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Training and assessing numerical abilities across the lifespan with intelligent systems: The example of Baldo

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Abstract

Baldo is an intelligent system that can be used to assess and train numerical abilities across the lifespan, especially for children and elderly. Baldo includes digital and tangible materials to propose activities, built on the game-based learning approach, strongly grounded on well-known and experimentally supported theories about numerical and mathematical cognition. Moreover, Baldo has a special module devoted to motivational and affective dimensions related to mathematic cognition and a tutoring system to select specific activities that are adequate to the actual and potential level of the people involved in the training pathway.

KEYWORDS
assessment, game-based learning, numerical cognition, training

1 | INTRODUCTION

For humans, it is important to encode and exploit information about numbers for their adaptation: indeed, numerical cognition is considered one of the four core knowledge systems at the foundation of human knowledge (Spelke & Kinzler, 2007), that are functional to face evolutionary challenges in dynamic environments. Dealing with numbers is not a matter for humans only: many other species (Boysen & Capaldi, 2014; Davis & Pérusse, 1988) are able to encode and exploit numerical information without any instruction about it. This indicates that there are some shared abilities, probably innate and some other skills, for example arithmetic ones that can be acquired with the proper instruction by human beings.

Counting, for example, is a numerical ability that can be observed in many species: extracting numbers from experience, distinguishing major from minor quantities, showing surprise if a calculation is wrong, distinguishing in a rapid and accurate way a small amount of objects and elements, the so-called subitizing (Kaufman et al., 1949), have been demonstrated in dolphins, ravens, rats, ants, bees, chimps and other species (Rambaugh et al., 1987). These mechanisms represent the common basis of numerical cognition and are not guided by conscious operation: they are shared by human adults, human infants and nonhuman species. Some studies suggest that this shared system to process numerical information becomes, in humans, the basis to build on more sophisticated strategies to deal with numbers, for example in the form of symbols. After formal instruction, humans can treat numbers in different forms, non-symbolic, symbolic and verbal. The well-known theory by Dehaene (Dehaene & Cohen, 1995) analyzes the connections between these different codes (mainly analogic, symbolic and verbal) and indicates that the ability to shift between them can be a pre-requisite for advanced mathematical knowledge.

This is important if we consider that children often encounter difficulties in learning different aspects of arithmetic and, because of this, develop a sense of frustration or even anxiety about maths (Ashcraft & Krause, 2007; Ashcraft & Ridley, 2005; Young et al., 2012). For this reason, there is a growing interest in interventions to promote the acquisition of basic numerical skills to reduce later mathematical difficulties (Caviola et al., 2012; Re et al., 2014). It is possible to promote these abilities by using digital and technological tools (Benavides-Varela

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et al., 2020). Moreover, it seems relevant to try to preserve these abilities at a later age, as a protective factor against intellectual decay (Arcara et al., 2017), as they help elderly people to keep their cognitive abilities as long as possible.

In this article, we describe an integrated digital and physical system, Baldo that relies on the findings and theories about numerical and arithmetical cognition, together with the related emotional aspects, to assess and train numerical abilities in the form of a game.

2 | NUMERICAL AND MATHEMATICAL COGNITION IN HUMAN DEVELOPMENT

Considering the development of numerical cognition in human beings, it is possible to trace a pathway that starts from the basic abilities (innate and shared with other species) and goes towards the formal education in school context that starts later during infancy. Many possible outcomes in numeric, and later mathematical, achievement are possible, even if the starting point is comparable: differences in math achievement are consistently reported at different age, between genders (Spelke, 2005) and in different cultures (Stevenson et al., 1993). These observed differences cannot be related to the presence or absence of the natural endowment to deal with numeric knowledge, as it is shared between humans (and other species), so it is likely that they depend on other factors, probably related to cultural and socio-cognitive factors, including education.

The theory of innate numerical abilities (Butterworth, 1999; Girelli et al., 2000), stated that humans have got, since the first weeks of their lives, innate numerical skills to classify small sets of elements (4–5 items); later, the culture teaches us how to use the numbers competency in a more advanced manner. Evidence indicates that human infants possess an intuitive sense of number, the so-called number sense (Dehaene, 2011). It relates to the approximate number system (ANS) (Gilmore et al., 2011, 2014; Halberda & Feigenson, 2008): a cognitive system that supports the estimation of the magnitude of a group with more than four elements without relying on language or symbols, together with the parallel individuation system or object tracking system for smaller magnitudes. Toddlers are sensitive to numerical properties, and they can detect differences between small numbers and have quantity expectations: they can be disappointed or astonished if they observe weird numerosity variations. According to Nys et al. (2013), humans share ANS with other species and it is the basis on which formal maths is acquired: there is also a reciprocal effect, as ANS benefits from formal instruction that provides strategies, procedures and so forth.

On the other side, it is entirely plausible that children can have numerical abilities at a varying degree: this can offer therefore different starting point to develop more abilities that are sophisticated. Number sense in infancy is strongly correlated to math skills in childhood: it is possible to predict math achievement starting from the number sense at very early ages. In an interesting study by Starr and colleagues (Starr et al., 2013), authors indicate that the number sense, before language acquisition, 'may serve as a developmental building block for the uniquely human capacity for mathematics'. In their experiment, they show that the performance on numerical preference scores at 6 months of age is correlated with math test scores at 3.5 years of age. The number sense can be an outstanding facilitating element in the acquisition of numerical symbols and mathematical abilities.

This study by Starr and colleagues traces another interesting and useful connection between number sense and mathematical abilities, because it clearly indicates that a stronger number sense predicts later stronger numerical abilities. This means that specific educational interventions, addressed to strengthen the number sense can improve mathematical achievement later.

2.1 | Pre-requisites for mathematics in education contexts

The basic abilities to deal with numbers can be the pre-requisites for mathematical and arithmetic abilities. Literature highlights consistently the importance of pre-requisites of calculation for the success in primary school and beyond (Hindman et al., 2010; Romano et al., 2010). The study by De Smedt et al. (2013) indicates that symbolic and non-symbolic numerical magnitude processing skills are correlated to individual differences in children's mathematical skills. For symbolic numbers (digits), weak performance correlates with low math achievement and dyscalculia whereas for non-symbolic formats (dots) conflicting findings at neurocognitive level have been reported: brain activation during number comparison correlates with children's mathematics achievement level. *Pre-requisites* and *later achievement* correlation opens various and complementary reflections. First, moving from pre-requisites to advanced math skills implies to rely more strongly on symbolic representation and less on symbolic ones, also the embodied elements are neglected in favour of symbolic ones. In kindergarten, children use their fingers or physical objects to count, but quickly maths becomes something abstract, mainly relying on working memory (Ashcraft & Kirk, 2001). In some case, this change can generate difficulties in treating numbers.

Moreover, encountering difficulties in maths can lead to negative association with numbers and calculus before school and this can even generate negative feelings towards math. A rich line of research has indicated that mathematics and related subjects can generate mathematical anxiety, a feeling of tension or fear, which appears when a person is faced with mathematical content (Morsanyi et al., 2016). It is still not clear what generates anxiety for math. A possible explanation can be found in the educational methodology, which becomes immediately too abstract, as introduced earlier.

These considerations lead us to focus on how to intervene: if pre-requisites can be strengthened and positive feeling towards math can be stimulated, an adequate and personalized intervention can enhance school achievement in math. We will focus on some examples of these interventions in the next paragraph.

2.2 | Fluid mental abilities in age-associated decline

In human experience, it is possible to observe a cognitive decline, that is non-pathological, associated to aging (Deary et al., 2009). Starting from early adulthood there is a decay in functions such as reasoning, memory, and executive functions that becomes more and more to be considered if we consider that population in Europe is becoming older. Even in absence of pathologies such as Alzheimer or dementia, there is a reduction of performance in cognitive tasks. Moreover, this issue can be difficult to study for the variability between people, the study design (cross-sectional) and the overlapping with undiagnosed or early pathologies.

Considering numerical abilities, there is a little age-associated decline, as happens for verbal ability and general, crystallized knowledge, but memory, executive functions, processing speed and reasoning start declining since middle age onwards (Hedden & Gabrieli, 2004; Park & Reuter-Lorenz, 2009): these functions, the fluid mental abilities, play a core role in keeping autonomy in everyday life. Moreover, when a single fluid ability decline, there is a tendency of the others to decline as well (Wilson et al., 2002).

Even if it is not clear which variables have an effect in preventing age-correlated decline (Lampit et al., 2014; Kuider et al., 2012), cognitive training has often been reported to improve cognitive performance.

In addition, in this case interventions can have a relevant effect in protecting fluid mental abilities, as we will see in the next paragraph.

3 | INTERVENTIONS TO SUPPORT NUMERICAL ABILITIES IN CHILDREN AND FLUID MENTAL ABILITIES IN ELDERLY

The chance to propose interventions to support the acquisition or improvement of numerical abilities has been extensively explored, leading to a wide number of studies (see Kadosh et al., 2013). The already cited work by De Smedt et al. (2013) provides neurocognitive data useful for educational interventions, which can have positive effects on children's numerical development both in typical and atypical populations. Following these indications, interventions to train the magnitude processing are likely to improve maths achievement. Obersteiner et al. (2013) focused on the exact and approximate number representation: also, in this case an appropriate training can enhance mathematical performance. The study by Mussolin et al. (2012) also addresses the dynamic between the ANS and symbolic representation, which have both correlations with mathematical abilities that are later acquired: they can be therefore the starting point for educational interventions. Park et al. (2016) investigated non-symbolic approximate arithmetic training and shows that it improves math performance in pre-schoolers: readiness, a multidimensional concept that identifies the competences that a child needs before entering school (Snow, 2006), can be decisive is improved by a training with a tablet on ANS. Libertus et al. (2016) found that verbal estimation variability, but not estimation accuracy, predicted formal math abilities even when controlling age, expressive vocabulary and ANS precision variables. It mediated the connection between ANS precision and overall math ability: variability in the ANS: number word mapping can be especially important for formal math abilities.

In another study led by the same authors (Libertus et al., 2013), it is shown that numerical approximation abilities correlate with and predict informal but not formal mathematics abilities. In the systematic review by Chen and Li (2014), the association between individual differences in non-symbolic number acuity and math performance is analysed showing that there is a connection. Other interventions propose different approaches, for example, Maertens et al. (2016) used training with numbers line, which is commonly used in primary schools in Italy. Some of the proposed interventions use Information and Communication Technologies tools: the study by Caviola et al. (2016) reports an example of computer-based interventions that can improve grades in math whose effect depend on the age. Sella describes with his colleagues (Sella et al., 2016) the videogame Number Race to favour the acquisition of numerical abilities and Praet shows the effectiveness of videogames. The importance of a precocious intervention is underlined by the already cited work by Kadosh et al. (2013) who describe various computer-based approaches. Lonnemann et al. (2011) studied the symbolic and non-symbolic distance effects in children and their connection with arithmetic skills. Data indicated that the relationship between numerical magnitude comparisons and arithmetic skills is not limited to symbolic stimuli, but that it can also be detected for non-symbolic dot patterns: both symbolic and non-symbolic number representations have a relevant role in the development of arithmetic skills.

In addition, for elderly people, computer-based interventions and training proved to be effective for executive functions (López-Martínez et al., 2011), memory (Flak et al., 2014) and so forth. These examples indicate that it is possible to improve numerical cognition and fluid mental abilities adopting a Game-Based and Technology-Enhanced Learning Approach that aims at stimulating the approximate number representation, the symbolic representation and the shift between the different systems.

4 | BALDO: AN INTELLIGENT GAME FOR TRAINING AND ASSESSING NUMERICAL ABILITIES

In this section, we describe a system with physical and digital card games named Baldo that has the same logic structure of Italian gaming cards but is also based on well-established theoretical contributions that we will briefly introduce.

4.1 | The theoretical base of Baldo

In the last years, the model on numerical cognition proposed by Dehaene (1992) has affirmed as the leading one. According to this model, called the triple code model, there are three representational codes for number: Arabic digits, verbal number words and analogue non-symbolic magnitude representations. These systems are different in many respects, including functional and neural level (Figure 1).

The visual Arabic number form corresponds to the numeric symbol, for example, 'four', the auditory verbal word form corresponds to the word 'four' and the analogue non-symbolic magnitude representation corresponds to the quantity, for example, four squares. The Arabic number code is involved in written exact arithmetic that is formally taught at school; the analogue non-symbolic magnitude representation is connected to the approximate number representation and allows to compare quantities and make approximate arithmetic; the verbal number representation is connected with counting and mental exact arithmetic (mental calculation). The capacity to deal with these codes and shift between them is strongly connected to numerical and mathematical abilities and can be the core for educational issues, as we have seen in the introduction. Considering the triple code model, the transition between the approximate number representation and the Arabic and verbal number representation can be uneven. In the work by Odic et al. (2015), it is shown how children map between number words and the ANS: the mapping between the ANS and verbal number representations is not functionally bidirectional in early development, and moreover the mapping direction from number representations to the ANS is established before the reverse. This indicates that the relation between words and corresponding numbers is built along development. The study led by Kolkam (Kolkman et al., 2013) indicates that for learning math there are important pre-requisites that are non-symbolic quantity skills, symbolic skills and the mapping between number symbols and non-symbolic quantities. These precursors develop in an interrelated way that may affect later math achievement, especially symbolic and mapping skills. Xenidou-Dervou et al. (2017) also focus on pre-requisites and underline that non-symbolic and especially symbolic magnitude comparison skills are important longitudinal predictors of mathematical achievement, even more that IQ and working memory. ANS, together with working memory and attention can be an element of a multiple functional components that can lead to ineffective numerical problem solving and dyscalculia (Fias et al., 2013).

Pre-requisites for math can be trained and the training should involve embodiment: Kiefer and Trumpp (2012) indicate that embodiment that can be stimulated using tangible materials, as cognition is embodied as it is based on perception, proprioception and experience. Cognitive functions are grounded in action and perception as a function of experience, including numerical and mathematical cognition: this has relevant implications for education because it indicates that appropriate sensory and motor interactions during learning are crucial for the human cognitive development.

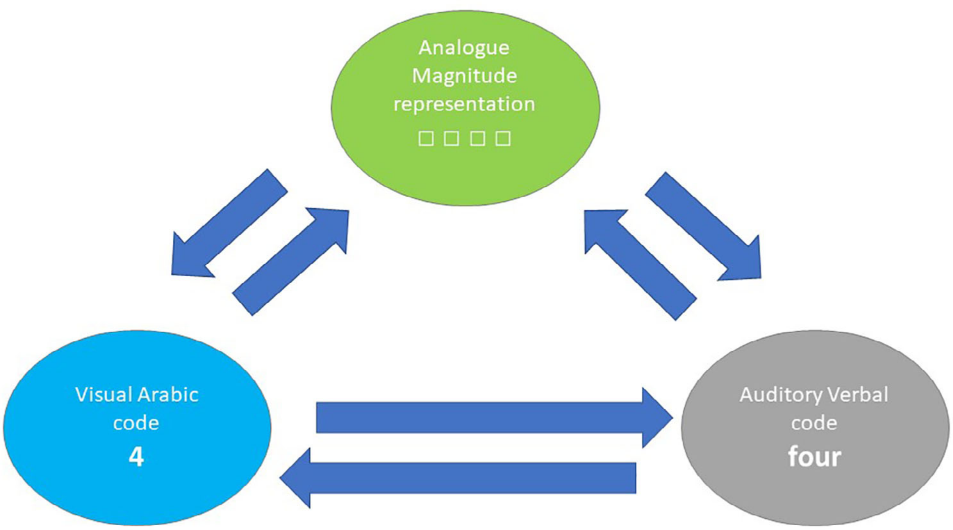

FIGURE 1 A schematic representation of triple code theory by Dehaene

4.2 | The game-based learning approach to improve math pre-requisites

Many recent studies indicate that game-based learning (GBL) can be effectively applied to the improvement of numerical cognition, as it has some relevant features that are particularly useful for this goal. Playing games implies learning as the players are forced to deal with goals, rules, adaptation, problem solving, interaction, that are often proposed to the learner in the form of a story (Dell'Aquila et al., 2016; Ponticorvo et al., 2017; Ponticorvo, Schembri, et al., 2019).

What is relevant for numerical cognition is that games are able to offer enjoyment, involvement, motivation, creativity and social interaction while conveying some useful information or content (Tobias et al., 2014). Moreover, it is possible to include tangibles elements to train numerical abilities (Ferrara et al., 2016). If we consider what has been discussed in the previous section, it is clear that some features of games and digital games, can be particularly useful in affecting positively learning, but also in producing positive feelings about maths and related subjects, including motivation and engagement (Abdul Jabbar & Felicia, 2015; Alsawaier, 2018) and increased time spent on task that improves learning outcomes (Karweit, 1984).

For math, this also means that the player (learner) will be motivated to practice also outside formal context. Jirout and Newcombe (2015) underline that this can be useful, as happens with semantic and linguistic skills: whereas language skills acquisition is supported at home by caregivers, math skills are stimulated in school context only. Nevertheless, if a little hint is given, for example, in the form of a mobile app, math skills improve. This means that skills related to mathematical achievement can be trained also in early childhood, even before school entrance, also in-home context (Berkowitz et al., 2015). Moreover, using games, players learn not buy being told, but by doing they experiment experiential learning that can help to remain focused, as learning actively makes less likely to become bored and to be more engaged emotionally. Moving to cognitive functions Drigas and Pappas (2015) report representative studies evaluating the effects of video games on mathematics achievement, memory, attention and cognitive skills. These studies indicate that video games can be useful in mathematics education, supporting the comprehension of fundamental concepts and motivating them to see positively mathematics. Generating positive feelings is the complementary pathway to promote mathematical achievement.

In the framework of GBL, these tools can be used for assessing mathematical abilities or related abilities such as reasoning (Ferrara et al., 2016) and soft-skills (Marocco et al., 2015) and emotional at different ages, becoming game-based assessment. In GBL, also for assessment, the game dimension allows to involve people so as to assess them on a longer timescale; it allows to observe people behaviour while they do something, rather than asking for self-reporting.

Playing games reduces the gap in numerical knowledge of children with different socio-economic status (Ramani & Siegler, 2011): the GBL approach, in this case, increased number line, magnitude comparison, numeral identification and arithmetic learning.

4.2.1 | Playing cards as a GBL approach to improve numerical abilities

Playing cards is a kind of games that can exploited to assess and improve numerical abilities in a framework of GBL and Game-based assessment (Kim & Shute, 2015; Mislevy et al., 2014, 2016; Oblinger, 2006).

Indeed, many studies indicate that board games and card games are effective methods to entertain children and for elder people playing cards is a very common activity associated to free time and amusement. The activity of playing cards trains several cognitive functions, such as memory, strategic thinking ability and theory of mind. Card games can be effective in stimulating brain and helping to fight memory loss (Fairfield et al., 2015). As we have hinted at before, playing cards, obviously, implies counting and dealing with number representation: for this reason, it can used for training and assessing numerical abilities. We therefore designed Baldo, the prototype we introduce in the next sub-section, to train numerical abilities using a card game with both digital and tangible elements, thus adopting a GBL and agent-based (Ponticorvo, Dell'Aquila, et al., 2019) approach.

4.3 | Baldo description

We developed Baldo, a card game named that follows the same logic structure of Italian gaming cards but is also based on the triple code model by Dehaene (2011). In the deck of cards, the three codes (textual, numerical and analogical) are mixed (Figure 2).

The numbering starts at zero and reaches none: This is linked to the decimal notation and is a small destabilization to stimulate in the minds of players a constant workout. There are special cards (basic arithmetic operators such as $+$, $-$, $\times$, / and $=$) that can be used to propose questions and mathematical problems on the table. Baldo exist in two versions: physical and digital.

In the Baldo physical side, the tangible cards allow active manipulation of concrete objects, which is relevant, in accordance with the Embodied and Situated Cognition (Clark, 2008) on the psychological side and to Montessori (2013) on the pedagogical side, to favour learning. In the

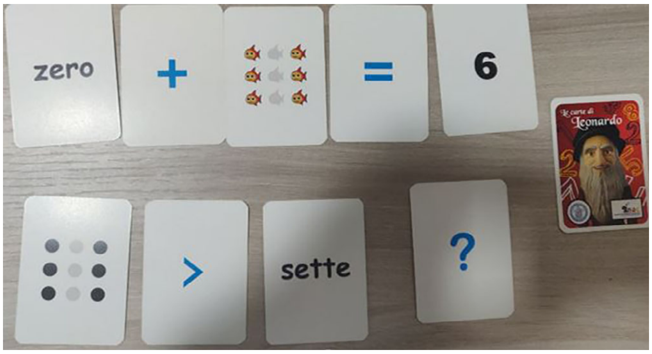

FIGURE 2 Baldo physical cards

FIGURE 3 A screenshot from Baldo digital app

digital version of the card game (Figure 3) players can choose between different decks (mixed codes, traditional card decks etc.) and play against an artificial agent trying different card games. These games include games from the Italian tradition.

The virtual opponent has an emotion module, able to adapt his facial expressions with the progress and his score in the game. This enables to consider the emotional dimension of the game and the connections between cognitive and emotional dimensions. In the digital version of Baldo, along with the card games to be played in multi-player mode, there is also a single-player game that consists in selecting the card with the highest value between two cards. It records the speed of the selection on 20 attempts. The application allows to test people, including children, on the shift speed between the different codes which is, as reported above, a pre-requisite for advanced mathematical skills. Together with assessment, this game is also useful to train the ability of moving between analogic representation, connected with the natural endowment described above and the symbolic one, connected to formal education in maths. During the gameplay, the player is forced to make rapid mental calculation with different codes, thus exercising in a fun context even without realizing it.

The artificial agent in Baldo selects the card to be played through an algorithm based on the Monte Carlo Tree Search (Figure 4). It simulates a bunch of hands and decides to play the card that could maximize its game.

In Baldo, digital app there is also an affective computing module, as our goal was to make the artificial intelligence component more flexible on the emotional side. Moreover, it makes possible to investigate the interaction between cognitive and emotional side. The module is represented in Figure 5

The game starts and the player plays according to turns; the emotional states varies if it is a first move or a response move and this interaction goes on until the end of the game. Depending on the game, the player must exercise the shift between different codes and determine a strategy according to the cards, the moment of the game and the overall strategy. This represents an effective assessment and training for numerical abilities and to protect fluid mental abilities, engaging both for children and for elderly people.

Considering Baldo intelligent system, it is conceived as an environment with interacting agents (Ponticorvo et al., 2017; Ponticorvo, Dell'Aquila, et al., 2019), where there is an artificial agent that replicated the actions of a psychologist/educator/parent or peer that must be aimed at encouraging/supporting the other player, the human player. This continues to engage pleasantly in the game and, consequently, to practice in an educational and/or rehabilitative card game. In our case, the psychologist's ability to successfully conduct the aims of the psycho-pedagogical intervention is based on two large families of psychological functions:

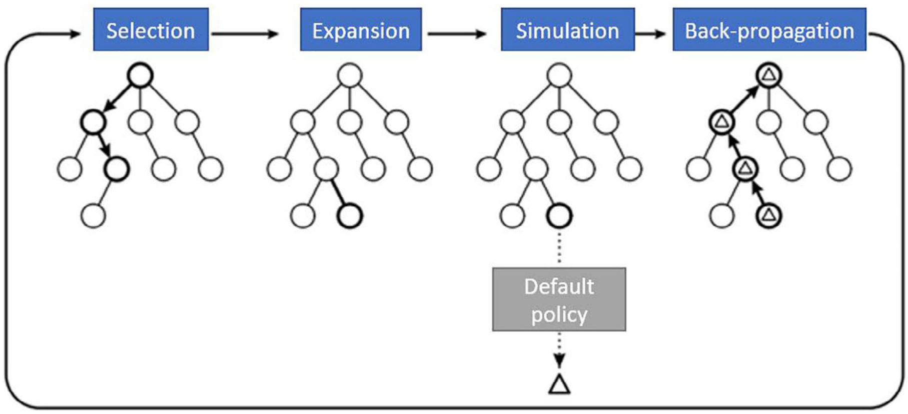

FIGURE 4 The algorithm for cards selection in Baldo

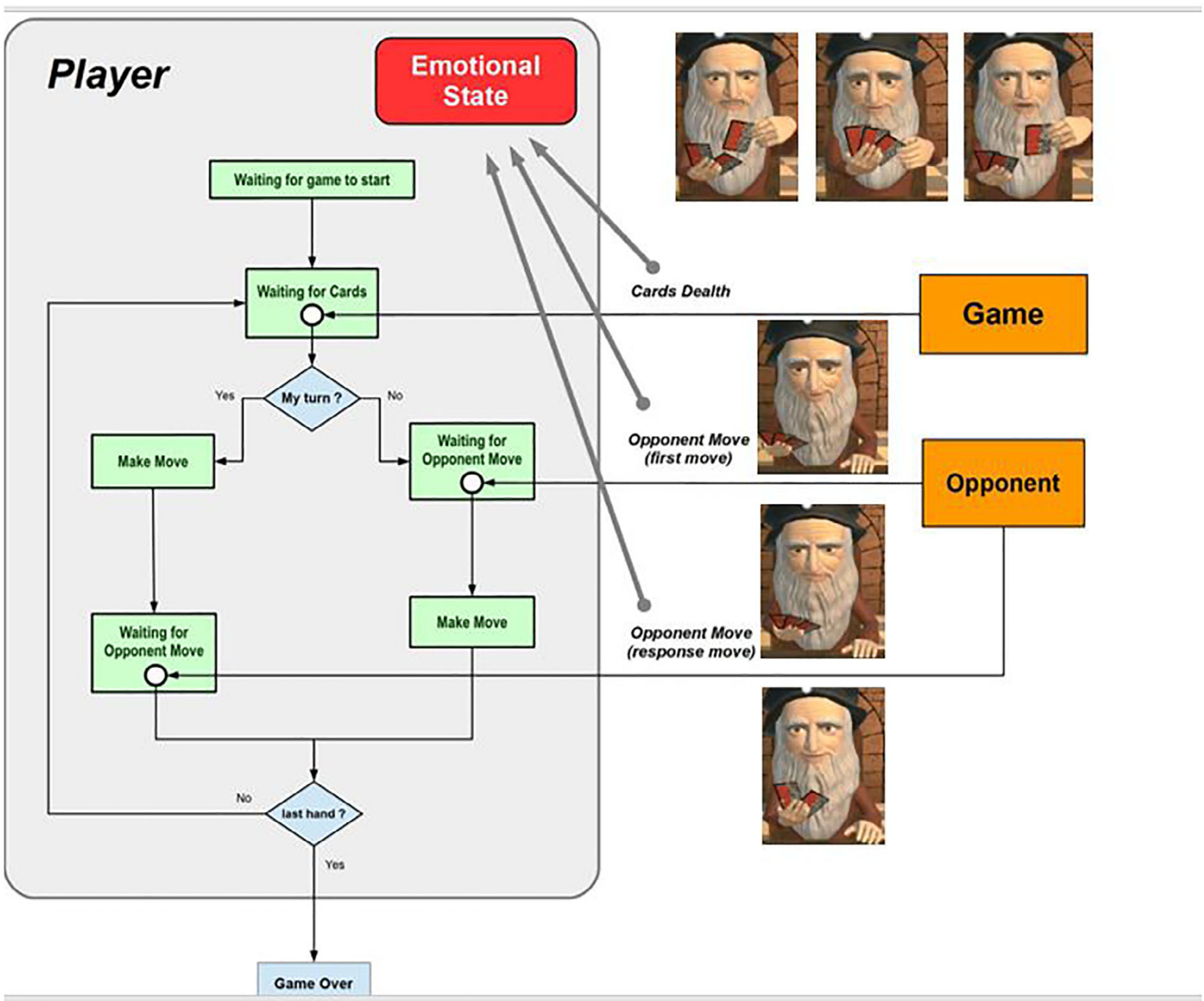

FIGURE 5 Baldo game flow

1. cognitive functions, that is the ability to reason in order to play the game of cards adequately, and therefore to make the best use of structured tools (cards): this relate to numerical abilities and fluid mental abilities;
2. psycho-social functions, that is, the ability to interact with the human player during the game also from a relational and emotional point of view. That is, the artificial agent must be able to use tools to express its emotional state and other psychological dimensions such as their own personality profile or empathic abilities.

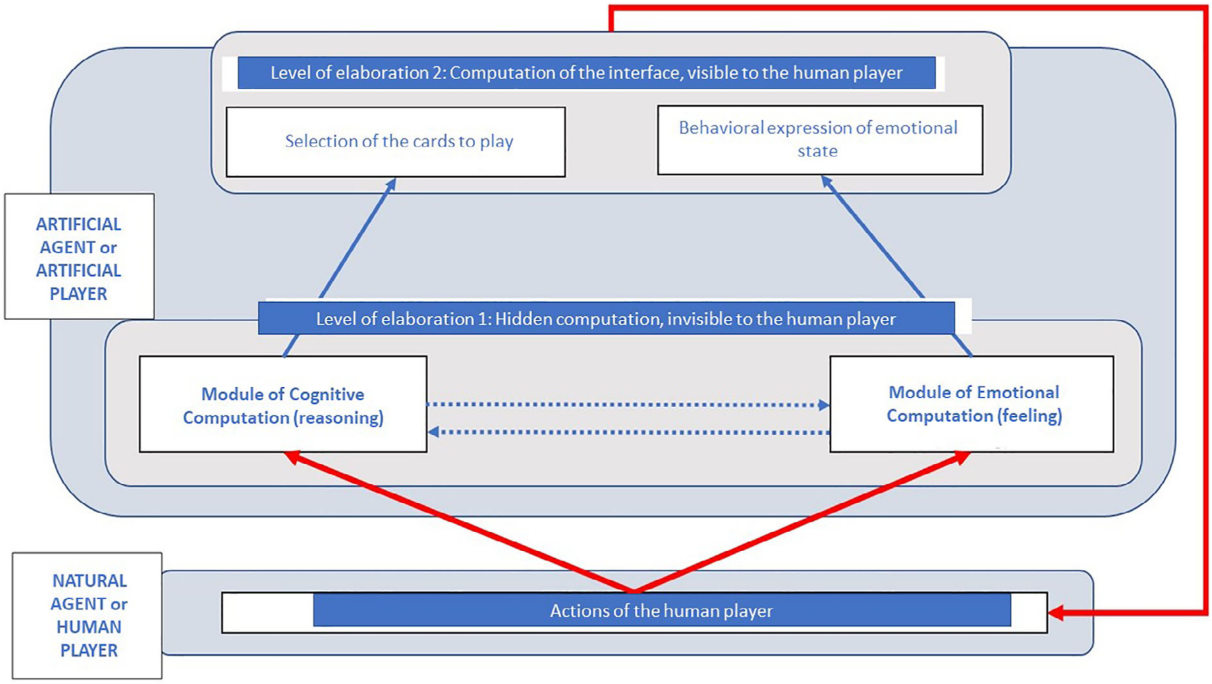

FIGURE 6 Baldo intelligent system

Given the characteristics of the recipients of the psychological intervention, the artificial intelligence system of the artificial agent must consist of both a system of production and expression of the emotional state and a system of rational and cognitive reasoning.

The general architecture of the Artificial Intelligence system of our agent is presented in Figure 6. It is divided into two sequential levels of processing, in the first level (hidden computation) there is a 'mental' computation module and in the second level (interface computation) is a 'behavioural' computation module.

The first level of processing collects information from the external environment (in our case the actions of the human player) and processes them by defining both the cognitive state (game action to be done) and the emotional state (the feeling of the moment) of the artificial agent in each game phase. The human player does not have direct access to the results of the first level processing; therefore, the level of computation is defined as hidden.

The second level of processing collects and processes the results of the first level of processing and produces actions visible to the human player (for which this level is defined as an interface) of the artificial agent to which the human player will have to respond/react.

From the psychological point of view, we can consider the first level of processing as the mind of the artificial agent whereas the second level of processing as the agent's behavioural expression apparatus.

The evaluation of Baldo is still running and is involving both children and elder adults. It is focused on the acceptability and pleasantness of the system and, with a longitudinal experimental design, the aim is to evaluate its effectiveness in improving mathematical abilities in children and preserving fluid mental abilities in elderly people.

5 | CONCLUSIONS AND FUTURE DIRECTIONS

In this article, we have described Baldo as an example of how to use intelligent systems to assess and train numerical abilities. We think that the card game proposed here represents an entertaining way to exercise mathematical abilities. It belongs to the field of edutainment in which a structured teaching such as maths can be faced in a ludic and involving manner. In our opinion, using cards both physical and digital can be a very effective way to assess numerical skills in the framework of GBL and embodied cognition. Putting together the physical and digital sides, allows to keep the embodied dimension, which is important for developing cognition and to join it with the chance to record almost every aspect of children game interaction or, more generally, player-game interaction, for example recording reaction times.

Learning approaches focused on embodiment underline that action can support educational objectives and, in the case of numerical cognition, help the transition between analogic and symbolic dimension, which is crucial in math learning. Another important aspect is related to the use of these tools in different context, included non-formal educational contexts such as home. Learning today takes place in a context of new interactions between formal and informal learning, the changing role of teachers, the impact of social media and the children active participation in learning activities. Learning cannot be confined in schools' walls or training classroom but happens in new contexts of interaction. Games are perfectly able to adapt to different contexts and different learning conditions, also because the smartphone version can be played almost everywhere and in every moment.

The next step will be to extensively test these tools with a longitudinal and experimental procedure that will follow groups of children at different ages to verify if the use of these tools can be valid and affordable to assess numerical abilities and later math school achievement. Moreover, Baldo will be developed to become a hybrid game joining physical and digital sides of the game: we will develop a real user playing with physical cards against an artificial digital player. The emotion module will be used in studies involving the social dimension and its connection with numerical abilities, as pre-requisites of calculation can be affected by emotional factors.

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CONFLICT OF INTEREST

The authors declare no potential conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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