B Decays into Two Charmed Mesons in the Isgur and Wise Theory.

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Summary. — From the experimental branching ratios for the decay $\bar{B} \to D^{(*)} \bar{D}_s^{(*)}$, evaluated within the Isgur and Wise theory, one gets $f_{D_s^*} = (246 \pm 47)$ MeV and $f_{D_s} = (241 \pm 36)$ MeV in good agreement with the value $f_{D_s} = (232 \pm 45 \pm 20 \pm \pm 48)$ MeV obtained from the measured $Br(D_s^+ \to \mu^+ \nu_\mu)$.

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1. – Introduction.

New symmetries for heavy quarks have been discovered as a consequence of QCD. In the no-recoil limit the spin of the heavy quarks is conserved since it is decoupled. Symmetry for heavy flavours comes out from the disappearance of their masses in the heavy-quarks effective Lagrangian [1].

As a consequence of these symmetries, all the form factors for the weak currents of the mesons containing one heavy quark (*heavy mesons*) are given in terms of the Isgur-Wise function $\xi(w^2)$ [2], as long as one neglects terms of order Λ_{QCD}/m_Q (m_Q is the mass of the heavy quark). The heavy-quarks effective theory (HQET) is not able to predict the Isgur-Wise function. By assuming particular q^2-behaviour for $\xi(w^2)$ different authors tried to obtain $\xi(w^2)$ from the experimental data on semi-leptonic and non-leptonic weak decays of B-mesons [3]. In a previous work we studied the Cabibbo-favoured $|\Delta C| = 1$ decays of B-mesons into two hadrons within the factorization approximation [4].

Here we consider the final states with two charmed particles: in this way we shall explore $\xi(w^2)$ in the range of values $(-0.79, -0.59)$ for w^2. A crucial point is the choice of the scale at which factorization is assumed. We shall follow the prescription of reference [5] to assume factorization for the effective Hamiltonian with the coefficients evaluated at a subtraction point $\bar{\mu}$ where $\alpha_s(\bar{\mu}) = 1$.

From the experimental rates for the channels studied we derive the values of f_{D_s} and $f_{D_s^*}$ which should be equal according with HQET, and compare them with the value of f_{D_s} derived from the measured branching ratio $\mathrm{Br}(D_s^+ \to \mu^+ \nu_\mu)$ [6].

In the following section we shall derive the predictions for the B decays into $D^{(*)}\bar{D}_s^{(*)}$ and in the final one we compare theory with experiment.

2. – Predictions for $\bar{B}$ decays in $D^{(*)}\bar{D}_s^{(*)}$ in Isgur and Wise theory.

The tree level weak Hamiltonian for the processes studied here is (upon neglecting Cabibbo-suppressed terms):

$$\mathscr{H}_{\text{tree}} = -i\,\frac{G_F}{\sqrt{2}}\,V_{bc}V_{sc}^*[\bar{c}_\alpha\gamma_\mu(1-\gamma_5)b_\alpha][\bar{s}_\beta\gamma^\mu(1-\gamma_5)c_\beta] + \text{h.c.} \tag{1}$$

The corresponding effective Hamiltonian at leading-log in QCD can be obtained using the effective theory for heavy quarks when the momenta of the virtual gluons in the loop are smaller than m_b.

The result can be written in the form [5]:

$$\mathscr{H}_{\text{eff}} = -i\,\frac{G_F}{\sqrt{2}}\,V_{bc}V_{sc}^*\sum_{j=1}^{4} C_j(\bar{\mu})\,O_j(\bar{\mu}), \tag{2}$$

where

$$\begin{cases} O_1 = [\bar{s}_\alpha\gamma_\mu(1-\gamma_5)\underline{b}_{\alpha v}^{(+)}][\bar{\underline{c}}_{\beta v'}^{(+)}\gamma^\mu(1-\gamma_5)\underline{c}_{\beta v''}^{(-)}], \\ O_2 = [\bar{s}_\beta\gamma_\mu(1-\gamma_5)\underline{c}_{\beta v''}^{(-)}][\bar{\underline{c}}_{\alpha v'}^{(+)}\gamma^\mu(1-\gamma_5)\underline{b}_{\alpha v}^{(+)}], \\ O_3 = [\bar{s}_\alpha\gamma_\mu(1-\gamma_5)\underline{b}_{\alpha v}^{(+)}][\bar{\underline{c}}_{\beta v'}^{(+)}\gamma^\mu(1+\gamma_5)\underline{c}_{\beta v''}^{(-)}], \\ O_4 = -2[\bar{s}_\beta(1+\gamma_5)\underline{c}_{\beta v''}^{(-)}][\bar{\underline{c}}_{\alpha v'}^{(+)}\gamma^\mu(1-\gamma_5)\underline{b}_{\alpha v}^{(+)}]. \end{cases} \tag{3}$$

The heavy fields are underlined and an upper $(-)$ $+$ labels the (anti)particles; the operators of ref. [5] have been Fierz-rearranged.

We shall assume factorization for the processes $\bar{B} \to D^{(*)}\bar{D}_s^{(*)}$ at the scale $\bar{\mu}$ with $\alpha_s(\bar{\mu}) = 1$ and take the following values for the $C_i(\bar{\mu})$ [5]:

$$\begin{cases} C_1(\bar{\mu}) = [-0.45 - i0.15], \quad C_3(\bar{\mu}) = [0.02 + i0.007], \\ C_2(\bar{\mu}) = [1.45 + i0.22], \quad C_4(\bar{\mu}) = [-0.06 - i0.009], \end{cases} \tag{4}$$

for

$$\alpha_s(m_W) = 0.12, \qquad \alpha_s(m_b) = 0.21, \qquad \alpha_s(m_c) = 0.29, \tag{5}$$

with $m_b = 5.2$ GeV and $m_c = 1.8$ GeV.

The matrix elements between charmed and beautiful mesons are given in terms of the universal form factor $\xi(w^2)$:

$$(6)\quad \begin{cases} \langle D(v')|\underline{\bar{c}}\gamma_\mu \underline{b}|B(v)\rangle = \sqrt{m_B m_D}\,\xi(w_D^2(q^2))(v_\mu + v'_\mu)\,, \\ \langle D^*(v',\varepsilon)|\underline{\bar{c}}\gamma_\mu\gamma_5 \underline{b}|B(v)\rangle = i\sqrt{m_B m_{D^*}}\,\xi(w_{D^*}^2(q^2))[\varepsilon_\mu^*(1+v\cdot v') - (\varepsilon^*\cdot v)v'_\mu]\,, \\ \langle D^*(v',\varepsilon)|\underline{\bar{c}}\gamma_\mu \underline{b}|B(v)\rangle = \sqrt{m_B m_{D^*}}\,\xi(w_{D^*}^2(q^2))\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu} v^\alpha v'^\beta\,, \\ \langle D^*(v',\varepsilon)|\underline{\bar{c}}\gamma_5 \underline{b}|B(v)\rangle = -i\sqrt{m_B m_{D^*}}\,\xi(w_{D^*}^2(q^2))(\varepsilon^*\cdot v)\,, \\ \langle D(v')|\underline{\bar{c}}\,\underline{b}|B(v)\rangle = \sqrt{m_B m_D}\,\xi(w_D^2(q^2))(v\cdot v'+1)\,, \end{cases}$$

where

$$(7)\qquad w_{D^{(*)}}^2(q^2) = (v-v')^2 = \frac{q^2-(m_B-m_{D^{(*)}})^2}{m_B m_{D^{(*)}}}\,.$$

The meson decay constants are defined by

$$(8)\quad \begin{cases} \langle D_s(p)|\underline{\bar{s}}\gamma_\mu\gamma_5 \underline{c}|0\rangle = -if_{D_s} p_\mu\,, \\ \langle D_s^*(p,\varepsilon)|\underline{\bar{s}}\gamma_\mu \underline{c}|0\rangle = m_{D_s^*} f_{D_s^*}\varepsilon_\mu^*\,, \end{cases}$$

and so

$$(9)\qquad \langle D_s(p)|\underline{\bar{s}}\gamma_5 \underline{c}|0\rangle = -if_{D_s} m_{D_s}\,.$$

By assuming factorization and a complete cancellation between QCD corrections of the semi-leptonic and non-leptonic Cabibbo-favoured b decays effective weak Hamiltonian, Bortoletto and Stone [7] have been able to make the Bjorken test [8] and to get $f_{D_s} = (276 \pm 45 \pm 44)$ MeV independently of the values of $|V_{bc}|$ and of the Isgur-Wise function. Here by keeping into account QCD corrections for $\mathscr{H}_{eff}$ and assuming the factorization at the scale $\bar{\mu}$ we find (for the sake of simplicity we do not write the colour, particle-antiparticle and velocity indices):

$$(10)\quad \langle D^+D_s^-|\mathscr{H}_{eff}|\bar{B}_d^0\rangle = -i\frac{G_F}{\sqrt{2}}V_{bc}V_{sc}^*\cdot$$
$$\cdot\left\{-\left(C_2+\frac{C_1}{N_c}\right)\langle D^+|\underline{\bar{c}}\gamma_\mu \underline{b}|\bar{B}_d^0\rangle\langle D_s^-|\underline{\bar{s}}\gamma^\mu\gamma_5 \underline{c}|0\rangle - \left(C_4+\frac{C_3}{N_c}\right)\langle D^+|2\underline{\bar{c}}\underline{b}|\bar{B}_d^0\rangle\langle D_s^-|\underline{\bar{s}}\gamma_5 \underline{c}|0\rangle\right\},$$

$$(11)\quad \langle D^{*+}D_s^-|\mathscr{H}_{eff}|\bar{B}_d^0\rangle =$$
$$= -i\frac{G_F}{\sqrt{2}}V_{bc}V_{sc}^*\left\{\left(C_2+\frac{C_1}{N_c}\right)\langle D^{*+}|\underline{\bar{c}}\gamma_\mu\gamma_5 \underline{b}|\bar{B}_d^0\rangle\langle D_s^-|\underline{\bar{s}}\gamma^\mu\gamma_5 \underline{c}|0\rangle + \right.$$
$$\left. + \left(C_4+\frac{C_3}{N_c}\right)\langle D^+|2\underline{\bar{c}}\gamma_5 \underline{b}|\bar{B}_d^0\rangle\langle D_s^-|\underline{\bar{s}}\gamma_5 \underline{c}|0\rangle\right\},$$

$$(12)\quad \langle D^+ D_s^{*-} | \mathcal{H}_{\text{eff}} | \bar{B}_d^0 \rangle =$$

$$= -i \frac{G_F}{\sqrt{2}} V_{bc} V_{sc}^* \left\{ \left(C_2 + \frac{C_1}{N_c} \right) \langle D^+ | \underline{c} \gamma_\mu \underline{b} | \bar{B}_d^0 \rangle \langle D_s^{*-} | \bar{s} \gamma^\mu \underline{c} | 0 \rangle \right\},$$

$$(13)\quad \langle D^{*+} D_s^{*-} | \mathcal{H}_{\text{eff}} | \bar{B}_d^0 \rangle =$$

$$= -i \frac{G_F}{\sqrt{2}} V_{bc} V_{sc}^* \cdot \left\{ \left(C_2 + \frac{C_1}{N_c} \right) \langle D^{*+} | \underline{c} \gamma_\mu (1 - \gamma_5) \underline{b} | \bar{B}_d^0 \rangle \langle D_s^{*-} | \bar{s} \gamma^\mu \underline{c} | 0 \rangle \right\}.$$

We assume the total screening of the factorizable part of the amplitudes proportional to $1/N_c$ due to the non-factorizable one, which seems to work well in B decays into two mesons with $|C| = 1$ [4] and in Cabibbo-allowed non-leptonic charm decays [9].

3. – Comparison between theory and experiment for $\bar{B}$ decays in $D^{(*)} \bar{D}_s^{(*)}$.

Once equations (4), (6) and (8), (9) are inserted into (10)-(13), the amplitudes depend on the product of the Cabibbo-Kobayashi-Maskawa matrix elements V_{bc} and V_{sc}^*, on the values of $\xi(w^2)$ in four points in the range $(-0.79, -0.59)$ for w^2 and on f_{D_s} and $f_{D_s^*}$. We take $|V_{sc}| = 0.975$, $|V_{bc}|$ and $\xi(w^2)$ are found by a fit to the semi-leptonic Cabibbo-favoured decays $\bar{B}_d^0 \to D^{+(*)} e^- \bar{\nu}_e$ in correspondence of the values found with different parametrizations for the Isgur-Wise function and the results are described in table I [10, 11].

The experimental rates for charged and neutral B decays have been measured by ARGUS and CLEO Collaborations [12]. The $\Delta I = 0$ property of the weak Hamiltonian defined in eq. (1) implies equal rates for the charged and neutral B

TABLE I. – *The values of $|V_{bc}|$ and $\xi(w^2)$ relevant to this paper are reported for various parametrizations. In the last column we put the resulting mean values relevant to this paper: $w_D^2(m_{D_s}^2) = -0.79$, $w_{D^*}^2(m_{D_s}^2) = -0.64$, $w_D^2(m_{D_s^*}^2) = -0.73$ and $w_{D^*}^2(m_{D_s^*}^2) = -0.59$.*

	$1 + bw^2 \cdot (1 + cw^2)$	$(1 - w^2/w_0^2)^{-1}$	$\exp[\beta w^2]$	$1 + (\rho^2/2) w^2$	Mean values
	$b = 1.28^{+0.31}_{-0.42}$ $c = 0.55^{+0.09}_{-0.21}$	$w_0 = 0.79^{+0.18}_{-0.13}$	$\beta = 0.90 \pm 0.22$	$\rho = 1.06 \pm 0.09$	$\xi(-0.79) = 0.50 \pm 0.05$ $\xi(-0.64) = 0.58 \pm 0.05$ $\xi(-0.73) = 0.53 \pm 0.05$ $\xi(-0.59) = 0.61 \pm 0.04$
$\lvert V_{bc} \rvert$	0.049 ± 0.007	0.048 ± 0.008	0.043 ± 0.005	0.039 ± 0.004	0.043 ± 0.003
χ^2 (χ^2/Ndf)	4.33 (0.62)	5.30 (0.66)	5.65 (0.71)	6.75 (0.84)	— —

TABLE II. – *The experimental values of* $\Gamma(\bar{B} \to (D^{(*)} \bar{D}_s^{(*)})$ *are found, by assuming equal values for the rates of the neutral and charged* B *according to the* $\Delta I = 0$ *property of the weak Hamiltonian defined in eq.* (1). *In the last column are the predicted values for* f_{D_s} *and* $f_{D_s^*}$.

Channels	Experimental rates $\cdot 10^{+12}$ MeV		f_{D_s} (MeV)	
$\Gamma(\bar{B}^0 \to D^+ D_s^-)$	4.2 ± 1.8	5.2 ± 1.6	232 ± 45	241 ± 36
$\Gamma(B^- \to D^0 D_s^-)$	9.5 ± 3.8			
$\Gamma(\bar{B}^0 \to D^{*+} D_s^-)$	6.0 ± 3.2	6.1 ± 2.6	257 ± 62	
$\Gamma(B^- \to D^{*0} D_s^-)$	6.2 ± 4.3			

Channels	Experimental rates $\cdot 10^{+12}$ MeV		$f_{D_s^*}$ (MeV)	
$\Gamma(\bar{B}^0 \to D^+ D_s^{*-})$	11.1 ± 7.9	8.8 ± 5.5	341 ± 114	246 ± 47
$\Gamma(B^- \to D^0 D_s^{*-})$	6.7 ± 7.6			
$\Gamma(\bar{B}^0 \to D^{*+} D_s^{*-})$	10.2 ± 6.0	11.8 ± 4.8	227 ± 51	
$\Gamma(B^- \to D^{*0} D_s^{*-})$	14.8 ± 8.1			

decays considered here and therefore we shall take the mean value of the experimental results for the two cases.

From these values we are able to derive, within the theoretical approach described in the previous section, the values (see also table II)

$$f_{D_s} = (241 \pm 36)\ \text{MeV}, \qquad f_{D_s^*} = (246 \pm 47)\ \text{MeV},$$

in good agreement with the HQET prediction $f_{D_s} = f_{D_s^*}$ and the value

$$f_{D_s} = (232 \pm 45 \pm 20 \pm 48)\ \text{MeV},$$

deduced from the branching ratio measured by the WA75 Collaboration [6]

$$\text{Br}(D_s^+ \to \mu^+ \nu_\mu) = (4.0^{+1.8}_{-1.4}{}^{+0.8}_{-0.6} \pm 1.8) \cdot 10^{-3}.$$

By assuming $SU(3)_{u,d,s}$ symmetry, one has $f_D = f_{D_s}$ which implies for the Cabibbo first-forbidden decays ($|V_{dc}| = 0.221$)

$$\text{Br}(D^+ \to \mu^+ \nu_\mu) = (5.2 \pm 1.5) \cdot 10^{-4},$$

consistent with the experimental upper limit $\text{Br}(D^+ \to \mu^+ \nu_\mu) < 7.2 \cdot 10^{-4}$ [10].

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