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Reviewed

 Article Cite Review PDF[Maezawa, Shun-ichi \(J-SUT\); Yazawa, Akiko \(J-SHINSGM-ST\)](#)**Special case of Rota's basis conjecture on graphic matroids.** (English summary)[Electron. J. Combin.](#) **29** (2022), no. **3**, Paper No. 3.63, 14 pp.

Classifications

[05C70 - Edge subsets with special properties \(factorization, matching, partitioning, covering and packing, etc.\)](#)[05B35 - Combinatorial aspects of matroids and geometric lattices](#)[05C05 - Trees](#)

Citations

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Review

In [R. Q. Huang and G.-C. Rota, *Discrete Math.* **128** (1994), no. 1-3, 225–236; [MR1271866](#)] it was conjectured that for given n bases B_1, B_2, \dots, B_n in a matroid of rank n , there exist n disjoint transversal bases of B_1, B_2, \dots, B_n . The matroids derived from graphs are very important and produce more general results in graph theory.

An *edge-colored graph* is a graph with an edge-coloring. For an edge-colored graph G , $C(G)$ denotes the set of colors used in G . An edge-colored graph is *rainbow* if no two edges have the same color. Rota's basis conjecture for graphic matroids can be stated as follows. Let G be an edge-colored connected multigraph with order $n \geq 3$. Suppose that $C(G)$ has $n - 1$ colors and the graph induced by the edges colored with c is a spanning tree for each color c . Then G has $n - 1$ mutually edge-disjoint rainbow spanning trees. This conjecture has not been completely solved yet.

In the paper under review, the authors take a new approach by considering constructions of edge-colored graphs, and solve the conjecture when edges colored with c induce a star for each color c (Theorem 3). This approach also plays an important role in solving the conjecture for general matroids. The problem addressed concerns the edge decomposition of an edge-colored graph into rainbow spanning trees.

There are some decomposition problems for the case in which the edge-colored graph is a complete graph such as the following conjecture by R. A. Brualdi and S. Hollingsworth [*J. Combin. Theory Ser. B* **68** (1996), no. 2, 310–313; [MR1417803](#)]: Let $m \geq 3$ be an integer and let K_{2m} be an edge-colored complete graph. Suppose that the graph induced by the edges colored with c is a perfect matching for each color c . Then the complete graph has m mutually edge-disjoint rainbow spanning trees.

In [R. A. Brualdi and S. Hollingsworth, *op. cit.*], this conjecture was solved when $m = 2$. Recently, the conjecture was solved for sufficient large m in [S. Glock et al., *J. Combin. Theory Ser. B* **146** (2021), 439–484; [MR4177962](#)] and the authors obtained a stronger conclusion than [R. A. Brualdi and S. Hollingsworth, *op. cit.*]. There are also other results about finding some edge-disjoint rainbow spanning trees in an edge-colored graph, but such studies for non-complete graphs are not yet developed.

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