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On chromatic symmetric homology and planarity of graphs. (English summary)

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Classifications

- **05C31 - Graph polynomials**
- 05C10 - Planar graphs; geometric and topological aspects of graph theory
- 05E05 - Symmetric functions and generalizations
- 20C30 - Representations of finite symmetric groups
- 55U15 - Chain complexes in algebraic topology

Citations

From References: 2
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Review

In [Adv. Math. **111** (1995), no. 1, 166–194; MR1317387], R. P. Stanley defined in the chromatic symmetric function of a graph, a remarkable combinatorial invariant which refines the chromatic polynomial. In [J. Combin. Theory Ser. A **154** (2018), 218–246; MR3718066], R. Sazdanović and M. Yip defined a new homological theory, called the chromatic symmetric homology of a graph G, based on the previous invariant. It is the homology of a chain complex involving graded representations of the symmetric group, associated to every subgraph of G with the same vertices as G, with respect to a suitable differential. This construction is inspired by Khovanov's categorification of the Jones polynomial [D. Bar-Natan, Algebr. Geom. Topol. **2** (2002), 337–370; MR1917056].

Chromatic symmetric homology is a strong invariant since it can distinguish pairs of graphs that have the same chromatic symmetric function. This is proved in [Adv. in Appl. Math. **150** (2023), Paper No. 102559; MR4604797], where A. Chandler et al. also studied properties of chromatic symmetric homology with integer coefficients and provided examples of graphs whose chromatic symmetric homology has torsion, leaving open the following conjecture: A graph G is non-planar if and only if its chromatic symmetric homology in bidegree $(1,0)$ contains $\mathbb{Z}_2$-torsion.

In the paper under review, this conjecture is partially proved, namely: for a finite non-planar graph G, the chromatic symmetric homology in bidegree $(1,0)$ contains $\mathbb{Z}_2$-torsion (Theorem 3). The strategy used by the authors is based on the application of Kuratowski's theorem: they show that the torsion elements in the homology of the complete graph K_5 and of the complete bipartite graph $K_{3,3}$ are mapped to torsion elements in the homology of the graphs that are obtained from them by the operations of edge subdivision and graph inclusion, i.e., all the non-planar graphs.

Reviewer: Ciampella, Adriana

References

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This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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