{ij}+b_i)u_i(t)-\sum_{j=1}^{N}a_{ij}u_j(t)\\&+k_{i1}(\lceil\delta_{pi}(t)\rfloor^{\gamma_1}+\lceil\delta_{pi}(t)\rfloor+\lceil\delta_{pi}(t)\rfloor^{\gamma_1'})\\&+k_{i2}(\lceil\delta_{vi}(t)\rfloor^{\gamma_2}+\lceil\delta_{vi}(t)\rfloor+\lceil\delta_{vi}(t)\rfloor^{\gamma_2'})\\&+\sum_{j=1}^{N}a_{ij}(w_i(t)-w_j(t))+b_i(w_i(t)-u_0(t))\Big).\end{aligned} \tag{16}$$

Now, substituting the control input (13) in (16), after some algebraic manipulation we have:

$$\begin{aligned}\dot{V} = -&\sum_{i=1}^{N}\Big(|\sigma_i(t)|^{p+1}+|\sigma_i(t)|^2+|\sigma_i(t)|^{q+1}+\kappa_i|\sigma_i(t)|\\&-\sum_{j=1}^{N}a_{ij}w_j(t)|\sigma_i(t)|-(\sum_{j=1}^{N}a_{ij}+b_i)w_i(t)|\sigma_i(t)|\\&-b_i|\sigma_i(t)|u_0(t)\Big).\end{aligned} \tag{17}$$

Now, consider the Assumptions 2 and 3, and select the control gains κ_i, $i=1,\cdots,N$ as in (14). In so doing, leveraging also Lemma 2, (17) can be recast as

$$\begin{aligned}\dot{V}(t) &\leq -\sum_{i=1}^{N}\Big(|\sigma_i(t)|^{p+1}+|\sigma_i(t)|^2+|\sigma_i(t)|^{q+1}\Big)\\&\leq -\Big(\sum_{i=1}^{N}|\sigma_i(t)|^2\Big)^{\frac{p+1}{2}}-N^{\frac{1-q}{2}}\Big(\sum_{i=1}^{N}|\sigma_i(t)|^2\Big)^{\frac{q+1}{2}}.\end{aligned} \tag{18}$$

Considering the Lyapunov function as in (15), inequality (18) can be finally re-written as

$$\dot{V}(t) \leq -2^{\frac{p+1}{2}}V(t)^{\frac{p+1}{2}}-2^{\frac{q+1}{2}}N^{\frac{1-q}{2}}V(t)^{\frac{q+1}{2}}. \tag{19}$$

Hence, according to the fixed-time stability theory (see Definition 1), the sliding surface $\sigma_i(t)$ $(\forall i=1,\cdots,N)$ converges to zero in a fixed-time $T_1 \leq \frac{1}{2^{\frac{1+q}{2}}(1-p)}+\frac{1}{2^{\frac{q+1}{2}}N^{\frac{1-q}{2}}(q-1)}$.
Note that, during the sliding motion, i.e. when $\sigma_i(t)=0$, we have that $\dot{\sigma}_i(t)=0$. This implies that, according to (11) and (12), for $t \geq T_1$, the reduced closed-loop error dynamics can be derived as

$$\begin{aligned}\dot{\delta}_{pi}(t) &= \delta_{vi}(t),\\ \dot{\delta}_{vi}(t) &= -k_{i1}(\lceil\delta_{pi}(t)\rfloor^{\gamma_1}+\lceil\delta_{pi}(t)\rfloor+\lceil\delta_{pi}(t)\rfloor^{\gamma_1'})\\&\quad -k_{i2}(\lceil\delta_{vi}(t)\rfloor^{\gamma_2}+\lceil\delta_{vi}(t)\rfloor+\lceil\delta_{vi}(t)\rfloor^{\gamma_2'}).\end{aligned} \tag{20}$$

From Lemma 1, the reduced closed-loop error system (20) is fixed-time stable at the origin. It follows that $\delta_{pi}(t)$, $\delta_{vi}(t) \to 0$ $(\forall i)$ in a settling time T_2 that is independent of any initial condition.
Accordingly, $\tilde{p}_i(t)$, $\tilde{v}_i(t) \to 0$ $(\forall i)$. Indeed, by introducing the global error vectors $\delta_p(t)=[\delta_{p1}(t),\delta_{p2}(t),\cdots,\delta_{pN}(t)]^\top \in \mathbb{R}^N$, $\delta_v(t)=[\delta_{v1}(t),\delta_{v2}(t),\cdots,\delta_{vN}(t)]^\top \in \mathbb{R}^N$ $\tilde{p}(t)=[\tilde{p}_1(t),\tilde{p}_2(t),\cdots,\tilde{p}_N(t)]^\top \in \mathbb{R}^N$ and $\tilde{v}(t)=[\tilde{v}_1(t),\tilde{v}_2(t),\cdots,\tilde{v}_N(t)]^\top \in \mathbb{R}^N$, according to (9) and (10), we can express the disagreement vector as function of (8) as

$$\begin{aligned}\delta_p(t) &= (\mathscr{L}+\mathscr{B})\tilde{p}(t),\\ \delta_v(t) &= (\mathscr{L}+\mathscr{B})\tilde{v}(t).\end{aligned} \tag{21}$$

Given the Assumption 1, since $\mathscr{L}+\mathscr{B}$ is a positive definite $M-$matrix, $\delta_p(t)$, $\delta_v(t) \to 0$ in a fixed-time implies that $\tilde{p}(t)$, $\tilde{v}(t) \to 0$.
Therefore, each vehicle within the platoon tracks the leader behavior while maintaining the formation in a fixed-time $T^\star \leq T_{max}=T_1+T_2$, depending on the proper choice of the control gains. In so doing the statement is proven. ■

V. Numerical Results

In this section, the effectiveness of the proposed distributed sliding mode control approach in guaranteeing the fixed-time leader-tracking consensus is validated considering an exemplar heterogeneous platoon composed of $N=5$ vehicles plus a leader and leveraging the Matlab/Simulink© simulation platform. In our operating scenario we assume that the leading vehicle moves according to a trapezoidal velocity profile. Specifically, it drives at an initial velocity of 15 $[m/s]$. At $t=15$ $[s]$ it begins accelerating with a constant acceleration of 2 $[m/s^2]$ until reaching the constant velocity of 25 $[m/s]$. Then, at $t=32$ $[s]$ it starts decelerating with a constant deceleration of -1.9 $[m/s^2]$ until reaching the final constant velocity of 10 $[m/s]$. External disturbances are chosen as $w_1(t)=0.2\sin(0.5t)$, $w_2(t)=0.2\sin(0.1t)$, $w_3(t)=$

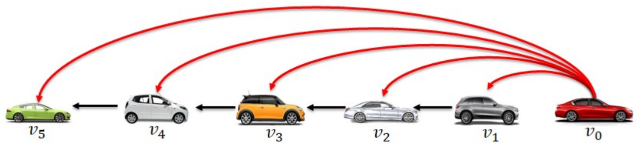


Fig. 1: Exemplar platoon of five heterogeneous vehicles plus a leader connected via the LPF topology.

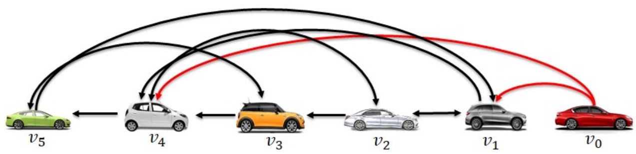
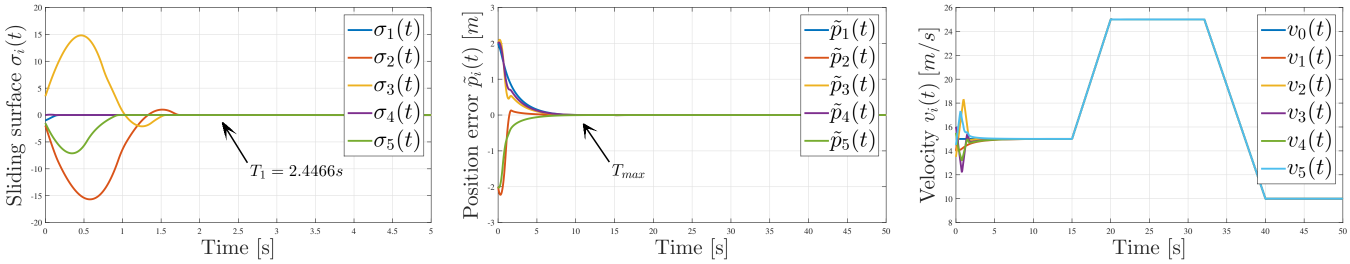


Fig. 2: Exemplar platoon of five heterogeneous vehicles plus a leader connected via a Random topology.

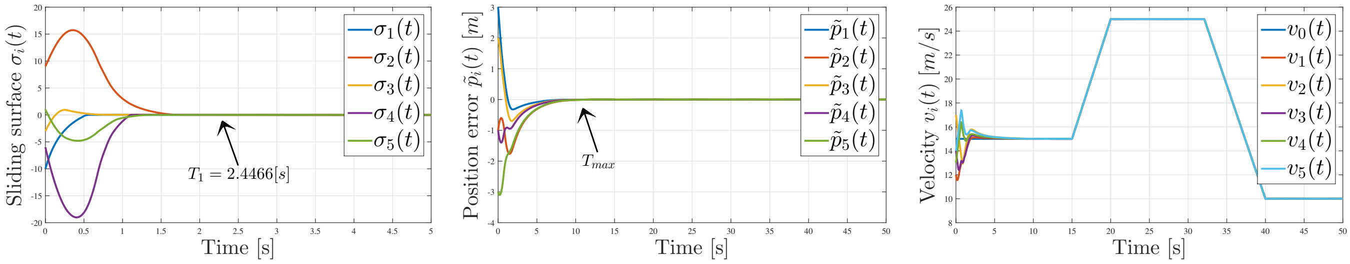
Mass m_i $[kg]$	$m_0=1400$, $m_1=1500$, $m_2=1445$, $m_3=1550$, $m_4=1200$, $m_5=1600$
Max acceleration $[ms^{-2}]$	5
Min acceleration $[ms^{-2}]$	-5
Desired spacing policy d_{ij} $[m]$	20
Control parameter p	0.5
Control parameter q	1.5
Control parameter γ_1	0.53
Control parameter γ_1'	1.85
Control parameter γ_2	0.7
Control parameter γ_2'	1.3
Control gains k_{i1} $[s^{-2}]$	0.1 $\forall i=1,\cdots,5$
Control gains k_{i2} $[s^{-1}]$	1.1 $\forall i=1,\cdots,5$

TABLE I: Vehicle and Control parameters.



(a) (b) (c)

Fig. 3: Leader-tracking performance under the control in (13). Leader-Predecessor-Follower topology scenario. Time history of: (a) sliding surface $\sigma_i(t)$ $(i = 1,2,3,4,5)$; (b) position error $\tilde{p}_i(t) = p_i(t) - p_0(t) - d_{i0}$ $(i = 1,2,3,4,5)$; (c) vehicles velocity $v_i(t)$ $(i = 0,1,2,3,4,5)$.



(a) (b) (c)

Fig. 4: Leader-tracking performance under the control in (13). Random topology scenario. Time history of: (a) sliding surface $\sigma_i(t)$ $(i = 1,2,3,4,5)$; (b) position error $\tilde{p}_i(t) = p_i(t) - p_0(t) - d_{i0}$ $(i = 1,2,3,4,5)$; (c) vehicles velocity $v_i(t)$ $(i = 0,1,2,3,4,5)$.

$0.3\sin(t)$, $w_4(t) = 0.6\sin(t)$ and $w_5(t) = 0.1\sin(0.1t)$.
The vehicle parameters as well as the control ones, selected according to Definition 1 and Lemma 1, are reported in Table I. To disclose the effectiveness of the control strategy and how it can ensure the leader-tracking in a fixed-time (which only depends on the proper choice of the control gains and not on any vehicles initial condition), here we consider two representative scenarios where different initial conditions as well as different communication networks are considered, namely: *i*) Leader-Predecessor-Follower (LPF) topology scenario (see Fig. 1); *ii*) Random topology scenario (see Fig. 2). Note that, the numerical analysis have involved other communication topologies. However, similar results have been obtained and, hence, they are here omitted for the sake of brevity.

A. *Leader-Predecessor-Follower topology scenario*

In the first driving scenario, vehicles are connected through the common Leader-Predecessor-Followers topology [2] (see Fig. 1), where each vehicle shares information with its predecessor and the leader. The position/velocity initial conditions for this scenario, as well as the control gains tuned according to Theorem 1 are listed in Table II.
Results in Fig. 3 confirm the theoretical derivation and disclose the effectiveness of the proposed fixed-time controller in guaranteeing the leader-tracking in a settling time $T^\star \leq T_{max} = T_1 + T_2 \approx 10$ $[s]$. Specifically, Fig. 3 (a) shows the time history of the sliding surface which converges to zero in a finite settling time $T_1 \approx 2.45$ $[s]$. Conversely, Fig.s 3 (b)-(c), disclosing the time histories of the position error $\tilde{p}_i(t)$ $(\forall i = 1,\cdots,5)$ and the vehicles velocities $v_i(t)$ $(\forall i = 0,1,\cdots,5)$, highlight that, once the manifold $\gamma_i = 0$ $(\forall i)$ is reached, each vehicle tracks the leader motion (see Fig. 3(c)) while maintaining the desired spacing distance (see Fig. 3(b)) in $T_2 \approx 7.55$ $[s]$.

B. *Random topology scenario*

In this operating scenario, we assume that vehicles are connected through the random communication topology depicted in Fig. 2, where the leader information is available just for a subset of vehicles, i.e. the first and fourth. The position/velocity initial conditions for this scenario, as well as the control gains tuned according to Theorem 1 are listed in Table III.
Fig. 4 confirms the theoretical derivation also for this operative scenario and disclose how, despite the changing of vehicles initial conditions and the network communication topology, the proposed control approach ensures the leader-tracking in the fixed-time $T^\star \leq T_{max} = T_1 + T_2 \approx 10$ $[s]$ (equal to one obtained into the L-P-F scenario). Note that this result

Initial position $[p_0(0),\cdots,p_5(0)]^\top$ $[m]$	$[100,82,58,42,22,-2]$
Initial velocity $[v_0(0),\cdots,v_5(0)]^\top$ $[ms^{-1}]$	$[15.0,14.0,13.5,16.0,15.5,14.5]$
Control gains $[\kappa_1,\cdots,\kappa_5]$ $[s^{-2}]$	$[5.7,5.94,6.14,6.8,6.06]$

TABLE II: Simulation parameters for the LPF topology scenario.

Initial position $[p_0(0),\cdots,p_5(0)]^\top$ $[m]$	$[103,86,62,45,21,0]$
Initial velocity $[v_0(0),\cdots,v_5(0)]^\top$ $[ms^{-1}]$	$[15,12,17,14,13,14]$
Control gains $[\kappa_1,\cdots,\kappa_5]$ $[s^{-2}]$	$[6.15,1.3,1.15,8.07,1.15]$

TABLE III: Simulation parameters for Random topology scenario.

is consistent with the theoretical derivation and proves the ability of the proposed distributed fixed-time controller in solving Problem 1 in a settling time which is fixed and independent from any initial conditions. Specifically, Fig. 4 (a) shows the time history of the sliding surface which converges to zero in a finite settling time $T_1 \approx 2.45$ $[s]$, despite the changing of initial conditions and the network connections. Conversely, Fig.s 4 (b)-(c), disclosing the time histories of the position error $\tilde{p}_i(t)$ $(\forall i = 1, \cdots, 5)$ and the vehicles velocities $v_i(t)$ $(\forall i = 0, 1, \cdots, 5)$, highlight that, once the manifold $\gamma_i = 0$ $(\forall i)$ is reached, each vehicle, under the action of the proposed controller, tracks the leader motion (see Fig. 4(c)) while maintaining the desired spacing distance (see Fig. 4(b)) in $T_2 \approx 7.55$ $[s]$.

VI. Conclusions

In this paper we have addressed and solved the fixed-time leader-tracking control problem for heterogeneous uncertain autonomous connected vehicles platoons via a distributed sliding-mode based control approach. Leveraging the fixed-time stability tools and the Lyapunov theory, we have analytically proved how the proposed distributed control strategy can ensure the leader-tracking in a fixed-time which only depends on the proper choice of the control gains, and not on any vehicles initial conditions. Moreover an estimation of this settling time has been provided. Numerical analysis, carried out considering two exemplar driving scenarios, have confirmed the theoretical derivation and have disclosed the effectiveness of the distributed fixed-time protocol in solving the leader-tracking problem in a finite settling time.

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