RESEARCH AND LEARNING ENVIRONMENT: THE ROLE OF THE TEACHER

Virginia Vaccaro[1]

Abstract

In this paper, starting from a didactical proposal about the introduction of the concept of natural number, already experienced in some primary school classes, we report about the subsequent reorganization of it in a new experience, quite different from the previous one. This transformation arises from the necessity of taking into account both: the pupils as they are, namely as members of a community with certain characteristics and not others; and the teacher herself, if she wants to actively take part in the learning project that she is going to realise.

1. Introduction

"… developing methods to help all children in suitable ways continues to be a focus of study for us all" (Tall, 2001). Ideas and methods on these subjects are presented in several scientific meetings and published in many specialised journals. In this way, theoretical research as well as reports on experimental educational activities are at disposal of both researchers and teachers. Articles on experimental activities mainly concern the children and the knowledge (2 poles of Brousseau's triangle) (Brousseau, 1986) and represent, together with textbooks, an important psychological support for many teachers, especially at the start of their career or in difficult situations. In fact, the publication of a proposal in a specialised journal often guarantees not only its coherence but also a wide practicability. But, sometimes, it may happen that a good teacher, in search of encouragement and inspiration, in presenting a pre-packaged didactical proposal to her/his class, without personal modifications, not only obtains results different from those expected but can also observe negative consequences both in terms of self esteem and of classroom climate (Ferrera, Furinghetti, 2002), (Vaccaro, 2001).

In Italy, the recent institution of university courses for future primary teachers together with deep changes in society, more and more technological and multiethnic, have been the reason why the teacher (the third pole of Brousseau's triangle) is becoming an object of investigation at both the initial and the inservice training levels. Since at both levels it is impossible to cover all disciplinary mathematical contents, the need arises to make efforts for "*recovering in the students an attitude toward mathematics, to convince, or re-*

[1] Università Federico II di Napoli; e-mail: vivaccar@unina.it

convince, them that it can be understood, a starting point from where they can begin to learn …" (Tortora, 2001). So, the most important goals of a training appear to enhance teachers' abilities: to plan suitable learning environments, to be flexible in guiding the learning process, to independently work on disciplinary contents, to make cognitive and metacognitive reflections on teachers' and pupils' activities. Maybe the lack of some of the listed abilities causes the failure of educational research. This hypothesis rests on the illusiveness of leaving educational itineraries unchanged in presence of social, cultural and historical differences: in particular, if one wants the pupils to become "*sharers of a collective behaviour*" (Sfard, 2001) and if one views the mind "*as essentially social in its origins and strictly dependent on historical and cultural factors*" (Vygotskij, 1978). It has long been known that it is necessary to analyse, from time to time, such variables as "*target audience, their mathematical background, the sources, the approach, the topics etc.*", inasmuch "*if, in another context, the variables are not identical, and they never are, the case has to be restated with possible modifications.*" (Arcavi, Bruckheimer, Ben-Zvi, 1982). Moreover, "*mathematical thinking is formed through a slowly personal and interpersonal development culturally addressed, and not through appropriation of a predefined reality"* (Guidoni, 2002). Therefore, a careful analysis of the context should guide the choice of the most appropriate learning environment, in fact "*the cognitive power, the learning capacities and attitudes are incremented maintaining the learning environment linked to the cultural context …*" (D'Ambrosio, 1995).

So, what to do in order to help "a good teacher, in search of encouragement and inspiration" to modify a didactical proposal by planning a suitable learning environment?

In order to try to give an answer to this crucial question, in the next two sections we follow through a teaching experiment. At the beginning, we were interested in the practicability and effectiveness of a specific didactical proposal, previously experimented by the author, in the general hypothesis that the theoretical framing and methodologies of a socio-cultural approach can help any teacher to understand and support a student's mathematical learning. But, early on, what we have said above became evident. So, we were faced with two related questions: if and how it was possible to enhance a teacher's ability to plan suitable learning environment; to find out some additional features for a didactical proposal being accessible and effective for who usually is alone with his/her work. In section 2 we sketch just a little of the first didactical proposal, while in the section 3, we give more details about the framework of it and we show that this additional feature is the starting point for teacher's free choices.

In the conclusions we make a comparison between the new proposal and the old one, we emphasize the social aspects of the new proposal and, finally, starting from this teaching experiment, we extrapolate an answer to the question

"if it is possible to enhance a teacher's ability …" as well as to the question "to find out some additional features … for who usually is alone with his/her work".

2. The didactical proposal to be re-contextualized

A committee of the Unione Matematica Italiana (UMI, Italian Mathematical Society) drew up a mathematics curriculum for primary school and presented it in *Matematica 2001*, *Mathematics for citizens*. This volume integrates research trends and results with teaching practice. Every proposal, already experimented, begins with a table relative to its mathematical characteristics: level, competences, themes (topics), content and process standards, external links. Then there follows all necessary didactical informations, for example: methodology and context, suggested activities, conceptual difficulties, tools etc……After all that, the proposal itself is described. One of these proposals, "Caoticus", concerning the content standard "Natural Number" for 1st or 2nd grade, was originated from the collaboration of the author with the teacher Lucia Arena, whose pupils come from different social backgrounds, and whose school is a permanent laboratory in which fairy tales are often used as a teaching resource. The story is set in Wonderland, the world of Lewis Carroll's Alice and shows Alice and Caoticus, a former toll keeper, busy in building natural numbers. Here is the beginning and some passages of the story.

I would like to tell you about when I felt like a mathematical genius. I was walking in Wonderland when I saw a cave and a light at the end of it…. I hadn't entered a cave but a tunnel! And the light had come from somewhere outside the tunnel…. I saw shepherds with strange clothes, farmers with simple tools, carts drawn by oxen, children throwing pebbles in a pond….The Cheshire Cat explained to me that the tunnel was the tunnel of time…!!...I entered this building and I saw such a mess as I have never seen before, not even in my own bedroom. There were all sort of things: fruit, wheat, vegetables, cheeses, barrels full of oil and vine, stones, wood, utensils, baskets, etc….These represented the taxes which the villagers paid to the chief villager according to their trade and according to their wealth… After a while a peasant came in with a basket of apples. The official (Caoticus) looked at a parchment to see how much this man should pay and saw that six things were drawn which seemed like fruit or stones…

While the story is being told the children are asked to do some activities: re-organise Caoticus' shop; put the objects in order; recognize the symbols used by Caoticus, use the fingers of their hands; what did Caoticus find out?; and so on…

The activities involve all the competences mentioned in the table that comes before, among which: measuring to find discrete variables, counting objects and comparing groups of objects, comparing and ordering numbers, reading and writing numbers in decimal notation, counting in ascending and descending order. The goals of these activities, for which Alice asks children's help, are recognizable as the main milestones, both historical and epistemological, of the transition from a finite set of discrete objects to symbol-as-numeral-adjective and from this to symbol-as-a-noun and hence as object of study. The story of the

journey, which unravels through the story of Caoticus, seeks to relive these transitions, present in every civilization, showing the effort that mankind has made in order to compare, to measure, to put in order every sort of things and then to leave stable signs of all these activities. The motivation of this proposal is that, even if children know and sometimes use numbers, it does not follow that they understand the concept of number. The competences involved in the proposal, have probably been previously developed in other contexts; however, the fact that they are all related within one story allows the pupils to understand the concept of number.

3. Description of the pathway of "adapting" the learning environment of the educational proposal to a different social and cultural context.

Initially, the research group was persuaded of the effective transferability of this didactical proposal (it may happen for some proposal!). So, encouraged by the successful experience of the teacher Lucia Arena and supported by the presence of a university researcher, Maria Elia, a teacher, decided to introduce the story of Alice in her classroom. However, Alice's call for help didn't engage her children nor the teacher herself! Therefore, if this had happened, as child psychology suggests, it meant that the story was not "inhabited" either by whoever told it or by whoever listened to it. The story did not manage to create that particularly pleasant situation from which knowledge can be acquired. At first, the teacher kept this difficulty to herself. Fortunately it then came out and it was possible for us to start with a different analysis of the proposal. It was necessary to throw out Alice, Caoticus and many other things, it was necessary to share with teacher the proposal's framework in order to emphasize the real aim of the activity, the real need of every steps. The result of this work was the framework of "Caoticus" described from the list of the following table:

1.create a messy environment with boxes, glasses or similar things containing discrete materials that can be counted in numbers not bigger than 10 (for each number there must be more than one container);
2.give rise to the need of ordering the environment for a particular purpose;
3.make comparisons of the contents of the boxes with respect to their quantities (remembering that it doesn't depend on the order);
4.prepare 10 desks;
5.ask the pupils to put on the same desk the containers with the same number of objects in order to obtain a local order (obviously one desk will be empty);
6.ask the pupils to put the desks in ascending order based on the quantities in the containers;
7.convince the pupils to put a label on the containers already checked that are on the same desk, in order to avoid somebody malicious causing confusion again, once the pupils were not there; ask the pupils to choose any symbol to represent the quantity and a word or a drawing for the

content (the use of the symbol as a numeral adjective arises from abstraction from the order);

8. in order to know what there is on each desk, attach to it a bigger label on which the children draw only the symbol already chosen to represent the same quantity in the containers and not the drawings or the names of things (the transition from symbol-as-numeral-adjective to symbol-as-a-name comes from abstraction from the quality);

9. ask someone unaware of the activity to guess the meaning of those symbols;

10. choose a conventional symbol to communicate with others.

Table 1. Framework of the proposal "Caoticus"

At this stage, pointing on the intuition that the cultural route is not exclusively scholastic, the teacher looked for resources outside the school, taking what the environment could offer. She was trying:

a) to identify reach social contexts close to the children's reality;

b) to analyse in depth their social, cultural and economic components;

c) to converte these components into educational opportunities;

d) to design a learning situation concerning mathematics as well as other topics.

The teacher worked in a suburb of Cava dei Tirreni, working-class and deprived, where it is still possible to find in the courtyards the tools for making ropes, an old job which is nowadays mechanised, in which entire families were involved and whose memory is still vivid for many people. In this research-action, the ropes become the semiotic mediator to work with, starting from the living history of the suburb carried out by the pupils with the involvement of their families. The activity was done by two second-year junior school classes. In the first phase of the activity a story was proposed which involved the children in looking for a solution to a problem:

".....Giovannino Perdigiorno[2], after a lot of wandering about… (remember? He had been in the Land of the Glass Men, in the Land of the Light Heads and also of the Engine Men)… stopped just there in the Land of the Men without Numbers. He was intrigued by the great mess and confusion which there was in the streets, in the shops, in the schools and especially in the rope and string factory which was in the town square. Giovannino stopped in front of the factory shop and carefully observed what was happening: a lot of people were coming and going. Some were workers, who were unloading big coils of different colours and sizes. Others were taking them away. You should understand that a string can be sly or thick depending on whether it is made from a few or many filaments wound together. There were very fat ropes to be used on big ships and very fine ones which could enter the eye of a needle. Giovannino went into the shop and he realised that the employee was tired, exasperated and … alone. What a mess! Giovannino asked for an explanation and the employee, called Lambertino, explained that the factory workers brought the coils of rope of different thickness and put them quickly on the high shelves of the shop. Whoever came to preleve the coils

[2] Giovannino is a character in a story by G. Rodari that the pupils already met in other school contexts

brought some corks which they gave to Lambertino who searched patiently for a coil of rope formed by as many filaments as the number of corks. Sometime he was lucky and immediately found what he wanted. At other times he took so long that a big impatient queue formed. The end of the day was the most dramatic moment. Lambertino had to check the remaining coils and to make an order for the missing ones for the next day. Absolutely impossible! Poor Lambertino worked until late at night! Giovannino understood the difficulties of the man who did not know the numbers and decided to give him a hand. Above all it was necessary to put the shop in order…

Reading the story allows the children to recognise the problem, to discuss the circumstances, to compare the real places with the fictional ones. The primary objective identified by the children is to reproduce Lambertino's shop with a lot of ropes: that is, a laboratory made from a lot of concrete things which could be manipulated and organised. An initial classification begins based on criteria chosen and shared by the class group (by the way, an activity not presented in Caoticus). The activity occurs in two stages. In the first instance, the children of one class, in the classroom-laboratory, made a classification based on the uses of the ropes.

1st Criterion. ***The uses of the ropes***: curtains, sport, kitchen, agriculture, shipping, fishing, …

Gianmarco suggested to write on the wall "Lambertino's shop" (he obviously felt the need to make the situation as real as possible). The job continued until the ropes, which were initially situated messy on the floor, were all gone. In the second instance, this group of children left the laboratory and were followed by a group from the other class. They checked the work done by their friends and introduced another classification according to a new criterion. The children distinguished ropes of a single colour from those made by a few colours or by a lot of colours….

2nd Criterion. ***The colours of the ropes***.

Classifications

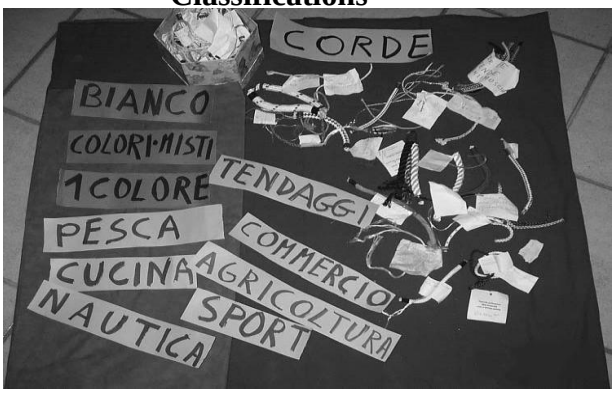

Fig.1

At this point a question arose: "Which type of classification would help Lambertino to have the best order in the shop and the best system for selling

ropes?" After a discussion, they decided to choose the classification based on the uses. As a consequence, in Lambertino's shop there would be shelves, settings, big boxes, …., each with a label describing a particular use. This arrangement solved the problem of the order in the shop but not the other one. In fact, every box was like Caoticus' shop. So, to satisfy the request to have the best system for selling ropes, there was a need to find a criterion to organise the ropes in every box. At this point, the teaching experiment followed Table 1, from beginning to end. Since corks and ropes were familiar objects to Lambertino as well as to children, it was decided to use them to single out the best way to help Lambertino to sell ropes.

Correspondence quantity-symbol

Fig. 2

Every class was divided into 3 groups of 6 children: each group, starting with a sample quantity of rope, corresponding to one cork or one stopper, built ropes of increasing thickness, by adding each time the same quantity, called "sample of rope". This stage of the job represents the moment of comparison and research of a one-to-one correspondence between equal quantities of different objects. Now, taking into account the ropes of one box, the quantity of "samples of rope" was the criterion to put in order the contents of that box. At last, to avoid using stoppers and corks, it was decided to invent symbols (see Fig. 2).

These activities were carried out by the children of the two classes at different times. In both classes the spontaneous use of fingers and of new symbols produced a merry climate of sharing. But the choice of different symbols and the subsequent discussions about the activities among the children of the two classes, emphasised the difficulty of interpreting the symbols chosen by each class to represent the same quantity, and gave rise to the necessity of finding a common symbol to represent the number in a conventional way.

4. Conclusions

The transposition of "Caoticus" succeeded in a satisfactory manner, in the sense that the new activity, built on the framework of Caoticus, reached the

expected goals and, this is an interesting aspect, also different ones. For example, in parallel with the laboratory activity, the children made visits to the region: they visited the last courtyard with a "wheel", the tool traditionally used to make ropes, they listened to the story of the local tradition, sensed the nostalgia felt by many persons for a recent past. These visits, stimulated by the laboratory activity, enriched the understanding of social, cultural and productive topics offered by the same region; they consolidated and organised children's knowledge, created relationships between the school and the community, enriched the cultural baggage of each child and children's attitude to co-operative learning.

For what strictly concerns the proposals, we have noted a different behaviour about the symbol of zero. The children who worked with Caoticus have had no problem to accept a desk without anything and to choose a symbol for it, in fact they chose a picture of the ghost of the duke of Barnabò without face and hands who, in secret, usually leaved letters, jokes and gifts in their class. In the class of the ropes, zero was born as a consequence of the sale of all ropes of a certain type, and so it was accepted, but the necessity of a symbol for it was less interesting for those children until the introduction of the number 10.

This teaching experiment involved a researcher and, among other primary school teachers, the two teachers Arena and Elia. This was a good opportunity to analyze the problem of transposition of a didactical proposal and to rise some crucial questions. The first one, as stated before, is: "to find out some additional features for a didactical proposal being accessible and effective for who usually is alone with his/her work". On the basis of our experience, we believe that any didactical proposal should be presented in such a way that: the mathematical objective be clearly stated, the free choices be carefully separated from the mathematical objective, the framework of the activity be suitably shown. The second question is: "is it possible to enhance a teacher's ability to plan suitable learning environment?". Our answer is yes, because we think that primary teachers are *manual workers, artisans, artists rather then theoreticians* (Hawkins, 1979). So if we give them teaching equipments like we suggest, then it is possible to appeal to a creativity that belongs to teachers as well as to their pupils. They can make the most advisable choices, invent the component factors, the framework, the architecture, taking into account both: the pupils as individuals and as members of a community with certain characteristics and not others; and the teacher her/himself if she/he wants to actively take part in the learning project that she/he is going to realise. The effort to establish the changes that must be brought about, is surely repaid by a professional and cultural growth and by a greater self-esteem.

To be able to widely use one's own social context is the exhausting challenge that everyday one ought to put oneself in order to produce suitable learning situations that not only are mathematical opportunity but also suitable

occasions to construct abilities to critically interpret reality and to intervene knowledgeably on it.

References

Arcavi A., Bruckheimer M., Ben-Zvi R. (1982). Maybe a mathematics teacher can profit from the study of the history of mathematics, *For the Learning of Mathematics*, 3

Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques, *Recherches en Didactique des Mathématiques*, vol. 7, n° 2, pp. 33-115

D'Ambrosio, U. (1995). Etnomatematica: teoria e pratica pedagogica, *L'Educazione Matematica*, Anno XVI, Serie IV, 2(3), 147-159.

Ferrera, G., Furinghetti, F. (2002). L'insegnante di fronte alle riviste professionali per l'insegnamento della matematica: stimoli e riflessioni, *L'Insegnamento della Matematica e delle Scienze Integrate*, vol. 25 B, N. 2, Aprile, pp.129-151.

Guidoni, P.(2001). Quale matematica per i ragazzi futuri cittadini. Quale fisica per … Quale scuola per …?, XXII Convegno Nazionale UMI-CIIIM, Ischia, pag.66.

Hawkins, D. (1979). Imparare a vedere, *Scienze dell'educazione* 19, Loescher, Torino (1974,The informed vision, *Agathon Press*, Inc., New York)

Sfard, H.(2001). There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning, *Educational Studies in Mathematics*, vol.46, pp.1-3

Tall, D. (2001). What mathematics is needed by teachers of young children?, *International Symposium Elementary Maths Teaching*, Edited by J. Novotná, M. Hejnỳ, pp.26-36

Tortora, R.(2001). Mathematics for children and for their teachers: a comparison, *International Symposium Elementary Maths Teaching*, Edited by J. Novotná, M. Hejnỳ, pp157-162

Vaccaro, V. (2001). L'apprendimento del numero e delle frazioni nella scuola elementare: esperienze ed attività di osservazione, *Atti del 3^{o} Internuclei Scuola dell'Obbligo*, Vico Equense, 1999, pp.67-72

Vygotskij, L. S. (1978). Mind in Society: The development of higher Psychological Processes, Harvard University Press, Cambridge MA.