

1. Introduction

We analyze the effect of *imperfect patent protection* on entry and licensing decisions of firms,¹ focusing on the case where the (potential) entrant is the patent holder of cost-reducing process innovation. The patent holder can offer an exclusive or a sole license for its technology to the incumbent.² In our framework, licensing is weakened by the possibility to imitate the new technology: the incumbent can refuse the offer and imitate, with a probability to be acquitted of patent infringement. The possibility of non-convicted imitation reduces the values of both exclusive and sole licenses. This changes the equilibria with respect to the full protection case, making in some cases the sole license more profitable. We prove that the exclusive licenses maximize innovator's expected profits only in the case of drastic innovation and patent protection high enough. In all the other cases, the innovator prefers entering the market proposing a sole license. This result has important consequences on the R&D investment decisions, and on social welfare. In the second part of the analysis, we extend the entry game introducing a pre-entry stage where the potential entrant decides whether and how much to invest in the R&D stochastic process leading to licensed innovation. We show that increasing the level of protection always increases R&D return appropriability; however, it does not always increase expected welfare but, boosting market structure from duopoly to monopoly, it creates a trade-off between dynamic efficiency and allocative efficiency.

1.1. Related literature

The theoretical literature on licensing is huge, dating back at least at Katz and Shapiro (1985), who shows how the possibility of licensing may raise prices and decrease the returns on innovation. Similar contributions in Kamien and Tauman (1986), Gallini and Wright (1990), San Martín and Saracho (2010), and Duchene et al. (2015). More sporadically, some authors analyzed the effect of licensing on social welfare: Fauli-Oller and Sandonis (2002), Chowdhury (2005), Sen and Tauman (2007), Bertran and Turner (2017). Others have analyzed the impact of licensing on the firm's incentive to innovate, obtaining, in general, that when a firm can license its innovation, the R&D effort is higher.³ Furthermore, increasing R&D has positive effects on welfare, when licensing is allowed (Colombo and Filippini, 2014), with some differences according to the type of licensing contract; e.g. fixed fee, or ad valorem royalties (San Martín and Saracho, 2010).

Our paper extends this literature introducing imperfect patent protection. We evaluate how the possibility of non-convicted imitation directly affects the profitability of licensing agreements, and indirectly impacts on the firms' incentive to innovate, the probability of innovation, and the expected welfare. Differently to previous contributes, increasing R&D return appropriability by strengthening the level of patent protection might be welfare decreasing.

¹Licensing is a contractual agreement between firms, including the possibility to exploit an asset protected by patents in return of license fees.

²An *exclusive license* ensures that no person or firm other than the named licensee can exploit the relevant intellectual property rights, i.e. use innovation. A *sole license* grants to the licensee the right to use the innovation in a non-exclusive way, since the licensor can exploit the innovation too.

³See, among the others, Salant (1984), Gallini and Winter (1985), Mukherjee and Mukherjee (2013), Colombo (2019).

2. The model

An incumbent (I) produces in monopoly with a constant marginal cost $c_I = c$, where $c \in (0, 1]$; a patent holder (E) owns a technological innovation that reduces the cost from c_I to $c_E = 0$. Firm E can commercially exploit innovation in three different ways. First, firm E can enter the market competing with the incumbent, refusing any licensing contract: thus, the innovator is more efficient and the duopoly is asymmetric. Second, firm E can enter the market and propose a sole license at a fixed fee L^e , compatible with a symmetric duopolistic competition where both firms use the more efficient technology, obtaining symmetric profits. Third, firm E can stay out of the market and propose an exclusive license at a fixed fee L^{ne} that preserves the incumbent's monopolistic position.⁴ We assume that the patent holder E has all the bargaining power, proposing take-it-leave-it licensing contracts.⁵ Firm I can refuse the proposal and violate the patent, imitating innovation. In this case, firm E asserts its patents against the infringer, and the Court (nature) convicts firm I of patent infringement with probability g . If firm E wins the trial, it obtains a fine F equal to firm I 's profit.⁶

The timing of the game is the following:

- at $t=1$, firm E enters (e) or not the market (ne);
- at $t=2$, firm E offers (l) or not (nl) a fixed-fee license (L^{ne} or L^e) to firm I ;
- at $t=3$, if at $t=2$ firm E offers the license, firm I accepts (a) or not (na);
- at $t=4$, if either at $t=2$ firm E does not propose the license or at $t=3$ firm I refuses the proposal, firm I imitates (m) or not the innovation (nm);
- at $t=5$, firms compete in the market;
- at $t=6$, if at $t=4$ firm I imitates the innovation, the Court convicts I to pay a fine F with probability g .

At $t = 1$, the game has two subgames: the non-entry and the entry one. In the non-entry subgame, the incumbent I maintains its monopolistic position, producing with efficient technology either in the case firms sign an exclusive license or in the case the incumbent imitates the innovation. In the entry subgame, we observe either a symmetric duopoly (both firms use the innovation) or an asymmetric duopoly (only the entrant uses innovation). The symmetric duopoly is efficient, and it occurs either when firm I accepts a sole license or when it refuses the agreement and imitates E .

We assume linear demand function, such that $p = 1 - q_I - q_E$, where p is the price and q_I and q_E the Incumbent and Entrant's quantities. We distinguish between drastic and non-drastic innovation, i.e., respectively with $c \in (0, \frac{1}{2})$ and $c \in [\frac{1}{2}, 1]$ where, in the case of drastic innovation, the incumbent's profit in the case of the asymmetric duopoly is null. Thus, the monopolistic profits, in case the incumbent adopts or not the innovation, are:⁷ $\Pi_I(0) = \frac{1}{4}$; $\Pi_I(c) = \frac{(1-c)^2}{4}$. In the case of quantity competition

⁴We focus on fixed-fee licensing since this tariff is preferred in the case of an outsider innovator (Kamien and Tauman, 1986). Moreover, sometimes the rival's output is difficult to observe, making the use of per-unit royalty impossible. Similarly, ad valorem royalties are more difficult to implement than fixed fee. Optimal two-part tariff and imperfect patent protection are explored in Capuano and Grassi (2019).

⁵The bargaining power of the firms defines the signed value of the license. Different bargaining mechanisms can be applied (see, *inter alia*, Watanabe and Muto, 2008; and Spulber, 2016).

⁶Some literature studies alternative fines schemes (see, *inter alia*, Krasteva, 2014): according to a 2012 PWC patent litigation study, fines in patent litigations accounted for 81.9% of damages awarded in 2006–2011. In our model, lower fines raise the negative impact of imperfect patent protection on the equilibrium licensing fixed-fee, on the innovator's expected profits, increasing the strength of our results.

⁷We use the following notation: $\Pi_j(c_j)$ is the monopolistic profit at a marginal cost $c_j = 0, c$; $\Pi_j(c_j, c_k)$ is

(à la Cournot), the symmetric duopolistic profits are: $\Pi_E(0, 0) = \Pi_I(0, 0) = \frac{1}{9}$. The asymmetric ones are:

$$\Pi_E(0, c) = \begin{cases} \frac{(1+c)^2}{9} & \text{if } c \in (0, \frac{1}{2}) \\ \frac{1}{4} & \text{if } c \in [\frac{1}{2}, 1] \end{cases} \quad \Pi_I(c, 0) = \begin{cases} \frac{(1-2c)^2}{9} & \text{if } c \in (0, \frac{1}{2}) \\ 0 & \text{if } c \in [\frac{1}{2}, 1] \end{cases}$$

In the case of imitation (labeled with m) we have to distinguish between two cases: (e) the patent holder enters the market, and (ne) it does not. In the first case, both firms compete with the more efficient technology and firm I (the imitator) pays a fine to firm E (the patent holder) with probability g equal to its profit. We have: $\Pi_E^{e,m}(0, 0) = \frac{1}{(3-g)^2}$ $\Pi_I^{e,m}(0, 0) = \frac{1-g}{(3-g)^2}$. In the second case, imitation occurs in a context where the imitator plays in monopoly. Profits are: $\Pi_E^{ne,m}(0, 0) = \frac{g}{4}$, $\Pi_I^{ne,m}(0, 0) = \frac{1-g}{4}$.

3. The SPNE of the game

We start considering the benchmark case of full patent protection (i.e., $g = 1$), where imitation is always convicted and non-profitable.

In the case of non-entry, the patent holder can license in exclusive the incumbent, which maintains its monopolistic position. Now, the incumbent will produce using the efficient technology, obtaining a market profit equal to $\Pi_I(0)$. In the case of no agreement, it produces with the inefficient technology, obtaining $\Pi_I(c)$. Thus, the optimal exclusive license (L^{ne}) leaves the incumbent with its outside option profit ($\Pi_I(c)$). In equilibrium, the patent holder's profit is equal to the value of the exclusive license.

$$\Pi_E^{ne,l,a} = L^{ne} = \Pi_I(0) - \Pi_I(c) = \frac{1}{4} - \frac{(1-c)^2}{4} \quad (1)$$

In the case of entry, firms compete in a duopoly. A sole license lets the incumbent produce with the more efficient technology, obtaining a market profit equal to $\Pi_I(0, 0)$. Otherwise, in the case of no agreement, it produces with the inefficient technology, obtaining $\Pi_I(c, 0)$. In equilibrium, the patent holder's profit is equal to the sum of its duopolistic profit and the value of the sole license.

$$\Pi_E^{e,l,a} = \Pi_E(0, 0) + L^e = 2\Pi_E(0, 0) - \Pi_I(c, 0) \quad (2)$$

$$= \begin{cases} \frac{2}{9} - \frac{(1-2c)^2}{9} & \text{if } c \in (0, \frac{1}{2}) \\ \frac{2}{9} & \text{if } c \in [\frac{1}{2}, 1] \end{cases} \quad (3)$$

Comparing Equations (1) and (2), we obtain the following result.

Result 1. *With full patent protection ($g = 1$), the patent holder maximizes its profit:*

- *entering the market and proposing a sole license*

$$L^e = \begin{cases} \frac{2}{9} - \frac{(1-2c)^2}{9} & \text{if } c \in (0, \frac{1}{2}) \\ \frac{2}{9} & \text{if } c \in [\frac{1}{2}, 1] \end{cases}$$

the duopolistic profit of firm j at a marginal cost $c_j = 0, c$ when rival's marginal cost is $c_k = 0, c$.

- staying out of the market and proposing an exclusive license

$$L^{ne} = \frac{1}{4} - \frac{(1-c)^2}{4} \quad \text{if } c \in \left(\frac{2}{3}, 1\right]$$

The intuition of the previous result is the following. In the case of no entry, the patent holder's profit, equal to the exclusive license fee L^{ne} , is negatively correlated with the incumbent's profit when no agreement occurs ($\Pi_I(c)$). For low values of c , the exclusive license fee is too low and the patent holder prefers to enter, adding the market profit to the license revenue. In other words, when c is too low (i.e., $c \in (0, \frac{2}{3})$), the patent holder maximizes its profits entering the market; when the innovation is very drastic (i.e., $c \in (\frac{2}{3}, 1]$), the patent holder stays out of the market, signing an exclusive license with the incumbent.

In the case of imperfect patent protection (i.e., $g \in [0, 1)$), imitation may be profitable. Imperfection affects the outside option profit of the licensee, impacting on the patent holder's decisions of R&D investment, market-entry, and licensing proposal.

To obtain the SPNE, we solve the game by backward induction by comparing the profits obtained in the previous section. We start from the case of non-drastic innovation, obtaining the following proposition:

Proposition 1. *When innovation is not drastic, for any level of patent protection firm E enters the market and firms E and I sign a sole license contract.*

Proof. In Appendix.

From Proposition 1, we infer the following result:

Result 2. *In the case of non-drastic innovation firms sign a sole license (L^e) and its value is:*

- $L^e = \frac{g}{9}$, when the patent protection is low; i.e., $\forall g \in [0, g_1]$.
- $L^e = \frac{4c(1-c)}{9}$, when the patent protection is high enough; i.e., $\forall g \in (g_1, 1]$.

where $g_1 = c(2-c)$ is the threshold value of the level patent protection g such that at $t = 4$ imitation is profitable after the patent holder's entry at $t = 1$ (entry subgame).

When the innovation is non-drastic, the incumbent's duopolistic profit is positive even though it does not adopt the innovation. This negatively affects the value of the license, either in the case of profitable imitation or not. The low value of the license, on the one hand, induces the incumbent to accept the agreement; on the other, it forces the patent holder to enter the market to add market profits to the ones by licensing. For any values of $c \in (0, \frac{1}{2})$, the value of the license fee is increasing in the level of patent protection until imitation is profitable ($g \in [0, g_1]$), then it is constant since the incumbent's outside option is equal to the asymmetric duopolistic profit $\Pi_I(c, 0)$.

In the drastic innovation case ($c \in [\frac{1}{2}, 1)$), we can state the following proposition:

Proposition 2. *When innovation is drastic:*

(i) *When the level of patent protection is low (i.e., $g \leq \min[g_1, 0.621]$), E enters the market and firms sign a sole license;*

(ii) *When the level of patent protection is high enough (i.e., $g > \min[g_1, 0.621]$), E does not enter the market proposing the exclusive license and I accepts, otherwise;*

where $g = 0.621$ is the threshold value such that at $t = 1$ entry is more profitable than non-entry when firms sign a license agreement (sole or exclusive).

Proof. In Appendix.

When the innovation is drastic, the patent holder prefers to stay out of the market and sign an exclusive license agreement only if the level of patent protection is high enough. For low values of g , the exclusive license fee is too low and the patent holder prefers to enter, adding the market profit to the license revenue. In other words, when g is too low (i.e., $g \leq \min[g_1, 0.621]$), the patent holder maximizes its profits entering the market; when g is high enough (i.e., $g > \min[g_1, 0.621]$), the patent holder stays out of the market, signing an exclusive license with the incumbent.

From Proposition 2, we deduce the following result:

Result 3. *In the case of drastic innovation the contracted license can be sole (L^e) or exclusive (L^{ne}) and its value is:*

$$\begin{aligned}
 & - L^e = \frac{g(3+g)}{9(3-g)^2}, \forall g \leq \min[g_1, 0.621]; \\
 & - L^{ne} = \begin{cases} \frac{g}{4} & \text{if } g \in [0, g_1] \cup [0.621, 1] \\ \frac{c(2-c)}{4} & \text{if } g \in (g_1, 1] \end{cases}
 \end{aligned}$$

Figure 1 describes Propositions 1 and 2. Either when the innovation is non-drastic or when it is drastic but the level of patent protection is low (white area), in equilibrium the patent holder enters the market and firms sign a sole license, competing in a symmetric duopoly. In the complementary set (grey area), the patent holder stays out of the market and firms sign an exclusive license. Thus, a monopoly emerges in equilibrium.

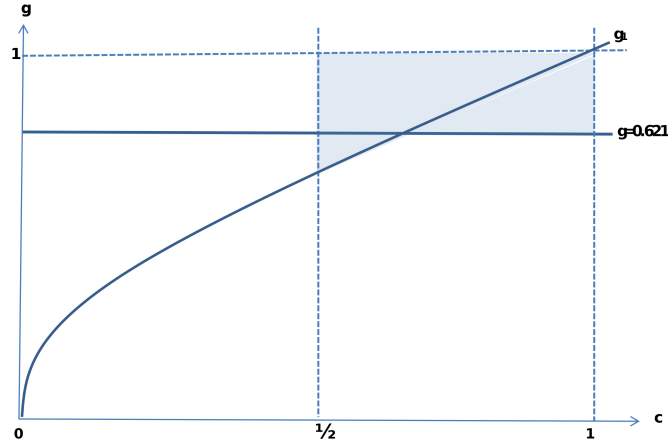


Figure 1. The market structure in the space c, g .

4. Welfare analysis

In this section, we extend our analysis considering a pre-entry stage ($t = 0$) where the potential entrant takes the R&D investment decision in order to develop the cost-reducing innovation that, in case of success, will be commercially exploited. We focus

on a simple case of stochastic innovation, where at $t = 0$ the potential entrant chooses the level of R&D expenditure ($\rho^2/2$) such that the probability of a cost-reduction from c to 0 is equal to $\rho \in [0, 1]$. In our framework, the expected R&D return on innovation (hereafter, R_E) is a proxy of the incentive to innovate. We compute R as the sum of the patent holder's market and license profits in the extended game (hereafter, Π_E), weighted by the probability to innovate ρ , net of the investment cost $\rho^2/2$;⁸ thus, we have: $R_E = \rho\Pi_E - \frac{\rho^2}{2}$. Maximizing R_E with respect to ρ , we obtain $\rho = \Pi_E$, and $R_E = \frac{\Pi_E^2}{2}$.

When innovation does not occur, expected profits are equal to the loss associated with the R&D expenditure. When innovation occurs, we distinguish two scenarios. In one case, the patent holder does not enter the market and firms sign an exclusive licensing contract. This occurs when the innovation is drastic and the level of patent protection is high enough (i.e.; $g \geq \min[g_1, 0.621]$). In the second case, the patent holder enters the market and firms sign a sole licensing contract. This occurs either when the innovation is non-drastic, or when the innovation is drastic and the level of patent protection is low enough (i.e.; $g < \min[g_1, 0.621]$).

Table 1 summarizes the equilibrium values of the following functions: the number of firms operating in the market ($N = 1, 2$), the type and the value of the license (L), the R&D return (R_E), the welfare in the case of succeeding (W_S), the welfare in the case of non-succeeding (W_{NS}),⁹ and the probability of innovation (ρ), associated with the three different subsets in the parameter space.

Table 1. Equilibrium values.

Parameter Set	N	L	Z_E	W_S	W_{NS}	ρ
$g \in [0.621, g_1]$	1	$L^{ne} = \frac{g}{4}$	$\frac{g^2}{32}$	$\frac{3}{8}$	$\frac{3(1-c)^2}{8}$	$\frac{g}{4}$
$c \geq 0.5$ and $g \geq g_1$	1	$L^{ne} = \frac{c(2-c)}{4}$	$\frac{c^2(2-c)^2}{32}$	$\frac{3}{8}$	$\frac{3(1-c)^2}{8}$	$\frac{c(2-c)}{4}$
$c < 0.5$	2	$L^e = \frac{4c(1-c)}{9}$	$\frac{(1+4c-4c^2)^2}{162}$	$\frac{4}{9}$	$\frac{3(1-c)^2}{8}$	$\frac{(1+4c-4c^2)}{9}$
$g \leq \min[g_1, 0.621]$	2	$L^e = \frac{g(3+g)}{9(3-g)^2}$	$\frac{(3g^2-3g+9)^2}{162(3-g)^4}$	$\frac{4}{9}$	$\frac{3(1-c)^2}{8}$	$\frac{3g^2-3g+9}{9(3-g)^2}$

The general form of the expected welfare is equal to

$$EW = \rho W_S + (1 - \rho)W_{NS} - \frac{\rho^2}{2} \quad (4)$$

It changes according to the market structure, becoming:

$$EW (g \in [0.621, g_1]) = \frac{12 - 24c + 6cg + 12c^2 - g^2 - 3c^2g}{32} \quad (5)$$

$$EW (c \geq 0.5 \text{ and } g \geq g_1) = \frac{10c^2 - 12c - 4c^3 + c^4 + 6}{16} \quad (6)$$

$$EW (c < 0.5) = \frac{400c^2 - 464c - 196c^3 + 44c^4 + 239}{648} \quad (7)$$

⁸Notice that, in case of failing in innovation (with probability $1 - \rho$), the potential entrant profit is null.

⁹In the case of non-succeeding in innovation, only the incumbent produces at a marginal cost equal to $c_I = c > 0 \implies W_{NS} = \frac{3(1-c)^2}{8}$.

$$EW(g \leq \min[g_1, 0.621]) = \frac{(5g^2 - 50g + 69)(g^2 - g + 3)}{72(3 - g)^2} + \frac{(2g^2 - 17g + 24)}{3(3 - g)^2} \frac{3(1 - c)^2}{8} \quad (8)$$

In any subset of the parameter space, the expected welfare function is increasing in the level of patent protection g .¹⁰ However, variations of the parameters cause discontinuities. In the following, in order to simplify the analysis, we compare the levels of the expected welfare, focusing on two arbitrary values of the pre-innovation marginal cost c : (i) $c = \frac{1}{3}$, non-drastic innovation; (ii) $c = \frac{2}{3}$, (very) drastic innovation.

When $c = \frac{1}{3}$, in equilibrium we face a duopoly with a sole license; increasing g over g_1 does not affect the expected welfare, that is $EW(c < 0.5)|_{c=\frac{1}{3}} = 0.18837$.

When $c = \frac{2}{3}$, increasing g over g_6 modifies the structure of the market that passes from duopoly to monopoly. Thus, the expected welfare is:

$$\begin{aligned} EW(g \leq \min[g_1, 0.621])|_{c=\frac{2}{3}} &= \frac{(136g^2 - 236g - 55g^3 + 5g^4 + 231)}{72(3 - g)^2} \\ EW(g \in [0.621, g_1])|_{c=\frac{2}{3}} &= \frac{4 - 3g^2 + 8g}{96} \\ EW(c \geq 0.5 \text{ and } g \geq g_1)|_{c=\frac{2}{3}} &= 0.091 \end{aligned}$$

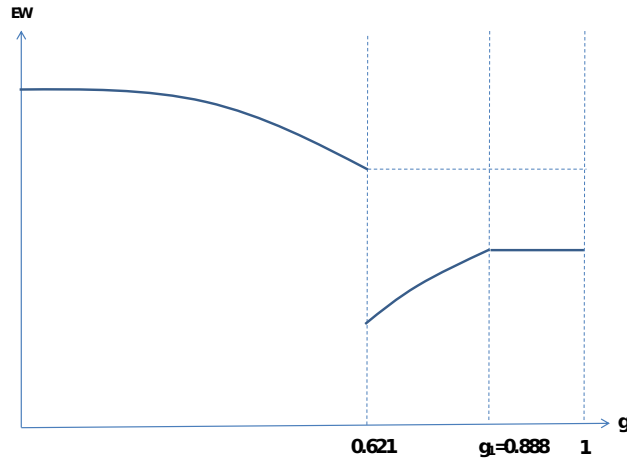


Figure 2. Expected welfare in the drastic innovation case ($c = 2/3$).

In the case of non-drastic innovation, the expected welfare is constant with respect to the level of patent protection, as well as the probability of innovation. In the case of drastic innovation, increasing the level of patent protection increases the probability of innovation but has a not univocal effect on the expected welfare. As depicted in Figure 2, for low levels of patent protection ($g \in [0, 0.621]$) a duopoly emerges in equilibrium and increasing g decreases EW . For a higher level of patent protection ($g \in (0.621, 1]$), a monopoly emerges. Increasing g over 0.621, the expected welfare has a downward

¹⁰Proof available on request.

jump due to the change in market structure, and it is increasing g . However, when a monopoly emerges, expected profits is lower than the values assumed when a duopoly emerges. The previous analysis leads us to the following results.

Result 4. *Increasing the level of patent protection: (i) in the case of non-drastic innovation, it does not affect the expected welfare; (ii) in the case of drastic innovation, it may decrease the expected welfare.*

Result 4 shows that, in some circumstances, increasing the level of patent protection may decrease the expected welfare. It happens in the case of drastic innovation when the structure of the market passes from a duopoly to monopoly. Although strengthening the patent system may lead to the generation of more innovation, too strong protection systems can trigger a high deadweight loss, due to the monopoly that patents grant. Therefore, stronger patent protection (for example, increasing the length of IPR protection obtained or enlarging the type of knowledge or creative expression protected), does not necessarily lead to higher expected social welfare. In other words, in equilibrium, there exists a trade-off between static and dynamic efficiency: the higher the level of patent protection, the higher the R&D expected returns. Hence, firms have a higher incentive to invest and the probability of succeeding in developing innovation increases. When the innovation is drastic, for levels of patent protection high enough, firms sign an exclusive license, the incumbent keeps its monopolistic position with an associated deadweight loss. Conversely, a low level of patent protection reduces R&D returns and expenditure, decreases the probability of succeeding in innovation, but, when innovation occurs, we expect oligopolistic competition and higher allocative efficiency.

5. Conclusion

The literature on innovation has often seen the imitation process is harmful to the development of innovation, detrimental to growth, consumers' and producers' surplus, and society at large. Thus, policies aimed at strengthening the patent system and discouraging imitation processes are associated with improvements in social welfare. However, the strengthening of patent systems could cause a decrease in the competition (and finally a reduction in the production and assimilation of new technologies), could create barriers to entry, increasing the costs of production by expansive licenses.

We exploit the trade-off between incentives to invest in R&D and competitiveness in the market presenting a licensing model where ex-ante R&D investment decisions are affecting by imperfect patent protection. The first result highlights how under non-drastic innovation, exclusive licensing is never profitable in equilibrium; i.e., it never emerges as part of the SPNE, and different levels of patent protection do not affect the post-innovation market structure (always duopolistic), just unfolding the profit distribution between firms. On the contrary, in the case of drastic innovation and levels of patent protection high enough, exclusive licensing can emerge as part of the SPNE. When patent protection is less effective, the incumbent has a positive incentive to imitate the entrant; this reduces the value of the license and boosts the innovator to enter the market and compete with the incumbent. The second result deals with welfare: imperfect patent protection affects welfare twofold. On the one hand, it boosts the entry of a new competitor increasing the allocative efficiency in the market; on the other hand, it reduces the license value, negatively affecting the R&D returns, as well as firms' incentives to innovate.

Appendix A

Proof of Proposition 1. Consider the non-entry subgame. At $t = 4$, firm I imitates the innovation iff $\Pi_I^{ne,l,na,m} \geq \Pi_I^{ne,l,na,nm} \Leftrightarrow \frac{1-g}{4} \geq \frac{(1-c)^2}{4} \Leftrightarrow g \leq c(2-c) = g_1$. We have two subcases: $g \leq g_1$ or $g > g_1$. Assume $g \leq g_1$, at $t = 3$, firm I accepts the license iff $\Pi_I^{ne,l,a} \geq \Pi_I^{ne,l,a,m} \Leftrightarrow \frac{1}{4} - L^{ne} \geq \frac{1-g}{4} \Leftrightarrow L^{ne} \leq \frac{g}{4}$; at $t = 2$, firm E proposes the license iff $\Pi_E^{ne,l,a} \geq \Pi_E^{ne,l,na,m} = \Pi_E^{ne,nl,m} \Leftrightarrow L^{ne} \geq \frac{g}{4}$. Assume $g > g_1$, at $t = 3$, firm I accepts the license iff $\Pi_I^{ne,l,a} \geq \Pi_I^{ne,l,na,nm} \Leftrightarrow \frac{1}{4} - L^{ne} \geq \frac{(1-c)^2}{4} \Leftrightarrow L^{ne} \leq \frac{c(2-c)}{4}$; at $t = 2$, firm E proposes the license iff $\Pi_E^{ne,l,a} \geq \Pi_E^{ne,l,na,nm} = \Pi_E^{ne,nl,nm} \Leftrightarrow L^{ne} \geq 0$. Consider the entry subgame. At $t = 4$, firm I imitates the innovation iff $\Pi_I^{e,l,na,m} \geq \Pi_I^{e,l,na,nm} \Leftrightarrow \frac{1-g}{(3-g)^2} \geq \frac{(1-2c)^2}{9} \Leftrightarrow g \leq \frac{1}{4c^2-4c+1} \left(12c^2 - 12c - \frac{3}{2}\sqrt{32c-32c^2+1} - \frac{3}{2} \right) = g_2$ where $\forall c \in [0, 1]$, $g_2 < 0 \Rightarrow g > g_2$ always. Since $g > g_2$, at $t = 3$, firm I accepts the license iff $\Pi_I^{e,l,a} \geq \Pi_I^{e,l,na,nm} \Leftrightarrow \frac{1}{9} - L^e \geq \frac{(1-2c)^2}{9} \Leftrightarrow L^e \leq \frac{4c(1-c)}{9}$; at $t = 2$, firm E proposes the license iff $\Pi_E^{e,l,a} \geq \Pi_E^{e,l,na,nm} = \Pi_E^{e,nl,nm} \Leftrightarrow \frac{1}{9} + L^e \geq \frac{(1+c)^2}{9} \Leftrightarrow L^e \geq \frac{c(2+c)}{9}$, where $\frac{c(2+c)}{9} \leq \frac{4c(1-c)}{9}$, $\forall c < \frac{2}{3}$. Consider now the entry decision at $t = 1$. Since $\forall c \in (0, \frac{1}{2}]$, $g_1 = c(2-c) > 0 > g_2$, then g can assume values such that: (a) $0 \leq g < g_1$; or (b) $g > g_1 \geq 0$. (a) When $0 \leq g < g_1$, $L^e = \frac{4c(1-c)}{9}$ and $L^{ne} = \frac{g}{4}$: firm E enters the market proposing a sole license (that will be accepted by firm I) iff $\Pi_E^{e,l,a} \geq \Pi_E^{ne,l,a} \Leftrightarrow \frac{1}{9} + \frac{4c(1-c)}{9} \geq \frac{g}{4} \Leftrightarrow g \leq \frac{4(1-4c^2+4c)}{9} = g_3$, where $g_3 > g_1$, $\forall c \in (0, \frac{1}{2}]$ (always). (b) When $g > g_1$, $L^e = \frac{4c(1-c)}{9}$ and $L^{ne} = \frac{c(2-c)}{4}$: firm E enters the market proposing a sole license (that will be accepted by firm I) iff $\Pi_E^{e,l,a} = \frac{1}{9} + \frac{4c(1-c)}{9} \geq \Pi_E^{ne,l,a} = \frac{c(2-c)}{4} \Rightarrow c \leq 0.62645$ (always). Combining results (a), (b) we have that firm E enters the market proposing a sole license $\forall (c, g) \in (0, \frac{1}{2}] \times [0, 1]$.

Proof of Proposition 2. From Proposition 1, in the non-entry subgame, firms sign an exclusive licensing agreement iff $L^{ne} = \frac{g}{4}$ when $g \leq g_1$, and iff $L^{ne} = \frac{c(2-c)}{4}$ when $g > g_1$. Consider now the entry subgame. At $t = 4$, firm I imitates the innovation iff $\Pi_I^{e,l,na,m} = \frac{1-g}{(3-g)^2} \geq 0 = \Pi_I^{e,l,na,nm} \Rightarrow g \leq 1 = g_4$ (always). At $t = 3$, firm I accepts the license iff $\Pi_I^{e,l,a} = \frac{1}{9} - L^e \geq \frac{1-g}{(3-g)^2} = \Pi_I^{e,l,na,m} \Rightarrow L^e \leq \frac{1}{9} - \frac{1-g}{(3-g)^2} = \frac{g(3+g)}{9(3-g)^2}$; at $t = 2$, firm E proposes the license iff $\Pi_E^{e,l,a} = \frac{1}{9} + L^e \geq \frac{g}{(3-g)^2} = \Pi_E^{e,nl,m} = \Pi_E^{e,l,na,m} \Rightarrow L^e \geq \frac{g}{(3-g)^2} - \frac{1}{9} = \frac{(15g-g^2-9)}{9(3-g)^2}$. Thus, firms sign a license agreement iff $L^e \leq \frac{g(3+g)}{9(3-g)^2}$ and $L^e \geq \frac{(15g-g^2-9)}{9(3-g)^2} \Rightarrow L^e \in \left[\frac{(15g-g^2-9)}{9(3-g)^2}, \frac{g(3+g)}{9(3-g)^2} \right]$; this occurs $\forall g \leq g = 0.878 = g_5$. Otherwise, no licensing agreement is signed.

Consider now the firm E 's entry decision at $t = 1$. Since $g_1 = c(2-c) < g_5$ when $c < 0.650$, we have four cases: (a) $g < \min[g_1, g_5]$; (b) $g_1 < g < g_5$; (c) $g > \max[g_1, g_5]$; and (d) $g_5 < g < g_1$. (a) When $g < \min[g_1, g_5]$, $L^e = \frac{g(3+g)}{9(3-g)^2}$ and $L^{ne} = \frac{g}{4}$: firm E enters the market proposing a sole license (that will be accepted by firm I) iff $\Pi_E^{e,l,a} = \frac{1}{9} + \frac{g(3+g)}{9(3-g)^2} \geq \Pi_E^{ne,l,a} = \frac{g}{4} \Rightarrow g \leq g_6 = 0.621 < g_5$; firm E does not enter the market proposing an exclusive license (that will be accepted by firm I) iff $g > g_6$.

(b) When $g_1 < g < g_5$, $L^e = \frac{g(3+g)}{9(3-g)^2}$ and $L^{ne} = \frac{c(2-c)}{4}$: firm E does not enter the market proposing an exclusive license (that will be accepted by firm I) iff $\Pi_E^{e,l,a} = \frac{1}{9} + \frac{g(3+g)}{9(3-g)^2} \leq \Pi_E^{ne,l,a} = \frac{c(2-c)}{4} \Rightarrow g \geq g_7 = \frac{(27c^2-54c-6\sqrt{36c-18c^2-7}+6)}{18c+9c^2+8}$ where $g_7 < 0$ (al-

ways). (c) When $g > MAX[g_1, g_5]$, profits are $\Pi^e = \frac{g}{(3-g)^2}$ and $\Pi^{ne} = L^{ne} = \frac{c(2-c)}{4}$: firm E does not enter the market proposing an exclusive license (that will be accepted by firm I) iff $\Pi^e = \frac{g}{(3-g)^2} \leq \Pi^{ne} = \frac{c(2-c)}{4} \Rightarrow g \leq g_8 = \frac{2+6c-3c^2+2\sqrt{6c-3c^2+1}}{2c-c^2}$ where $g_8 > 1$ (always). (d) When $g_5 < g < g_1$, profits are $\Pi^e = \frac{g}{(3-g)^2}$ and $\Pi^{ne} = L^{ne} = \frac{g}{4}$: firm E does not enter the market proposing an exclusive license (that will be accepted by firm I) iff $\Pi^e = \frac{g}{(3-g)^2} \leq \Pi^{ne} = \frac{g}{4} \Rightarrow g \in [0, 1]$ (always).

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