PROMOTING ARGUMENTATION AS A SOCIALLY SITUATED PRACTICE THROUGH INFORMAL MATHEMATICAL PROJECT

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The paper presents a retrospective analysis of La Tombola Infernale, a project situated within Informal Mathematics Education and grounded in the Design-Based Research methodological framework. The project engages students in designing and playing a Dante-inspired tombola game, through which they explore both mathematical concepts and the Inferno canticle of the Divine Comedy. Drawing on Ferrari and Saccoletto's (2023) theoretical perspective on argumentation as a socially situated practice, the study investigates how this specific informal learning context supports the development of students' argumentative processes. The analysis focuses on one design cycle—implemented across different age groups and settings—highlighting the role of peer interaction, linguistic resources, and material artefacts in fostering collective meaning-making and the progressive formalisation of modular arithmetic.

INTRODUCTION

The Covid-19 pandemic emergency severely strained school systems worldwide and widened disparities in inclusion, participation, and meaningful learning among students from various socio-economic backgrounds (Bakker, Cai & Zenger, 2021). During this period, distance learning, characterized by screen-mediated relationships, exacerbated issues such as limited internet and digital device access. In Italy, the time dedicated to distance education was uneven, with regions like Campania experiencing over a year of online instruction. These growing inequalities prompted new educational experiments still referenced today to improve relationships within and outside schools. One such initiative is La Tombola Infernale Project (TIP), a Design-Based Research (DBR) study (Bakker, 2018; Cobb et al., 2003) within Informal Mathematics Education (IME) (Nemirovsky, Kelton, and Civil, 2017). Aligning with IME principles, it adopts an interdisciplinary approach merging Dantean literature and mathematics to bridge formal and informal teaching through mutualism, fostering participatory research to enhance math education (Eshach, 2007). This paper offers a retrospective review of a DBR cycle where students from primary to high school of unprivileged contexts explore modular arithmetic (mod , a key component of the project. The analysis underscores the role of linguistic resources in argumentative processes, as discussed by Ferrari and Saccoletto (2023).

METHODOLOGICAL FRAMEWORK

The TIP is developed within the DBR framework. This methodology involves studying long-term interventions that foster ongoing, cyclical processes of action and reflection involving researchers and teachers. These research cycles alternate between designing,

implementing, and analysing activities (Bakker, 2018). A key goal of the present research project is to bridge the gap between teaching practices and theoretical research in mathematics education, especially by aligning formal instruction with research on informal learning contexts (Eshach, 2007). Although the project has undergone several developments since its inception, the present contribution provides a study of the initial experimental phase. This phase, according to DBR approach, follows a single cycle comprising three stages: design, implementation, and analysis. The study retrospectively examines an introductory experimental activity aimed at integrating two theories of mathematics teaching and learning: IME (Nemirovsky et al., 2017), which serves as the *orienting framework*, and argumentation as a socially situated practice (Ferrari & Saccoletto, 2023) as the *domain-specific* framework. This leads to the main research question: *How does an informal mathematics education project, like TIP, affect students' explanations and argumentative activities when exploring new mathematical concepts?*

The orienting framework: Informal Mathematics Education

IME promotes mathematical learning in non-traditional, interest-driven contexts and supports knowledge construction through problem-solving, collaboration, and discussion, while reducing the emotional load of formal schooling. Its key features include the absence of traditional assessment, fluid disciplinary boundaries, and the use of alternative learning spaces, fostering engagement, communication, and argumentation (Nemirovsky et al., 2017). The TIP is situated within this framework and draws on the concept of Boundary Objects (Star & Griesemer, 1989), using Dante's Inferno as a cultural artifact to bridge mathematics and literature and to promote inclusive and meaningful learning. The Divine Comedy, rich in scientific and mathematical references, was chosen as a starting point to enable students to explore mathematical concepts and content from other disciplines through the material construction and reinterpretation of a popular game such as tombola.

The domain-specific framework: argumentation as a socially situated practice

Ferrari and Saccoletto (2023) propose a theoretical perspective that conceptualises argumentation as a linguistic and social process rather than as an isolated product. The authors adapt the argumentative theory of Mercier and Sperber (2017), originally developed within cognitive and evolutionary psychology, to mathematics education and situate their work within a social constructionist view of knowledge. In this perspective, argumentation emerges through interactions among students and between students and the teacher and includes not only the initial production of arguments but also reactions, objections, requests for clarification, and reformulations, all of which are inseparable from the participants and the context. Language is not merely a vehicle for argumentation but constitutes its very structure, as arguments are conceived as oral or written texts whose quality and effectiveness are deeply shaped by students' linguistic resources. They identify three main functions of argumentation in the mathematics classroom—*certifying, convincing,* and *explaining*—placing particular

emphasis on *convincing*, understood in relational terms, and *explaining*, conceived as the construction of intelligibility for others. By introducing the notion of *epistemic vigilance* (Mercier & Sperber, 2017), the authors highlight the fundamentally social function of argumentation: the teacher's role is reconceptualised not as an immediate arbiter of correctness, but as a facilitator of epistemic vigilance who supports students in critically evaluating, questioning, and negotiating claims, rather than passively accepting them. Applied to classroom settings, this perspective underscores the need to design learning environments in which peers are the primary audience for argumentation and in which questioning and critical engagement are legitimised.

DESIGN-BASED RESEARCH OF THE LA TOMBOLA INFERNALE PROJECT

TIP is an interdisciplinary educational pathway conceived by a third-sector association in collaboration with the Department of Mathematics and Applications "R. Caccioppoli" and the Department of Human Sciences of the University of Naples Federico II. The aim is to engage students in creating a thematic tombola game based on Dante's Inferno, conceived as the product to be developed during the DBR study (Bakker, 2018; Cobb et al., 2003). The activity was targeted at two student groups and implemented at two different times. Firstly, it involved 15 students spanning from third-year lower secondary to the first two years of upper secondary. It will constitute Group 1. Secondly, two fifth-grade classes participated, constituting Group 2. Some elements were defined at the outset of the pathway by the researchers and the association's mathematics teachers. The main steps common to both groups were: 1) Designing the equipment: constructing the main board and designing the tombola cards, through which students engaged in combinatorial reasoning; designing a container for the numbers in the shape of an upside-down truncated cone, mirroring Dante's Inferno (Fig. 1); and devising the rules of the game for this Dante-themed version of tombola; 2) Designing the interdisciplinary structure of the game: choosing a criterion for associating the 90 tombola numbers with an equal number of characters from the Inferno; selecting 90 infernal characters, and creating a special card for each of them, with a description of the character on one side and, on the other, a students' drawing of the character, a tercet from the Inferno about the character, and some arithmetic curiosities related to the associated number (Fig. 2).

Fig. 1: The containers shaped like a truncated cone with the circles of Hell.

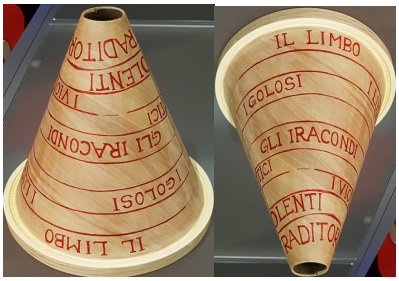
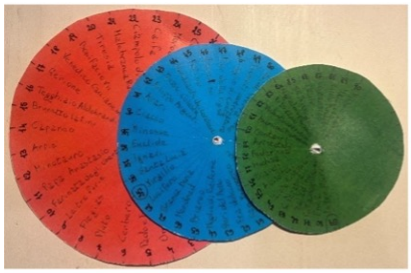


Fig. 2: An example of a character card, relating to Filippo Argenti

In agreement with the IME, these activities were carried out in various sites of naturalistic and cultural interest across the city. As noted at the outset, the project took place during the pandemic, when most schools in Naples were closed. Despite these constraints, all activities were conducted in person, in compliance with safety regulations, using parks, streets, gardens, and public squares as learning environments, thereby recognizing the city as an open-air learning space.

Concerning the mathematical content, within the DBR *domain-specific* framework of argumentation as a socially situated practice (Ferrari & Saccoletto, 2023), an initial experimental activity was implemented. This activity aimed to enable students to investigate the mathematical criterion used by the teacher-researcher designers to link the 90 tombola numbers with 90 characters from Dante's Inferno. The criterion relies on modular arithmetic modulo 34, as Inferno has 33 cantos plus an introductory one. The 90 characters were selected in the following way: three characters for each canto from I to XXII, and two characters for each canto from XXIII to XXXIV. Each character was assigned a number between 1 and 90, chosen so that the number is congruent modulo 34 to the canto number where the character appears. For instance, Medusa was linked to the number 77 because it is congruent to 9 modulo 34, and Medusa appears in Canto IX. For Group 2, many features changed compared to the Group 1: the sessions were no longer extracurricular, the work was carried out with the entire class rather than a heterogeneous group of students differing in age, grade, and school track, the students were younger, the lessons took place in the school classrooms, and the project was planned together with the teachers of the two involved classes. Within this context the project was replicated, but during co-design with the classroom teachers, it was decided to add a new fundamental element: three overlapping and concentric circular rotating cardboard discs, each divided into 34 circular sectors of equally size (Fig. 3). It works as a semiotic artefact (Bartolini Bussi & Mariotti, 2008) to support the understanding of the modular arithmetic-based criterion, for associating numbers with characters and, above all, for deducing the information that can be derived from this association. The artefact was built in this way: the outer red disc features numbers 1-34 along its perimeter, the middle blue disc 35-68, and the inner green disc 69-90, with the corresponding character names written radially. By aligning numbers 1, 35, and 69, each character matches the number of the Dantean canto in which they first appear, indicated on the outer circle.

Fig. 3: The artefact designed by the teachers for the Group 2.



Implementation

The implementation was organized into three phases for Group 1. In contrast, Group 2 only completed the first two phases, where the semiotic artefact played a central role, as will be discussed in the analysis section.

In Phase 1, students were prompted to imagine a criterion for linking the 90 tombola numbers with chosen characters from the Inferno, aiming for some inferential insight. Working in small groups, they devised different association criteria. Each group presented its idea, which was then collectively summarised and written on the board. The teacher, as a facilitator of epistemic vigilance, moderated a whole-class discussion to assess the validity of these criteria. In Phase 2, they were given a sheet showing the 90 characters already matched with the 90 numbers according to the modular arithmetic criterion, of which they were unaware, and were asked to study: i) the information provided by the number with respect to the character, ii) the criterion underlying the association. In Phase 3, the students were asked to present to an external audience the activities that led to the construction of the game—including the one described in this section—at two institutional events, held at the end of the project at the University of Naples Federico II. This phase fostered argumentation, as preparing presentations required explicit articulation, triggering deeper explaining processes (Ferrari & Saccoletto, 2023).

For Group 2, the artefact was crucial in revealing the core mathematics behind the TIP. It was first given to the children in a disassembled state, with the circles separated, a sample fastener for assembly, and no instructions. Through intuitive exploration, paired work, and class discussions guided by the teachers, students independently reassembled the artefact, learned how to operate it, and identified what information they could derive from the number and character links.

Analysis

In Phase 1, the students of the Group 1 first engaged in small-group and then in a whole-class mathematical discussion, proposed different possible criterions for associating the 90 tombola numbers with the selected Inferno characters. In particular, they proposed five possible criteria, which were written on the board by one student: (1) number of occurrences of the name of the characters, (2) number of sins, (3) physical characteristics, (4) number associated to the historical period of the characters, (5) number of the circle in which the characters appear.

The teacher and the students commented on all the proposed criterion, and during the discussion, activating a collective epistemic vigilance, decided which must be rejected marked them with a no on the board: (1) the idea of using the number of occurrences of each character as the associated number was rejected because multiple characters could share the same number of occurrences, (2) the same argument was used to reject the number of sins as a possible criterion, (3) the proposal to use the number of relevant physical characteristics was discarded because it was difficult to apply and risked lacking objectivity, (4) the criterion based on the historical period was also rejected, as it did not provide a clear, unambiguous ordering for all characters. In contrast, (5) The number of the circle was also rejected as a criterion, since, in this case as well, multiple characters within the same circle would share the same number. During this mathematical discussion, the activation of epistemic vigilance (Ferrari & Saccoletto, 2023) within the informal setting facilitates a more genuine debate than in typical classroom contexts. In the latter, numerous researchers (see e.g. Plamm & Pins, 2024) highlight how students tend to rely on the teacher's statements or textbook content, and how the teacher's authority often overshadows critical debate.

In Phase 2, Group 1 using a sheet that showed 90 characters already matched with 90 numbers according to a modular arithmetic criterion (unknown to them), first worked in small groups, then took part in a teacher-led whole-class discussion to compare their preliminary ideas about (i) the information given by the number relative to the character and (ii) the criterion underlying the association. As in Phase 1, the collective understanding that emerged was sketched on the board by a student. Comparing their reasoning with peers appears to promote shared validation of conjectures developed independently by each group. Students identified both the criterion and the information provided by the number—specifically, the character's canto—using arguments effective for convincing and explaining (Ferrari & Saccoletto, 2023). They first observed and wrote on the board: "Each character from 1 to 34 is associated with the canto to which it belongs." They then added: "Upon reaching the 34th character, to begin the second column of characters, we add 34 to the obtained number and For the third column, we add 34 to the number obtained in the second column (e.g., 34 + 35 = 69)." Their explanations of congruence modulo 34 reveal a cyclical vision of the relationships through 34-column patterns.

In Phase 3, in the Group 1, students' institutional presentations show a shift toward more rigorous language. For instance, student A. explained the criterion to the audience through the example of Saladin (no. 72):

> A.: […] If we divide 72 by 34 […], we obtain 2 with a remainder of 4. And it is precisely in the fourth canto that we encounter Saladin.

Unlike the expressions in which a cyclical view via 34-row columns emerged at the end of Phase 2, student A. properly used the word "remainder" for the number 4, catching another vision of the congruence modulo 34 in terms of the remainder of the division of 34. This transition illustrates a move toward greater formalization of

language, aimed at making the mathematical content more accessible to the audience. Moreover, student A. chose to use an example in order to be even more explicit and persuasive, in which it is possible to recognize the *explanatory function* (Ferrari & Saccoletto, 2023).

Fig. 4: The disc with the inscription "Canto Reader" added by the students.

The analysis of the activity in Group 2 focuses on the processes of mediation of the semiotic artefact (Bartolini Bussi & Mariotti, 2008) and on how that element activated functions of argumentation. The active discovery approach led the children to develop various ways of using the artefact and to name it accordingly as the "Canto Reader" (Fig. 4). However, it is worth noting that the initial version of the proposed artefact did not include the white disc. During the activities, two students suggested adding it in order to cut out a circular sector, so that the empty space could be used to indicate a specific number and to show how the three numbers on the three discs, congruent modulo 34, are associated with three characters appearing in the same canto — namely, "*in the number Inferno canto signed on the red disc*," as one of the students stated while manipulating the artifact. The proposal to add the white disc to the artefact can therefore be interpreted as serving the students' need to enact the *explaining* and *convincing functions* theorized by Ferrari and Saccoletto (2023).

CONCLUSIVE REMARKS

The contribution explores the potential of TIP and illustrates how an Informal Mathematics Education setting, grounded in a Design-Based Research approach, can foster rich argumentative practices around a novel mathematical concept for students, such as modular arithmetic. By mobilising a culturally resonant context such as Dante's Inferno and introducing boundary objects such as the thematic tombola and the rotating discs, the project created conditions in which students' explanations and arguments emerged as socially situated, linguistically mediated activities. Across both Groups, the emphasis on epistemic vigilance and peer-oriented discussion supported the transition from intuitive, non-formal strategies to more rigorous mathematical language, as exemplified by Group 1 students' evolving ways of expressing congruence modulo 34. Moreover, Group 2 students' active refinement of the artefact—such as the proposal to add the white disc—highlights their need to perform explanatory and persuasive functions in line with Ferrari and Saccoletto's (2023) framework and shows how design choices can be shaped by learners' argumentative

purposes. These findings suggest that integrating IME principles with a focus on argumentation as a socially situated practice can contribute to more inclusive, participatory, and conceptually meaningful mathematics learning experiences, particularly in contexts marked by educational inequalities. Future research may extend this work by exploring how subsequent DBR cycles can consolidate and generalise these results across different mathematical domains and school settings. Finally, we want to stress how the pandemic deeply affected many lives, particularly those of adolescents, exposing widespread vulnerabilities and highlighting the fragility of the education system. This experience underscores the need for meaningful educational innovation also for reimagining education beyond the traditional classroom, recognizing the cities as an open-air learning space capable of sustaining in-person learning even in times of crisis.

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