

## MAPPING THE GALAXY COLOR–REDSHIFT RELATION: OPTIMAL PHOTOMETRIC REDSHIFT CALIBRATION STRATEGIES FOR COSMOLOGY SURVEYS

DANIEL MASTERS<sup>1</sup>, PETER CAPAK<sup>2</sup>, DANIEL STERN<sup>3</sup>, OLIVIER ILBERT<sup>4</sup>, MARA SALVATO<sup>5</sup>, SAMUEL SCHMIDT<sup>6</sup>, GIUSEPPE LONGO<sup>7</sup>, JASON RHODES<sup>3,8</sup>, STEPHANE PALTANI<sup>9</sup>, BAHRAM MOBASHER<sup>10</sup>, HENK HOEKSTRA<sup>11</sup>, HENDRIK HILDEBRANDT<sup>12</sup>, JEAN COUPON<sup>9</sup>, CHARLES STEINHARDT<sup>1</sup>, JOSH SPEAGLE<sup>13</sup>, ANDREAS FAISST<sup>1</sup>, ADAM KALINICH<sup>14</sup>, MARK BRODWIN<sup>15</sup>, MASSIMO BRESCIA<sup>16</sup>, AND STEFANO CAVUOTI<sup>16</sup>

<sup>1</sup> Infrared Processing and Analysis Center, California Institute of Technology, Pasadena, CA 91125, USA

<sup>2</sup> Spitzer Science Center, California Institute of Technology, Pasadena, CA 91125, USA

<sup>3</sup> Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA

<sup>4</sup> Aix Marseille Université, CNRS, LAM (Laboratoire d'Astrophysique de Marseille) UMR 7326, F-13388, Marseille, France

<sup>5</sup> Max-Planck-Institut für extraterrestrische Physik, Giessenbachstrasse, D-85748 Garching, Germany

<sup>6</sup> Department of Physics, University of California, Davis, CA 95616, USA

<sup>7</sup> Department of Physics, University Federico II, via Cinthia 6, I-80126 Napoli, Italy

<sup>8</sup> Kavli Institute for the Physics and Mathematics of the Universe, The University of Tokyo, Chiba 277-8582, Japan

<sup>9</sup> Department of Astronomy, University of Geneva ch. dcogia 16, CH-1290 Versoix, Switzerland

<sup>10</sup> Department of Physics and Astronomy, University of California, Riverside, CA 92521, USA

<sup>11</sup> Leiden Observatory, Leiden University, P.O. Box 9513, 2300 RA, Leiden, The Netherlands

<sup>12</sup> Argelander-Institut für Astronomie, Universität Bonn, Auf dem H'ugel 71, D-53121 Bonn, Germany

<sup>13</sup> Department of Astronomy, Harvard University, 60 Garden Street, MS 46, Cambridge, MA 02138, USA

<sup>14</sup> Massachusetts Institute of Technology, Cambridge, MA 02139, USA

<sup>15</sup> Department of Physics and Astronomy, University of Missouri, Kansas City, MO 64110, USA

<sup>16</sup> Astronomical Observatory of Capodimonte—INAF, via Moiariello 16, I-80131, Napoli, Italy

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### ABSTRACT

Calibrating the photometric redshifts of  $\gtrsim 10^9$  galaxies for upcoming weak lensing cosmology experiments is a major challenge for the astrophysics community. The path to obtaining the required spectroscopic redshifts for training and calibration is daunting, given the anticipated depths of the surveys and the difficulty in obtaining secure redshifts for some faint galaxy populations. Here we present an analysis of the problem based on the *self-organizing map*, a method of mapping the distribution of data in a high-dimensional space and projecting it onto a lower-dimensional representation. We apply this method to existing photometric data from the COSMOS survey selected to approximate the anticipated *Euclid* weak lensing sample, enabling us to robustly map the empirical distribution of galaxies in the multidimensional color space defined by the expected *Euclid* filters. Mapping this multicolor distribution lets us determine where—in galaxy color space—redshifts from current spectroscopic surveys exist and where they are systematically missing. Crucially, the method lets us determine whether a spectroscopic training sample is representative of the full photometric space occupied by the galaxies in a survey. We explore optimal sampling techniques and estimate the additional spectroscopy needed to map out the color–redshift relation, finding that sampling the galaxy distribution in color space in a systematic way can efficiently meet the calibration requirements. While the analysis presented here focuses on the *Euclid* survey, similar analysis can be applied to other surveys facing the same calibration challenge, such as DES, LSST, and *WFIRST*.

*Key words:* dark energy – dark matter – galaxies: distances and redshifts – large-scale structure of universe – methods: statistical

### 1. INTRODUCTION

Upcoming large-scale surveys such as LSST, *Euclid*, and *WFIRST* will measure the three-dimensional cosmological weak lensing shear field from broadband imaging of billions of galaxies. Weak lensing is widely considered to be one of the most promising probes of the growth of dark matter structure, as it is sensitive to gravitation alone and requires minimal assumptions about the coupling of dark matter and baryons (Bartelmann & Schneider 2001; Weinberg et al. 2013). Moreover, weak lensing tomography is sensitive to the dark energy equation of state through its impact on the growth of structure with time (Hu & Tegmark 1999). However, it is observationally demanding: in addition to requiring accurately measured shapes for the weak lensing sample, robust redshift estimates to the galaxies are needed in order to reconstruct the three-dimensional matter distribution. Because it is infeasible to obtain spectroscopic redshifts (spec-*z*'s) for the huge numbers

of faint galaxies these studies will detect, photometric redshift (photo-*z*) estimates derived from imaging in some number of broad filters will be required for nearly all galaxies in the weak lensing samples.

Photo-*z* estimation has become an indispensable tool in extragalactic astronomy, as the pace of galaxy detection in imaging surveys far outstrips the rate at which follow-up spectroscopy can be performed. While photo-*z* techniques have grown in sophistication in recent years, the requirements for cosmology present novel challenges. In particular, cosmological parameters derived from weak lensing are sensitive to small, systematic errors in the photo-*z* estimates (Huterer et al. 2006; Ma et al. 2006). Such biases are generally much smaller than the random scatter in photo-*z* estimates (Dahlen et al. 2013), and are of little consequence for galaxy evolution studies; however, they can easily dominate all other uncertainties in weak lensing experiments (Newman et al. 2015). In

addition to weak lensing cosmology, accurate and well-characterized photo- $z$ 's will be crucial to other cosmological experiments. For example, baryon acoustic oscillation (BAO) experiments that rely on redshifts measured from faint near-infrared grism spectra will often have to resort to photo- $z$ 's in order to determine the correct redshift assignment for galaxies with only a single detected line. Well-characterized photo- $z$  estimates will be needed to correctly account for any errors thus introduced.

There are two key requirements placed on the photo- $z$  estimates for weak lensing cosmology. First, redshift estimates for individual objects must have sufficient precision to correct for intrinsic galaxy shape alignments as well as other potential systematics arising from physically associated galaxies that may affect the interpretation of the shear signal. While not trivial, meeting the requirement on the precision of individual photo- $z$  estimates ( $\sigma_z < 0.05(1+z)$ ) for *Euclid*, Laureijs et al. (2011) should be achievable (Hildebrandt et al. 2010). The second, more difficult, requirement is that the overall redshift distributions  $N(z)$  of galaxies in  $\sim 10$ – $20$  tomographic bins used for the shear analysis must be known with high accuracy. Specifically, the mean redshift  $\langle z \rangle$  of the  $N(z)$  distribution must be constrained to better than  $2 \times 10^{-3}(1+z)$  in order to interpret the amplitude of the lensing signal and achieve acceptable error levels on the cosmological parameter estimates (Huterer et al. 2006; Amara & Réfrégier 2007; Laureijs et al. 2011). Small biases in the photo- $z$  estimates, or a relatively small number of objects with catastrophically incorrect photo- $z$ 's, can cause unacceptably large errors in the estimated  $N(z)$  distribution. Photo- $z$  estimates alone are not sufficient to meet this requirement, and spectroscopic calibration samples will be needed to ensure low bias in the  $N(z)$  estimates. The significant difficulties associated with this requirement are summarized by Newman et al. (2015).

The most straightforward approach to constrain  $N(z)$  is to measure it directly by random spectroscopic sampling of galaxies in each tomographic redshift bin (Abdalla et al. 2008). The total number of spectra needed to meet the requirement is then set by the central limit theorem. For upcoming ‘‘Stage IV’’ cosmology surveys (LSST, *Euclid*,<sup>17</sup> and *WFIRST*) it is estimated that direct measurement of  $N(z)$  for the tomographic bins would require total spectroscopic samples of  $\sim 30,000$ – $100,000$  galaxies, fully representative in flux, color, and spatial distribution of the galaxies used to measure the weak lensing shear field (e.g., Ma & Bernstein 2008, Hearin et al. 2012). Moreover, the spectroscopic redshifts would need to have a very high success rate ( $\gtrsim 99.5\%$ ), with no subpopulation of galaxies systematically missed in the redshift survey. Newman et al. (2015) note that current deep redshift surveys fail to obtain secure redshifts for  $\sim 30\%$ – $60\%$  of the targeted galaxies; given the depths of the planned dark energy surveys, this ‘‘direct’’ method of calibrating the redshifts seems to be unfeasible.

Because of the difficulty of direct spectroscopic calibration, Newman et al. (2015) argue that the most realistic method of meeting the requirements on  $N(z)$  for the dark energy experiments may be some form of spatial cross-correlation of photometric samples with a reference spectroscopic sample, with the idea that the power in the cross-correlation will be highest when the samples match in redshift (Newman 2008;

Schmidt et al. 2013; Rahman et al. 2015). This approach shows significant promise, but is not without uncertainties and potential systematics. For example, it requires assumptions regarding the growth of structure and galaxy bias with redshift, which may be covariant with the cosmological inferences drawn from the weak lensing analysis itself. Further work may clarify these issues and show that the technique is indeed viable for upcoming cosmological surveys. However, it seems safe to say that this method cannot *solely* be relied on for the weak lensing missions, particularly as at least two approaches will be needed: one to calibrate  $N(z)$  for the tomographic bins, and another to test and validate the calibration.

In light of these arguments, it is clear that targeted spectroscopic training and calibration samples will have to be obtained to achieve the accuracy in the  $\langle z \rangle$  estimates of tomographic bins required by the weak lensing missions. Moreover, careful optimization of these efforts will be required to make the problem tractable. Here we present a technique, based on the simple but powerful *self-organizing map* (SOM) (Kohonen 1982, 1990), to map the empirical distribution of galaxies in the multidimensional color space defined by a photometric survey. Importantly, this technique provides us with a completely data-driven understanding of what constitutes a representative photometric galaxy sample. We can thereby evaluate whether a spectroscopic sample used for training and calibration spans the full photometric parameter space; if it does not, there will be regions where the photo- $z$  results are untested and untrained. Machine learning—based photo- $z$  algorithms, in particular, depend critically on representative spectroscopic training sets, and their performance will be degraded in regions of color space without spectroscopic coverage<sup>18</sup> (Collister & Lahav 2004; Hoyle et al. 2015).

We show that the empirical color mapping described here can be used to optimize the training and calibration effort by focusing spectroscopic effort on regions of galaxy parameter space that are currently poorly explored, as well as regions with a less certain mapping to redshift. Alternatively, we can use the technique to identify and discard specific regions of color space for which spectroscopy will prove to be too expensive, or for which the redshift uncertainty is too large. In effect, the method lets us systematize our understanding of the mapping from color to redshift. By doing so, the number of spectroscopic redshifts needed to calibrate  $N(z)$  for the weak lensing tomographic bins can be minimized. This approach will also naturally produce a ‘‘gold standard’’ training sample for machine learning algorithms.

The technique we adopt also provides insight into the nature of catastrophic photo- $z$  failures by illustrating regions of color space in which the mapping between color and redshift becomes degenerate. This is possible because the self-organized map is topological, with nearby regions representing similar objects, and widely separated regions representing dissimilar ones. In addition, template-fitting photo- $z$  codes can potentially be refined with the map, particularly through the development of data-based priors and by using the empirical color mapping to test and refine the galaxy template sets used for fitting.

<sup>18</sup> This dependence on training sample representativeness tends to be obscured by the photo- $z$  versus spec- $z$  plots most often used to illustrate the quality of photo- $z$  algorithms, which (of necessity) only show results for the subset of galaxies with known spectroscopic redshifts. Of course, those are also the galaxies for which similar training objects exist.

<sup>17</sup> <http://www.euclid-ec.org>

Here our focus is on the *Euclid* survey, one of the three Stage IV dark energy surveys planned for the next decade, the other two being LSST and *WFIRST*. *Euclid* will consist of a 1.2 meter space telescope operating at L2, which will be used to measure accurate shapes of galaxies out to  $z \sim 2$  over  $\sim 15,000 \text{ deg}^2$  with a single, broad (*riz*) filter. These observations will reach an AB magnitude of  $\simeq 24.5$  ( $10\sigma$ ). In addition to these observations, a near-infrared camera on *Euclid* will obtain *Y*, *J*, and *H* band photometry to an AB magnitude  $\simeq 24$  ( $5\sigma$ ), which, together with complementary ground-based optical data, will be used for photo- $z$  determination. The mission will also constrain cosmological parameters using BAO and redshift space distortions (RSD), using redshifts obtained with a low-resolution grism on the near-infrared camera. A more detailed description of the survey can be found in Laureijs et al. (2011).

For this work, we assume that *Euclid* will obtain *ugrizYJH* photometry for photo- $z$  estimation. We select galaxies from the COSMOS survey (Scoville et al. 2007) that closely approximate the *Euclid* weak lensing sample, with photometry in similar bands and at similar depths as the planned *Euclid* survey. While our focus is on *Euclid*, the method we present is general and directly applicable to other weak lensing surveys facing the same calibration problem.

This paper is organized as follows. In Section 2 we give an overview of the methodology used to map the galaxy multicolor space. In Section 3 we discuss the galaxy sample from the COSMOS survey used to approximate the anticipated *Euclid* weak lensing sample. In Section 4 we describe the self-organizing map algorithm and its implementation for this application. In Section 5 we discuss the map in detail, including what it reveals about the current extent of spectroscopic coverage in galaxy multicolor space. In Section 6 we address the problem of determining the spectroscopic sample needed to meet the weak lensing requirement, and in Section 7 we conclude with a discussion.

## 2. OVERVIEW: QUANTIFYING THE EMPIRICAL DISTRIBUTION OF GALAXIES IN COLOR SPACE

Galaxies with imaging in a set of  $N$  filters will follow some distribution in the multidimensional space (of dimension  $N - 1$ ) defined by the unique colors measured by the filters. These colors together determine the shape of the low-resolution spectral energy distribution (SED) measured by the filters. Henceforth, we will call the position a galaxy occupies in color space simply its color, or  $C$ . For example, the *Euclid* survey is expected to have eight bands of photometry (*ugrizYJH*),<sup>19</sup> and therefore a galaxy’s position in color space is uniquely determined by seven colors:  $u - g$ ,  $g - r$ , ...,  $J - H$ . Galaxy color is the primary driver of photometric redshift estimates: template-based methods predict  $C$  for different template/redshift/reddening combinations and assign redshifts to galaxies based on where the models best fit the observed photometry, while machine learning methods assume the existence of a mapping from  $C$  to redshift, and attempt to discover it using spectroscopic training samples.

Our goal here is to empirically map the distribution of galaxies in the color space defined by the anticipated *Euclid* broadband filters. We refer to this distribution as  $\rho(C)$ . Once

<sup>19</sup> This will be the case in the region overlapping with the LSST survey. We note that *Euclid* will also have a broad (*riz*) filter that will be used for the weak lensing shape measurements; our assumption here is that it will not add significant value to the photo- $z$  estimates.

we understand how galaxies are distributed in color space, optimal methods of sampling the distribution with spectroscopy can be developed to make an informed calibration of the color-redshift relation.

The general problem of mapping a high-dimensional data distribution arises in many fields. Because the volume of the data space grows exponentially with the number of dimensions, data rapidly become sparse as the dimensionality increases. This effect—the so-called “curse of dimensionality” (Bellman 1957)—makes normal data sorting strategies impractical. A number of algorithms, collectively referred to as nonlinear dimensionality reduction (NLDR), have been developed to address this problem by projecting high-dimensional data onto a lower-dimensional representation, thus facilitating visualization and analysis of relationships that exist in the data.

We adopt the SOM algorithm, described in more detail in Section 4. As emphasized by Geach (2012), self-organized mapping is a powerful, empirical method to understand the multidimensional distributions common in modern astronomical surveys. Two primary motivations for choosing this technique over others are the relative simplicity of the algorithm and the highly visual nature of the resulting map, which facilitates human understanding of the data.

## 3. APPROXIMATING THE EUCLID WEAK LENSING SAMPLE WITH COSMOS DATA

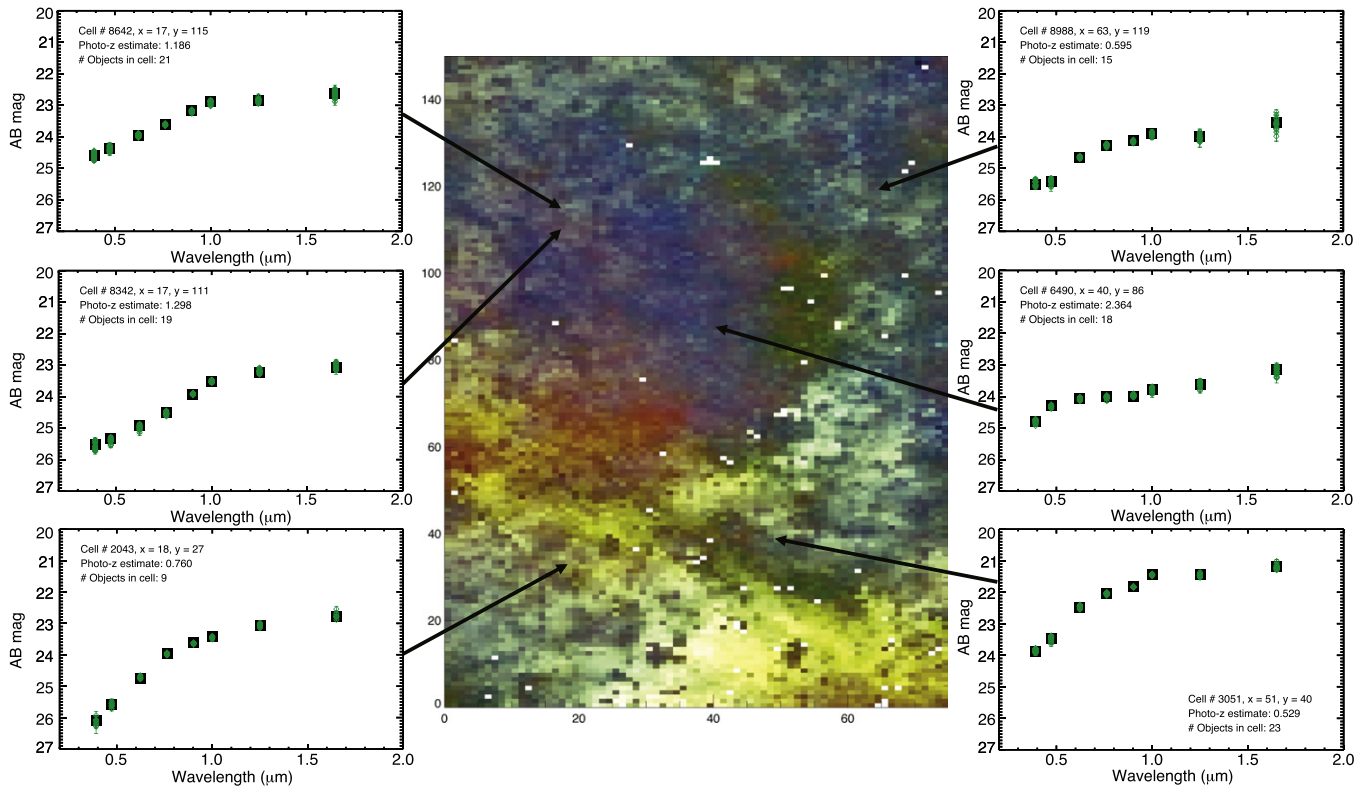
We use multiwaveband data from the COSMOS survey (Capak et al. 2007) to provide a close approximation to the expected *Euclid* weak lensing data. Photo- $z$  estimates for the *Euclid* sample will rely on three near-infrared filters on the telescope (*YJH*), reaching an AB depth of 24 mag ( $5\sigma$ ) for point sources, as well as complementary ground-based imaging in the optical, which we assume will consist of *ugriz* imaging with LSST (in the northern sky the ground-based imaging data may be restricted to *griz*, affecting the analysis somewhat but not changing the overall conclusions).

To provide a close analog to the expected *Euclid* data, we use COSMOS *u* band imaging from CFHT, *griz* imaging from Subaru Suprime Cam, and *YJH* imaging from the UltraVista survey (McCracken et al. 2012), spanning a  $1.44 \text{ deg}^2$  patch of COSMOS with highly uniform depth. We apply a flux cut to the average flux measured across the Subaru *r*, *i*, and *z* bands to match the expected depth limit of the single, broad visible filter *Euclid* will use for the weak lensing shear measurement. The resulting “*Euclid* analog” sample consists of 131,609 objects from COSMOS.

## 4. MAPPING GALAXY COLOR SPACE WITH THE SELF-ORGANIZING MAP

The SOM (Kohonen 1982, 1990) is a neural network model widely used to map and identify correlations in high-dimensional data. Its use for some astronomical applications has been explored previously (see, e.g., Naim et al. 1997; Brett et al. 2004; Way & Klose 2012; Fustes et al. 2013; Carrasco Kind & Brunner 2014; Greisel et al. 2015). The algorithm uses unsupervised, competitive learning of “neurons” to project high-dimensional data onto a lower-dimensional grid. The SOM algorithm can be thought of as a type of nonlinear principal component analysis, and is also similar in some respects to the k-means clustering algorithm (MacQueen 1967). In contrast to these and other methods, the SOM preserves the





**Figure 1.** The 7-color self-organized map (SOM) generated from  $\sim 131k$  galaxies from the COSMOS survey, selected to be representative of the anticipated *Euclid* weak lensing sample. In the center is the  $75 \times 150$  map itself, which encodes the empirical *ugrizYJH* spectral energy distributions (SEDs) that appear in the data. The map is colored here by converting the *H*, *i*, and *u* band photometry of the cells to analogous RGB values, while the brightness is scaled to reflect the average brightness of galaxies in different regions of color space. On the sides we show examples of 8-band galaxy SEDs represented by particular cells, whose positions in the map are indicated with arrows. The cell SEDs are shown as black squares. The actual SEDs (shifted to line up in *i*-band magnitude) of galaxies associated with the cells are overlaid as green diamonds. Between 9 and 23 separate galaxy SEDs are plotted for each of the cells shown, but they are similar enough that they are hard to differentiate in this figure. A key feature of the map is that it is topological in the sense that nearby cells represent objects with similar SEDs, as can be seen from the two example cells shown in the upper left. Note that the axes of the SOM do not correspond to any physical quantity, but merely denote positions of cells within the map and are shown to ease comparison between figures.

topology of the high-dimensional data in the low-dimensional representation. Similar objects are thus grouped together on the self-organized map, and clusters that exist in the high-dimensional data space are reflected in the lower-dimensional representation. This feature makes the maps visually understandable and thus useful for identifying correlations that exist in high-dimensional data. More detailed descriptions of the algorithm and its variants can be found in a number of references (see, e.g., Vesanto 2002; Carrasco Kind & Brunner 2014).

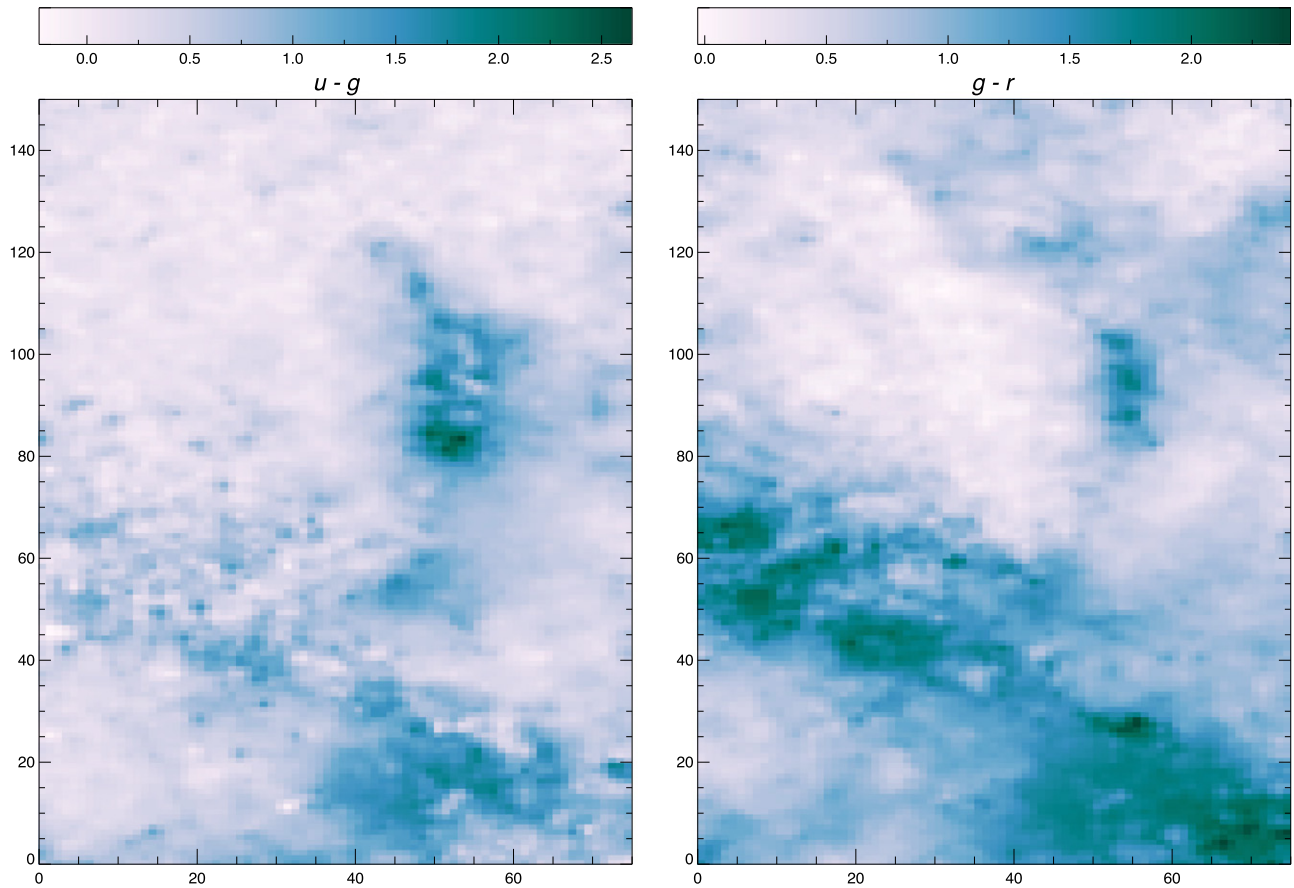
The SOM consists of a fixed number of cells arranged on a grid. The grid can be of arbitrary dimension, although two-dimensional grids are most common as they are the easiest to visualize. Each cell in the grid is assigned a weight vector  $\mathbf{w}$  having the same number of dimensions as the training data. This vector can be thought of as pointing to a particular region of the multidimensional parameter space occupied by the data. The weight vectors are initialized prior to training, either randomly or by sampling from the input data. The training of the map is unsupervised, in the sense that the output variable of interest (here, redshift) is not considered. Only the input attributes (galaxy photometry) drive the training. We note that any measured galaxy property (size, magnitude, shape, environment, surface brightness, etc.) could be used in the training. We consider only colors here, as these are the primary drivers of the photo-*z* estimates, and the quantities most

physically tied to redshift. The other properties mentioned can still be used after the map has been created to identify and help break redshift degeneracies within particular regions of galaxy color space.

Training proceeds by presenting the map with a random galaxy from the training sample, which the cells “compete” for. The cell whose weight vector most closely resembles the training galaxy is considered the winner, and is called the Best Matching Unit, or BMU. The BMU as well as cells in its neighborhood on the map are then modified to more closely resemble the training galaxy. This pattern is repeated for many training iterations, over which the responsiveness of the map to new data gradually decreases, through what is known as the learning rate function. Additionally, the extent of the neighborhood around the BMU affected by new training data shrinks with iteration number as well, through what is known as the neighborhood function. These effects cause the map to settle to a stable solution by the end of the training iterations.

To compute the winning cell for a given training object, a distance metric must be chosen. Most often, the Euclidean distance between the training object  $\mathbf{x}$  and the cell weight vector  $\mathbf{w}_k$  is used. With data of dimension  $m$ , this distance is given by:

$$d_k^2 = d_k^2(\mathbf{x}, \mathbf{w}_k) = \sum_{i=1}^m (x_i - w_{k,i})^2. \quad (1)$$



**Figure 2.** Variation of two colors along the self-organizing map:  $u - g$  on the left and  $g - r$  on the right. In the language of machine learning, these are “features” in the data that drive the overall structure of the map. The well-known Lyman break is evident for galaxies at  $2.5 \lesssim z \lesssim 3$  in  $u - g$  and  $3 \lesssim z \lesssim 4$  in  $g - r$  (around  $x = 50$ ,  $y = 90$ ). The regions with red  $g - r$  color spreading diagonally across the lower part of the map are a combination of passive galaxies and dusty galaxies at lower redshift.

However, dimensions with intrinsically larger error than others will be overweighted in this distance metric. To account for this, we instead use the reduced  $\chi^2$  distance between the training object and the cell weight vector. With  $\sigma_{x_i}$  representing the uncertainty in the  $i$ th component of  $\mathbf{x}$ , this becomes

$$d_k^2 = d_k^2(\mathbf{x}, \mathbf{w}_k) = \frac{1}{m} \sum_{i=1}^m \frac{(x_i - w_{k,i})^2}{\sigma_{x_i}^2}. \quad (2)$$

The BMU is the cell minimizing the  $\chi^2$  distance. Once the BMU has been identified, the weight vectors of cells in the map are updated with the relation

$$\mathbf{w}_k(t+1) = \mathbf{w}_k(t) + a(t)H_{b,k}(t)[\mathbf{x}(t) - \mathbf{w}_k(t)]. \quad (3)$$

Here  $t$  represents the current timestep in the training. The learning rate function  $a(t)$  is a monotonically decreasing function of the timestep (with  $a(t) \leq 1$ ), such that the SOM becomes progressively less responsive to new training data. With  $N_{\text{iter}}$  representing the total number of training iterations, we adopt the following functional form for  $a(t)$ :

$$a(t) = 0.5^{(t/N_{\text{iter}})}. \quad (4)$$

The term  $H_{b,k}(t)$  is the value of the neighborhood function at the current timestep for cell  $k$ , given that the current BMU is cell  $b$ . This function is encoded as a normalized Gaussian

kernel centered on the BMU:

$$H_{b,k}(t) = e^{-D_{b,k}^2/\sigma^2(t)}. \quad (5)$$

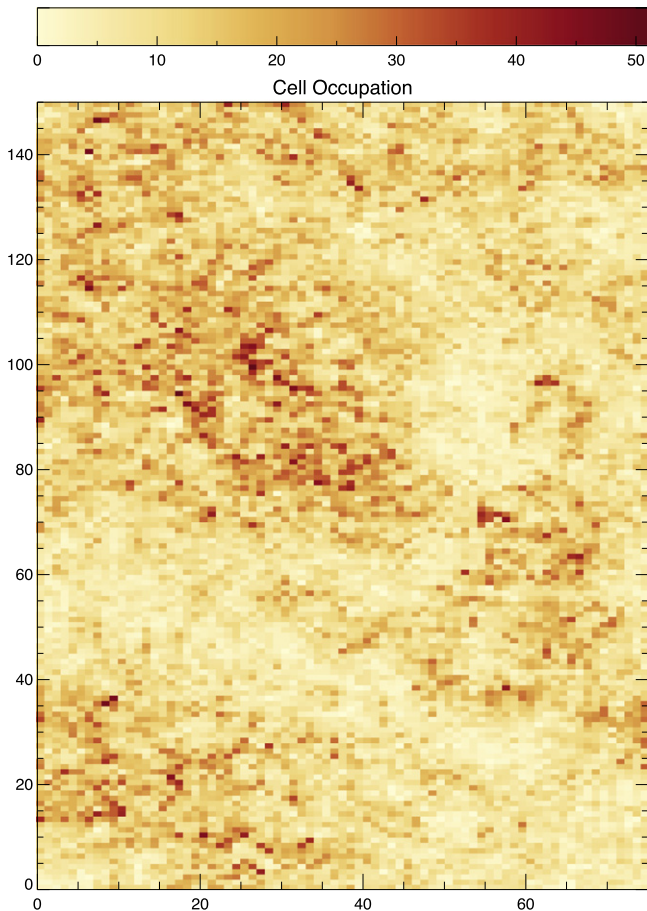
Here  $D_{b,k}$  is the Euclidean distance on the map separating the  $k$ th cell and the current BMU. The width of the Gaussian neighborhood function is set by  $\sigma(t)$  and is given by

$$\sigma(t) = \sigma_s(1/\sigma_s)^{(t/N_{\text{iter}})}. \quad (6)$$

The starting value,  $\sigma_s$ , is large enough that the neighborhood function initially encompasses most of the map. In practice, we set  $\sigma_s$  equal to the size (in pixels) of the smaller dimension of the rectangular map. The width of the neighborhood function shrinks by the end of training such that only the BMU and cells directly adjacent to it are significantly affected by new data.

#### 4.1. Optimizing the Map for the Photo- $z$ Problem

There is significant flexibility in choosing the parameters of the SOM. Parameters that can be modified include the number of cells, the topology of the map, the number of training iterations, and the form and evolution of the learning rate and neighborhood functions. Perhaps most influential is the number of cells. The representative power of the map increases with more cells; however, if too many cells are used the map will overfit the data, modeling noise that does not reflect the true data distribution. Moreover, there is a significant computational



**Figure 3.** SOM colored by the number of galaxies in the overall sample associating with each color cell. The coloration is effectively our estimate of  $\rho(\mathbf{C})$ , or the density of galaxies as a function of position in color space.

cost to increasing the number of cells. On the other hand, if too few cells are used, individual cells will be forced to represent larger volumes of color space, in which the mapping of color to redshift is less well defined.

We explored a range of alternatives prior to settling on the map shown throughout this work. A rectangular map was chosen because this gives any principal component in the data a preferred dimension along which to align. Our general guideline in setting the number of cells was that the map should have sufficient resolution such that the individual cells map cleanly to redshift using standard photo- $z$  codes. With 11,250 cells, the map bins galaxies into volumes, or “voxels,” of color space of comparable size as the photometric error on the data, with the result that variations within each color cell generally do not result in significant change in photo- $z$  estimates. As we discuss in Section 6, the true spread in galaxy redshifts within each color cell is an important quantity to understand for the calibration of  $N(z)$ .

#### 4.2. Algorithm Implementation

We implemented the SOM algorithm in C for computational efficiency. The number of computations required is sizable and scales with both the total number of cells and the number of training iterations. Optimizations are certainly possible, and may be necessary if this algorithm is to be applied to much larger photometric datasets. We initialized the values of the cell

weight vectors with random numbers drawn from a standard normal distribution. The number of training iterations used was  $2 \times 10^6$ , as only minimal improvements in the map were observed for larger numbers of iterations. At each iteration, a random galaxy was selected (with replacement) from the training sample to update the map.

We applied the algorithm based on seven galaxy colors:  $u - g$ ,  $g - r$ ,  $r - i$ ,  $i - z$ ,  $z - Y$ ,  $Y - J$ , and  $J - H$ , which are analogous to the colors that will be measured by *Euclid* and used for photo- $z$  estimation. The errors in the colors are computed as the quadrature error of the photometric errors in the individual bands. If a training object has a color that is not constrained due to bad photometry in one or both of the relevant bands, we ignore that color in the training iteration. Only the well-measured colors for that object are used both to find the BMU and update the corresponding colors of the cell weight vectors. If a color represents an upper/lower limit, we penalize the  $\chi^2$  distance for cells that violate the limit when computing the BMU, with a penalty that varies depending on the size of the discrepancy between the limit and the cell color value.

#### 4.3. Assessing Map Quality

Ideally, the SOM should be highly representative of the data, in the sense that the SEDs of most galaxies in the sample are well-approximated by some cell in the map. To assess the representativeness of the map we calculate what is known as the average quantization error over the entire training sample of  $N$  objects:

$$\epsilon_q = \frac{1}{N} \sum_{i=1}^N \|\mathbf{x}_i - \mathbf{b}_i\|. \quad (7)$$

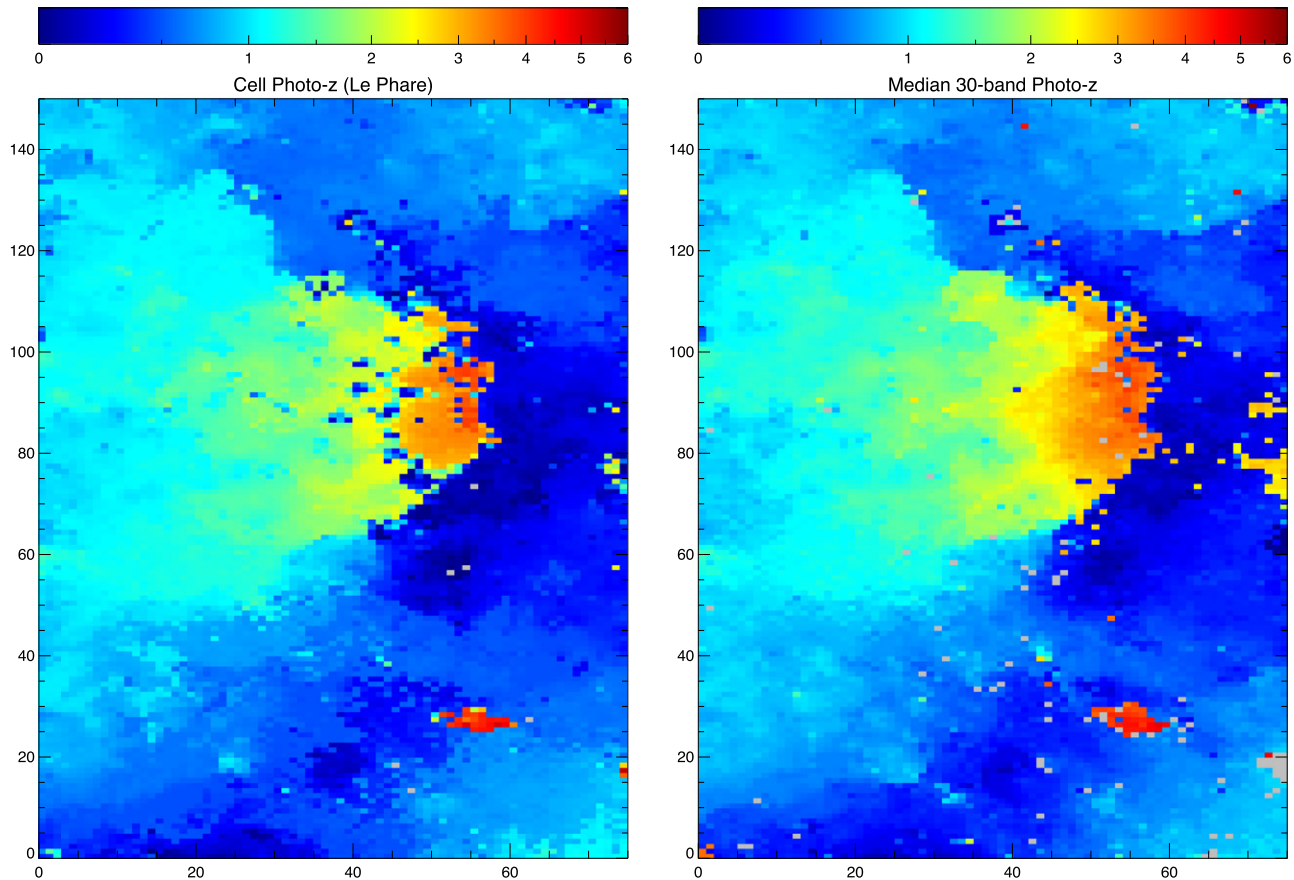
Here  $\mathbf{b}_i$  is the best matching cell for the  $i$ th training object. We find that the average quantization error is 0.2 for the sample. The quantization error is the average vector distance between an object and its best-matching cell in the map.<sup>20</sup> Therefore, with seven colors used to generate the map, the average offset of a particular color (e.g.,  $g - r$ ) of a given galaxy from its corresponding cell in the map is  $0.2/\sqrt{7} = 0.08$  mag. Note that the map provides a straightforward way of identifying unusual or anomalous sources. Such objects will be poorly represented by the map due to their rarity—in effect, they are unable to train their properties into the SOM. Simply checking whether an object is well represented by some cell in the map is therefore a way of testing whether it is “normal,” and may be useful for flagging, for example, blended objects, contaminated photometry, or truly rare sources.

## 5. ANALYZING THE COLOR SPACE MAP

Figure 1 provides an overview of the SOM generated from COSMOS galaxies, which encodes the 8-band SEDs that appear in the data with non-negligible frequency. Note that the final structure of the map is to some extent random and depends on the initial conditions combined with the order in which training objects are presented, but the overall topological

<sup>20</sup> We do not use the  $\chi^2$  distance for this test because of the discrete nature of the cells. Bright objects, or those with smaller photometric errors, will have artificially higher  $\chi^2$  separation from their best-matching cell (even if the photometry matches well), making the metric less appropriate for assessing the representativeness of the map.





**Figure 4.** Photo- $z$  estimates across the map, computed in two ways. Left: photo- $z$ 's computed directly for each cell by applying the *Le Phare* template fitting code to the 8-band photometry represented by the cells. Right: photometric redshifts for the cells computed as the median of the 30-band COSMOS photo- $z$ 's of the objects associated with each cell.

structure will be similar from run to run; this was verified by generating and comparing a number of maps.<sup>21</sup> Figure 2 illustrates the variation of two colors ( $u - g$  and  $g - r$ ) across the map, demonstrating how these features help drive the overall structure. In the following analysis we probe the map by analyzing the characteristics of the galaxies that associate best with each cell in color space.

### 5.1. The Distribution of Galaxies in Color Space, $\rho(\mathbf{C})$

In Figure 3 we show the self-organized map colored by the number of galaxies associating best with each cell. This coloration is effectively our estimate of  $\rho(\mathbf{C})$ , the density of galaxies as a function of position in color space. An important caveat is that the density estimate derived from the COSMOS survey data is likely to be affected to some degree by cosmic variance (and perhaps, to a lesser extent, by shot noise). The true  $\rho(\mathbf{C})$  can ultimately be constrained firmly with the wide-area survey data from LSST, *Euclid*, and *WFIRST*. However, the COSMOS-based  $\rho(\mathbf{C})$  should be a close approximation of what the full surveys will find.

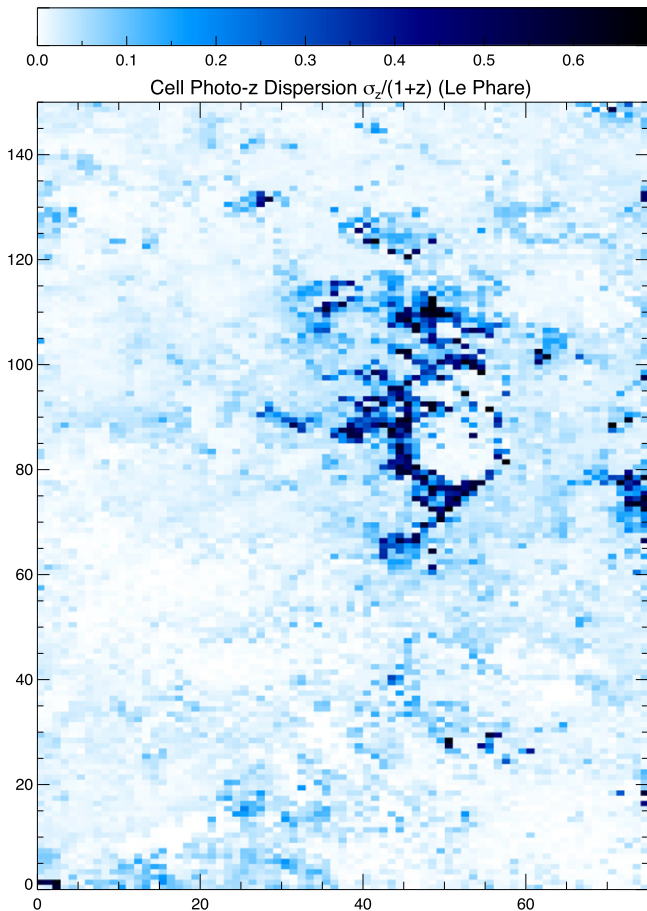
<sup>21</sup> See Appendix B for examples of alternate maps made with different initial conditions and training orders.

### 5.2. Photometric Redshift Estimates across the Map

Because the cells in the SOM represent galaxy SEDs that appear in the data, we can compute photometric redshifts for them to see how they are distributed in redshift. We used the *Le Phare* template fitting code (Arnouts et al. 1999; Ilbert et al. 2006) to compute cell photo- $z$ 's. We used the cell weight vectors (converting the colors to photometric magnitudes normalized in  $i$ -band) as inputs for *Le Phare*, assigning realistic error bars to these model SEDs based on the scatter in the photometry of galaxies associated with each cell. The result of the photo- $z$  fitting is shown on the left side of Figure 4.

We also estimate redshifts on the map by computing the median photo- $z$  of the galaxies associated with each cell, using the 30-band photo- $z$  estimates provided by the COSMOS survey (Ilbert et al. 2009). These photo- $z$  estimates take advantage of more photometric information than is contained in the eight *Euclid*-like filters used to generate the map. Nevertheless, as can be seen on the right side of Figure 4, the resulting map is quite smooth, indicating that the eight *Euclid* bands capture much of the relevant information for photo- $z$  estimation contained in the 30-band data.

Redshift probability density functions (PDFs) generated by the *Le Phare* template fitting can be used to estimate redshift uncertainty across the map, letting us identify cells that have high redshift variance or multiple redshift solutions, as well as cells with a well-defined mapping to redshift. In Figure 5 we



**Figure 5.** Dispersion in the photo- $z$  computed with the *Le Phare* template fitting code as a function of color cell. As can be seen, high dispersion regions predominantly fall in localized areas of color space near the boundary separating high and low redshift galaxies.

show the photo- $z$  dispersion results from the *Le Phare* code. The dispersion is the modeled uncertainty in the redshift assigned to each cell, based on the spread in the cell’s redshift PDF. Figure 5 shows that there are well-defined regions in which the modeled uncertainties are much higher, and that these regions tend to cluster around sharp boundaries between low- and high-redshift galaxies. Note that these boundaries are inherent to the data and indicate regions of significant redshift degeneracy. A possible improvement in this analysis is to more rigorously estimate the photometric uncertainty for each cell using a metric for the volume of color space it represents; we defer this more detailed analysis to future work.

### 5.3. Current Spectroscopic Coverage in COSMOS

One of the most important results of the mapping is that it lets us *directly* test the representativeness of existing spectroscopic coverage. To do so, we used the master spectroscopic catalog from the COSMOS collaboration (M. Salvato et al. 2016, in preparation). The catalog includes redshifts from VLT VIMOS (zCOSMOS, Lilly et al. 2007; VUDS, Le Fèvre et al. 2015), Keck Multi-object Spectrograph for Infrared Exploration (MOSFIRE) (N. Scoville et al., 2015 in preparation; MOSDEF, Kriek et al. 2014), Keck Deep Extragalactic Imaging Multi-object Spectrograph (DEIMOS) (Kartaltepe et al. 2010, G. Hasinger et al. 2015, in preparation), *Magellan* IMACS (Trump et al. 2007), Gemini-S (Balogh et al. 2014),

Subaru FMOS (Silverman et al. 2014), as well as a non-negligible fraction of sources provided by a number of smaller programs. It is important to note that the spectroscopic coverage of the COSMOS field is not representative of the typical coverage for surveys. Multiple instruments with different wavelength coverages and resolutions were employed. Moreover, the spectroscopic programs targeted different types of sources: from AGN to flux-limited samples, from group and cluster members to high-redshift candidates, etc., providing an exceptional coverage in parameter space.

In the left panel of Figure 6, we show the map colored by the median spectroscopic redshift of galaxies associated with each cell, using only galaxies with the highest confidence redshift assignments (corresponding to  $\sim 100\%$  certainty). The gray regions on the map correspond to cells of color space for which no galaxies have such high confidence spectroscopic redshifts; 64% of cells fall in this category. In the right panel of Figure 6 we show the same plot, but using all confidence  $\gtrsim 95\%$  redshifts in the master catalog. Significantly more of the galaxy color space is covered with spectroscopy when the requirement on the quality of the redshifts is relaxed, with only 51% of color cells remaining gray. However, for calibration purposes very high confidence redshifts will be needed, so that the right-hand panel may be overly optimistic. As can be seen in both panels, large and often continuous regions of galaxy color space remain unexplored with spectroscopy.

It should be noted that Figure 6 is entirely data-driven, demonstrating the direct association of observed SED with observed redshift. An interesting possibility suggested by this figure is that the color–redshift relation may be smoother than expected from photo- $z$  variance estimates from template fitting (e.g., Figure 5). High intrinsic variance in the color–redshift mapping should result in large cell-to-cell variation in median spec- $z$ , whereas the actual distribution appears to be rather smooth overall.

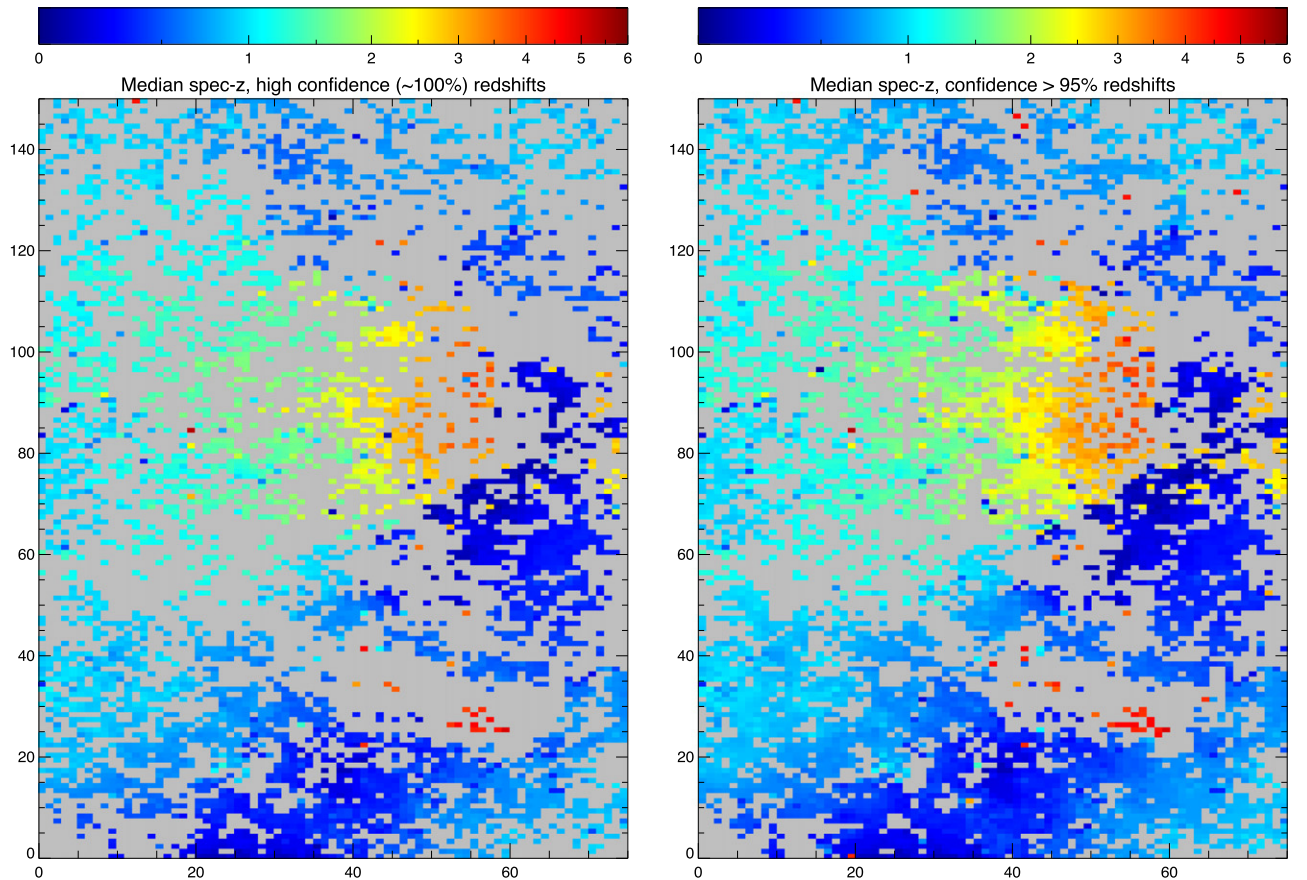
### 5.4. Magnitude Variation across Color Space

Not surprisingly, the median galaxy magnitude varies strongly with location in color space, as illustrated in Figure 7. This variation largely determines the regions of color space that have been explored with spectroscopy, with intrinsically fainter galaxies less likely to have been observed. In fact, as we will discuss further in Section 6.6, the majority of galaxies in unexplored regions of color space are faint, star-forming galaxies at  $z \sim 0.2$ – $1.5$ , which are simply too “uninteresting” (from a galaxy evolution standpoint) to have been targeted in current spectroscopic surveys. Such sources will, however, be critically important for weak lensing cosmology.

## 6. TOWARD OPTIMAL SPECTROSCOPIC SAMPLING STRATEGIES FOR PHOTO- $z$ CALIBRATION

We have demonstrated that the SOM, when applied to a large photometric dataset, efficiently characterizes the distribution of galaxies in the parameter space relevant for photo- $z$  estimation. We now consider the problem of determining the spectroscopic sample needed to calibrate the  $\langle z \rangle$  of the tomographic redshift bins to the required level for weak lensing cosmology. We show that allocating spectroscopic efforts using the color space mapping can minimize the spectroscopy needed to reach the requirement on the calibration of  $N(z)$ .





**Figure 6.** Left: the median spectroscopic redshift of galaxies associating with each SOM cell, using only very high confidence ( $\sim 100\%$ ) redshifts from the COSMOS master spectroscopic catalog (M. Salvato et al. 2015, in preparation). The redshifts come from a variety of surveys that have targeted the COSMOS field; see the text for details. Gray regions correspond to parts of galaxy color space for which no high-confidence spectroscopic redshifts currently exist. These regions will be of interest for training and calibration campaigns. Right: the same figure, but including all redshifts above  $\gtrsim 95\%$  confidence from the COSMOS spectroscopic catalog. Clearly, more of the color space is filled in when the quality requirement is relaxed, but nevertheless large regions of parameter space remain unexplored.

### 6.1. Estimating the Spectroscopic Sample Needed for Calibration

Obtaining spectroscopic redshifts over the full color space of galaxies is obviously beneficial, but the question arises: precisely how many spectra are needed in different regions of color space in order to meet the dark energy requirement? Here we provide a framework for understanding this question in terms of the color space mapping.

First we note that each color cell has some subset of galaxies that best associate with it; let the total number of galaxies associating with the  $i$ th cell be  $n_i$ . We refer to the true redshift probability distribution of these galaxies as  $P_i(z)$ . For the sake of this argument we assume that a tomographic redshift bin for weak lensing will be constructed by selecting all galaxies associating with some subset of the cells in the SOM. Let the total number of cells used in that tomographic bin be  $c$ . Then the true  $N(z)$  distribution for galaxies in the resulting tomographic redshift bin is

$$N(z) = \sum_{i=1}^c n_i P_i(z). \quad (8)$$

The mean of the  $N(z)$  distribution is given by

$$\langle z \rangle = \frac{\int z N(z) dz}{N_T}, \quad (9)$$

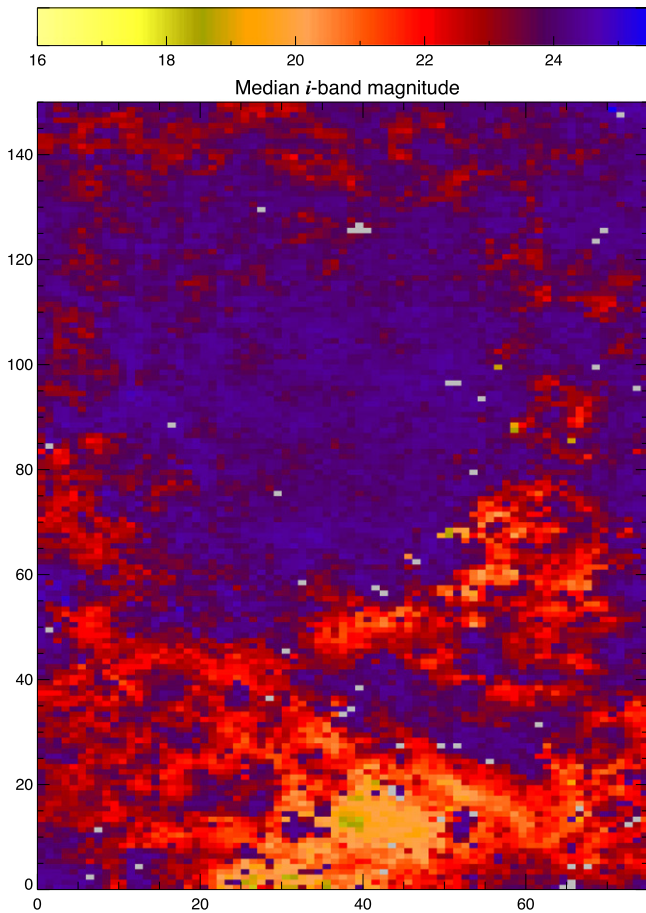
where the integral is taken over all redshifts and  $N_T$  is the total number of galaxies in the redshift bin. Inserting Equation (8) into (9), we find that the mean redshift of the bin can be expressed as

$$\begin{aligned} \langle z \rangle &= \frac{1}{N_T} \int z [n_1 P_1(z) + \dots + n_c P_c(z)] dz \\ &= \frac{1}{N_T} [n_1 \langle z_1 \rangle + \dots + n_c \langle z_c \rangle]. \end{aligned} \quad (10)$$

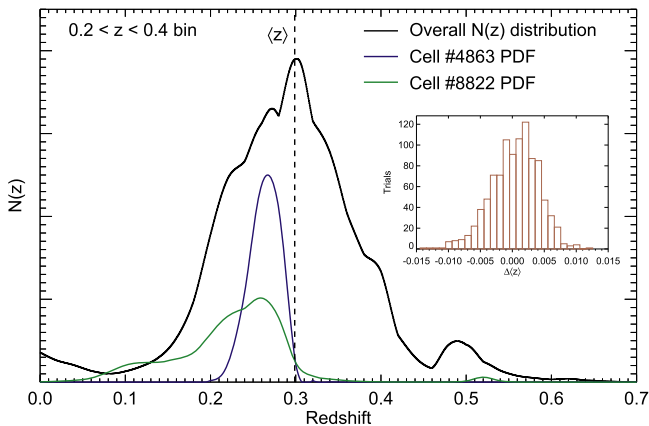
Equation (10) is the straightforward result that the mean redshift of the full  $N(z)$  distribution is proportional to the sum of the mean redshifts of each color cell, weighted by the number of galaxies per cell. The uncertainty in  $\langle z \rangle$  depends on the uncertainty of the mean redshift of each cell, and is expressed as

$$\Delta \langle z \rangle = \frac{1}{N_T} \sqrt{\sum_{i=1}^c n_i^2 \sigma_{z_i}^2}. \quad (11)$$

Equation (11) shows quantitatively what is intuitively clear, namely that the uncertainty in  $\langle z \rangle$  is influenced more strongly by cells with both high uncertainty in their mean redshift and a significant number of galaxies associating with them. This indicates that the largest gain can be realized by sampling more heavily in denser regions of galaxy color space, as well as those regions with higher redshift uncertainty. Conversely, cells with



**Figure 7.** Map colored by the median  $i$ -band magnitude (AB) of galaxies associating with each cell. The strong variation of magnitude with color is not unexpected, and largely explains the absence of spectra in particular regions of galaxy color space.



**Figure 8.** Modeled  $N(z)$  distribution for the 0.2–0.4 redshift bin. The  $N(z)$  distribution is constructed using Equation (8), treating the  $P_i(z)$  functions estimated for each cell from *Le Phare* as truth, and with  $n_i$  values from Figure 3. In addition, two random cell PDFs that contributed to the overall  $N(z)$  of the tomographic bin are shown, one (cell #4863) with a relatively narrowly-peaked distribution and the other (cell #8822) with more redshift uncertainty. We ran Monte Carlo simulations of spectroscopically sampling the  $N(z)$  distributions in various ways to estimate the uncertainty in  $\langle z \rangle$ ; see Table 1. The inset plot shows the distribution of errors in the estimated  $\langle z \rangle$  over 1000 Monte Carlo trials for the simple strategy of obtaining one spectrum per color cell and using Equation (10) to estimate  $\langle z \rangle$ . The uncertainty in the mean is the standard deviation of this distribution, yielding  $\sigma_{\langle z \rangle} / (1 + \langle z \rangle) = 0.0028$ .

very high redshift dispersion could simply be excluded from the weak lensing sample (although caution would be needed to ensure that no systematic errors are introduced by doing so).

If we assume that the  $c$  color cells have roughly equal numbers of galaxies and that  $\sigma_{\langle z_i \rangle}$  is roughly constant across cells, then Equation (11) becomes

$$\Delta \langle z \rangle = \sigma_{\langle z_i \rangle} / \sqrt{c}. \quad (12)$$

With  $\sigma_{\langle z_i \rangle} \sim 0.05(1 + \langle z \rangle)$ , we find  $\sim 600$  color cells with this level of uncertainty would be needed to reach the *Euclid* calibration requirement for the redshift bin. With one spectrum per cell required to reach this level of uncertainty in  $\sigma_{\langle z_i \rangle}$ , this estimate of the number of spectra needed is in rough agreement with that of Bordoloi et al. (2010), and much lower than estimates for direct calibration through random sampling. Note that the mean redshifts  $\langle z_i \rangle$  for each color cell used in Equation (10) should be based on spectroscopic redshifts, to ensure that the estimates are not systematically biased. The error in a cell’s mean redshift estimate,  $\sigma_{\langle z_i \rangle}$ , will depend on the dispersion in the  $P_i(z)$  distribution for the cell, and will scale inversely with the square root of the number of spectra obtained to estimate it.

The preceding analysis treats the photo- $z$  calibration as a stratified sampling problem, in which the overall statistics of a population are inferred through targeted sampling from relatively homogeneous subpopulations. The gain in statistical precision from using Equation (10) to estimate  $\langle z \rangle$  can be attributed to the systematic way in which the full color space is sampled, relative to blind direct sampling. However, stratified sampling will only outperform random sampling in the case that the subpopulations being sampled do in fact have lower dispersion than the overall distribution—i.e., in the case that the  $P_i(z)$  distributions for the color cells have lower redshift dispersion than the  $N(z)$  distribution of all the galaxies in a tomographic bin.

## 6.2. Simulating Different Sampling Strategies

Now we attempt to more realistically estimate the spectroscopic coverage needed to achieve the requirement in our knowledge of  $\langle z \rangle$ . To begin, we assume that the cell redshift PDFs from *Le Phare* are reasonably accurate, and can be taken to represent the true  $P_i(z)$  distributions for galaxies in each color cell. (This assumption is, of course, far from certain, and simply serves as a first approximation.) With the known occupation density of cells of the map (Figure 3), we can then use Equation (8) to generate realistic  $N(z)$  distributions for different tomographic bins. For this illustration, we break the map up into photo- $z$ -derived tomographic bins of width  $\Delta z = 0.2$  over  $0 < z < 2$  (although *Euclid* will most likely use somewhat different bins in practice). An example of one of the  $N(z)$  distributions modeled in this way is shown in Figure 8.

The uncertainty in the estimated  $\langle z \rangle$  of these  $N(z)$  distributions can then be tested for different spectroscopic sampling strategies through Monte Carlo simulations, in which spectroscopy is simulated by randomly drawing from the  $P_i(z)$  distributions. (Alternatively, given our knowledge of the individual  $\sigma_{\langle z_i \rangle}$  uncertainties, Equation (11) can be used directly. In fact, the results were checked in both ways and found to be in agreement).

The results of three possible sampling strategies are given in Table 1. The simplest strategy tested (“Strategy 1”) is to obtain



**Table 1**  
Simulated Uncertainty in  $\langle z \rangle$  for Representative Redshift Bins for Different Sampling Strategies

Redshift Bin	Strategy 1 <sup>a</sup>		Strategy 2 <sup>b</sup>			Strategy 3 <sup>c</sup>	
	#Spectra	$\sigma_{\langle z \rangle}/(1 + \langle z \rangle)$	#Spectra	% Sample Lost <sup>d</sup>	$\sigma_{\langle z \rangle}/(1 + \langle z \rangle)$	#Spectra	$\sigma_{\langle z \rangle}/(1 + \langle z \rangle)$
0.0–0.2	659	0.0034	627	4.2	0.0024	723	0.0028
0.2–0.4	1383	0.0028	1314	4.6	0.0015	1521	0.0020
0.4–0.6	2226	0.0014	2115	3.9	0.0007	2448	0.0010
0.6–0.8	2027	0.0018	1926	4.3	0.0005	2229	0.0012
0.8–1.0	1357	0.0021	1290	4.4	0.0009	1491	0.0013
1.0–1.2	1705	0.0011	1620	4.6	0.0005	1875	0.0008
1.2–1.4	559	0.0029	532	4.4	0.0015	613	0.0021
1.4–1.6	391	0.0044	372	3.3	0.0021	429	0.0031
1.6–1.8	268	0.0064	255	2.7	0.0050	294	0.0055
1.8–2.0	164	0.0093	156	2.1	0.0085	180	0.0088
Total #spectra:	10739		10207			11793	

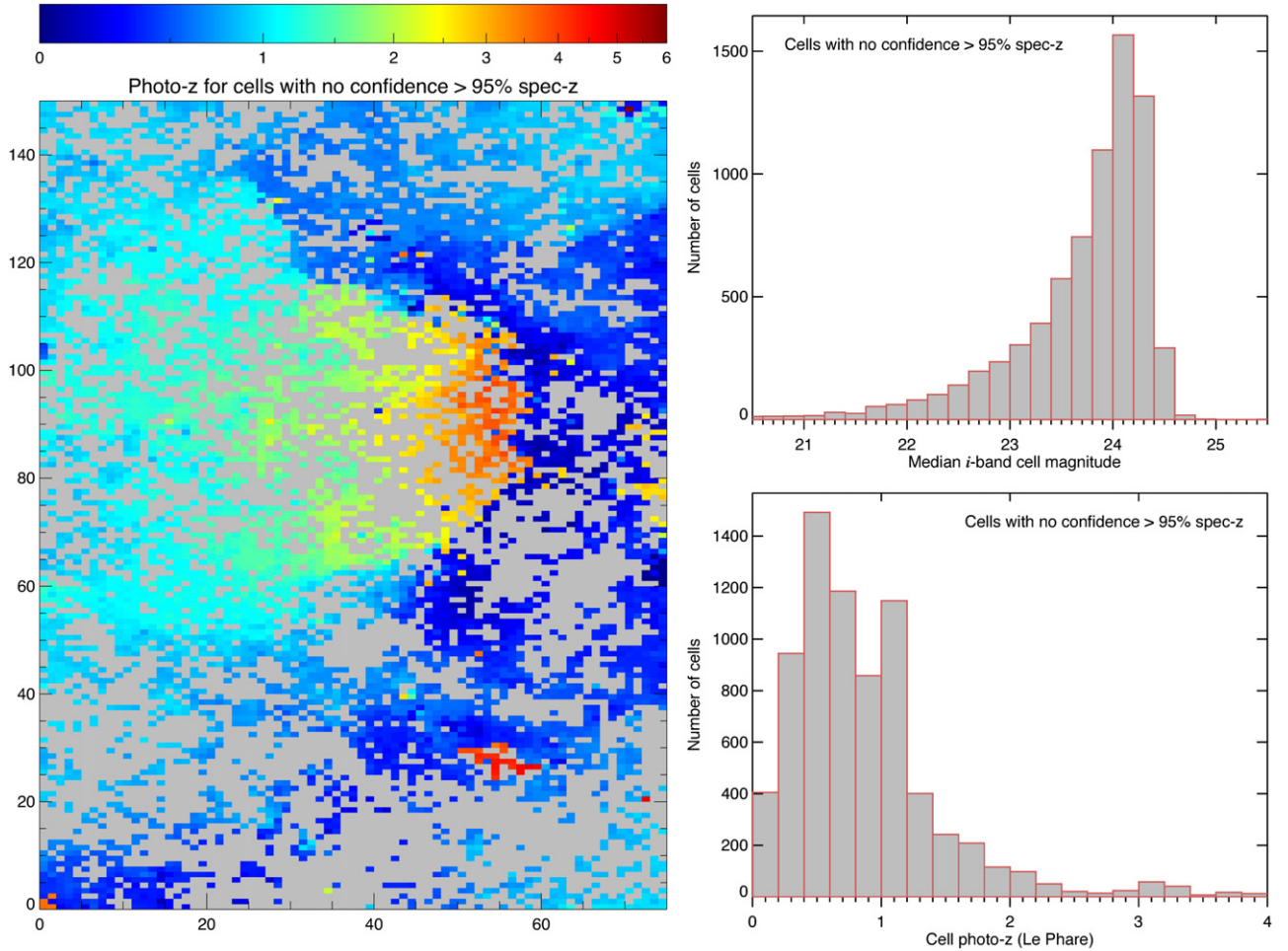
**Notes.**

<sup>a</sup> Obtaining one spectrum per color cell to estimate  $\langle z_i \rangle$ , with  $\langle z \rangle$  computed using Equation (10).

<sup>b</sup> Again obtaining one spectrum per color cell to estimate  $\langle z_i \rangle$ , but rejecting the 5% of cells with the highest redshift uncertainty.

<sup>c</sup> Obtaining three spectra per color cell for the 5% of cells with the highest redshift uncertainty, one spectrum per cell for the other 95%.

<sup>d</sup> The fraction of galaxies lost from the weak lensing sample for that tomographic bin due to excluding 5% of the most uncertain color cells.



**Figure 9.** Left: the inverse of the right panel of Figure 6, illustrating the distribution and photometric redshifts of color cells currently containing no galaxies with confidence  $>95\%$  redshifts. Right, top: magnitude distribution of cells unsampled by spectroscopy, where the cell magnitude is defined as the median  $i$ -band magnitude (AB) of galaxies associating with the cell. Right, bottom: photo- $z$  distribution of unsampled cells, computed with *Le Phare* on the 8-band data representative of the *Euclid* photometry. The majority of the color space regions currently unsampled by spectroscopy correspond to faint galaxies ( $i$ -band  $\sim 23$ – $24.5$  AB) at  $z \sim 0.2$ – $1.5$ .



one spectrum per color cell in order to estimate the cell mean redshifts. Equation (10) is then used to compute the overall mean of the tomographic bin. We expect to meet the *Euclid* requirement,  $\Delta\langle z \rangle \leq 0.002(1 + \langle z \rangle)$ , for 3/10 bins (and come close in the others) with this approach, which would require  $\sim 11\text{k}$  spectra in total.

The second strategy tested is similar to the first, in that one spectrum per cell is obtained. However, galaxies associated with the 5% of the cells in each bin with the highest redshift uncertainty are rejected from the weak lensing sample, and these cells are ignored in the sampling. This significantly reduces the uncertainty in the  $\langle z \rangle$  estimates, with 6/10 bins meeting the requirement; moreover, it reduces the total number of spectra needed by 5%. However, it comes at the cost of reducing the number of galaxies in the weak lensing sample.

The third strategy is to sample the 5% of the cells with the highest redshift uncertainty with three spectra each in order to estimate their mean redshifts with greater accuracy, again obtaining one spectrum for the other 95% of the cells. This strategy again lowers the uncertainty in the  $\langle z \rangle$  estimates substantially, but at the cost of increased spectroscopic effort, requiring  $\sim 12\text{k}$  spectra in total. The additional spectra needed may also prove to be the more difficult ones to obtain, so the effort needed cannot be assumed to scale linearly with the number of spectra.

These examples are simply meant to be illustrative of the possible strategies that can be adopted for the spectroscopic calibration. More refined strategies are possible—for example, an optimal allocation of spectroscopic effort could be devised that scales the number of spectra in a given region of color space proportionately to the redshift uncertainty in that region, while rejecting limited regions of color space that are both highly uncertain and difficult for spectroscopy. Additional spectroscopy may need to be allocated to the higher redshift bins, for which there tend to be fewer cells overall as well as higher dispersion within cells. Tomographic bins could also be intentionally generated to minimize the uncertainty in  $\langle z \rangle$ . The simpler examples shown here do illustrate that, if we believe the cell  $P_i(z)$  estimates from template fitting, the *Euclid* calibration requirement  $\Delta\langle z \rangle \leq 0.002(1 + \langle z \rangle)$  is achievable with  $\sim 10\text{--}15\text{k}$  spectra in total (roughly half of which already exist).

### 6.2.1. Is Filling the Map with Spectroscopy Necessary?

The number of spectra needed derived above assumes that at least one spectrum per SOM color cell is necessary to estimate the  $\langle z_i \rangle$  for that cell. However, if a particular region of color space is very well understood and maps smoothly to redshift, sparser spectroscopic sampling in that region together with interpolation across cells might be sufficient. Equivalently, groups of neighboring cells with low redshift uncertainty that map to roughly the same redshift could potentially be merged using a secondary clustering procedure, thus lowering the overall number of cells and the number of spectra required. These considerations suggest that, while the exact number of spectra required to meet the calibration requirement is uncertain, the results presented above are likely to represent upper limits.

### 6.3. Estimating the True Uncertainty in the Color–Redshift Mapping

The analysis above highlights the important role played by the true uncertainty in the mapping from color to redshift for some number of broadband filters. A single spectroscopic redshift gives us an estimate of a cell’s mean redshift with an uncertainty that depends on the true dispersion in  $P_i(z)$  for the cell. Unfortunately, we cannot know this distribution precisely without heavily sampling the cell with spectroscopy, which is impractical (we can, however, model it with different photo- $z$  codes).

Given the importance of the uncertainty in the mapping of color to redshift in different parts of color space, strategies to constrain this uncertainty efficiently should be considered. One possibility is that a limited amount of ancillary photometry can effectively identify the redshift variation within cells. The reason this could work is that objects with very different redshifts but similar *Euclid* colors are likely to be distinguishable in other bands (e.g., IR or FUV). Moreover, well-defined and distinct magnitude distributions for objects in the same region of color space could indicate and help break a color–redshift degeneracy.

Another interesting possibility is that the uncertainty in  $P_i(z)$  in different parts of color space can be constrained *from the map itself*, as it is filled in with spectroscopy. This is because the cell-to-cell redshifts would be expected to show high variation in parts of color space where the relation has high intrinsic variation, and vary more smoothly in regions where the relation is well-defined. We defer a detailed analysis of this possibility to future work.

### 6.4. Effect of Photometric Error on Localization in Color Space

Photo- $z$  uncertainty is due both to the inherent uncertainty in the mapping from some number of broadband colors to redshift, as well as to the uncertainty in the colors themselves due to photometric error. It is well-known that photometric redshift performance degrades rapidly at low signal-to-noise for the latter reason.

*Euclid* and other dark energy surveys will also observe deep calibration fields, in which the survey depth is  $\sim 2$  mag deeper than the main survey. These will preferentially be the fields with spectroscopic redshifts used for training and calibration. Because of the photometric depth, the photometric error will be negligible in these fields, and the uncertainty in mapping color to redshift will be due to inherent uncertainty in the relation.

Even if the relation between color and redshift is mapped as fully as possible in the deep fields, photometric error in the shallower full survey will introduce uncertainties by allowing galaxies to scatter from one part of color space to another. The errors thus introduced to the tomographic redshift bins can be well characterized using the multiple observations of the deep fields, and folded into the estimates of  $\sigma_{\langle z_i \rangle}$ . The ultimate effect on the  $N(z)$  estimates will depend on the S/N cut used for the weak lensing sample.

### 6.5. Cosmic Variance

One of the primary difficulties with direct measurement of the  $N(z)$  distribution for tomographic redshift bins is the need for multiple independent sightlines in order to avoid cosmic

variance-induced bias in the  $N(z)$  estimates. Systematically measuring the color–redshift relation as described here, however, largely sidesteps the problem posed by cosmic variance. This is because the true  $\rho(\mathcal{C})$  distribution can be inferred from the full survey (which will be unaffected by cosmic variance or shot noise), while the calibration of  $P(z|\mathcal{C})$  can be performed on data from a small number of fields, as long as galaxies in those fields span the overall galaxy color space sufficiently.

### 6.6. Galaxies in Under-sampled Regions of Color Space

From the preceding analysis, a reasonable step toward calibration of the photo- $z$ 's for cosmology is to target the regions of multicolor space currently lacking spectroscopy (the gray regions in Figure 6). It is therefore important to understand the nature of the galaxies in these regions, in order to predict the spectroscopic effort needed.

Of the 11,250 cells in the SOM presented here, roughly half currently have no objects with high-confidence spectroscopic redshifts. The distribution of these cells on the map, as well as their photometric redshift estimates, are displayed on the left side of Figure 9. The right side of Figure 9 shows the overall magnitude and photometric redshift distribution of the unsampled cells of color space. Most unsampled cells represent galaxies fainter than  $i = 23$  (AB) at redshifts  $z \sim 0.2$ – $1.5$ , and  $\sim 83\%$  of these are classified as star-forming by template fitting. These magnitude, redshift, and galaxy type estimates directly inform our prediction of the spectroscopic effort that will be required to calibrate the unsampled regions of galaxy color space.

Generally speaking, these galaxies have not been targeted in existing spectroscopic surveys because they are faint and not considered critical for galaxy evolution studies. However, they are abundant and thus important for weak lensing cosmology. In Appendix A we give a detailed estimate of the observing time that would be needed to fill in the empty parts of color space with a fiducial survey with Keck, making use of the Low Resolution Imaging Spectrograph (LRIS), DEIMOS, and MOSFIRE. We find that  $\sim 40$  nights would be required if we reject the 1% most difficult cells—a large time allocation, but not unprecedented in comparison with other large spectroscopic surveys. This is significantly less than the  $\sim 100$  nights needed to obtain a truly representative sample without prior knowledge of the color distribution (Newman et al. 2015). For both LSST and *WFIRST* the calibration sample required is likely to be significantly larger, due to the greater photometric depths of these surveys in comparison with *Euclid*. Therefore, methods to improve the sampling as proposed here will be even more important to make the problem tractable for those surveys.

## 7. DISCUSSION

Statistically well-understood photometric redshift estimates for billions of galaxies will be critical to the success of upcoming Stage IV dark energy surveys. We have demonstrated that self-organized mapping of the multidimensional color distribution of galaxies in a broadband survey such as *Euclid* has significant benefits for redshift calibration. Importantly, this technique lets us identify regions of the photometric

parameter space in which the density of galaxies  $\rho(\mathcal{C})$  is non-negligible, but spectroscopic redshifts do not currently exist. These unexplored regions will be of primary interest for spectroscopic training and calibration efforts.

Applying our SOM-based analysis to the COSMOS field, we show that the regions of galaxy parameter space currently lacking spectroscopic coverage generally correspond to faint ( $i$ -band magnitude (AB)  $\gtrsim 23$ ), star-forming galaxies at  $z < 2$ . We estimated the spectroscopy required to fill the color space map with one spectrum per cell (which would come close to or achieve the required precision for calibration) and found that a targeted,  $\sim 40$  night campaign with Keck (making use of LRIS, DEIMOS and MOSFIRE) would be sufficient (Appendix A). It should be noted that this analysis is specific to the *Euclid* survey. The calibration needs of both LSST and *WFIRST* are likely to be greater, due to the deeper photometry that will be obtained by those surveys.

We demonstrated that systematically sampling the color space occupied by galaxies with spectroscopy can efficiently constrain the  $N(z)$  distribution of galaxies in tomographic bins. The precise number of spectra needed to meet the bias requirement in  $\langle z \rangle$  for cosmology depends sensitively on the uncertainty in the color–redshift mapping. Template-based estimates suggest that this uncertainty is rather high in some regions of *Euclid*-like color space. However, the smoothness of the spectroscopic redshift distribution on the map suggests that the template-based uncertainties may be overestimated, which would reduce the total number of spectra needed for calibration.

Assuming that the uncertainties in  $P(z|\mathcal{C})$  from template fitting are accurate, we demonstrate that the *Euclid* requirement on  $\Delta \langle z \rangle$  should be achievable with  $\sim 10$ – $15$ k total spectra, about half of which already exist from various spectroscopic surveys that have targeted the COSMOS field. Understanding the true uncertainty in  $P(z|\mathcal{C})$  will likely prove critical to constraining the uncertainty in  $\langle z \rangle$  for the tomographic bins, and we suggest that developing efficient ways of constraining this uncertainty should be prioritized.

The topological nature of the SOM technique suggests other possible uses. For example, a potentially very useful aspect of the SOM is that it lets us quantify the “normality” of an object by how well-represented it is by some cell in the map. Rare objects, such as AGNs, blended sources, or objects with otherwise contaminated photometry could possibly be identified in this way. We also note that the mapping, by empirically constraining the galaxy colors that appear in the data, can be used both to generate consistent priors for template fitting codes as well as test the representativeness of galaxy template sets. These applications will be explored in future work.

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**Table 2**

Assumed Continuum Sensitivity per  $R \sim 1500$  Resolution Element for Selected Keck Instruments on a Flat-spectrum 24th Magnitude (AB) Object in a One-hour Exposure

Instrument	Band	SNR	Number of Nights
LRIS	I	1.5	7
DEIMOS	I	2.0	19
MOSFIRE	Y	0.7	4
MOSFIRE	J	0.6	1
MOSFIRE	H	0.5	7
MOSFIRE	K	0.4	1

**Note.** In the last column we give the estimated number of nights required for each instrument in a fiducial survey designed to complete the *Euclid* color space calibration.

the European Southern Observatory Very Large Telescope, Paranal, Chile, under Large Program 185.A-0791. This work is based in part on data products made available at the CESAM data center, Laboratoire d’Astrophysique de Marseille.

## APPENDIX A

### ESTIMATING THE OBSERVING TIME REQUIRED FOR THE *EUCLID* CALIBRATION

Given the  $<0.2\%$  accuracy in  $\langle z \rangle$  required for the *Euclid* tomographic bins, and following the analysis presented above, a nearly optimal approach would be to obtain one spectrum per SOM cell, while rejecting  $\sim 1\%$  of the cells requiring the longest spectroscopic observations. Taking existing spectroscopy into account, a total of  $\sim 5k$  new spectroscopic redshifts would be needed. We estimate that these spectra could be obtained in  $\sim 40$  nights with Keck, as outlined below.

To quantify the required exposure time, we constructed a fiducial survey on the Keck telescope with the LRIS (Oke et al. 1995), the DEIMOS (Faber et al. 2003), and the MOSFIRE (McLean et al. 2012) instruments. This telescope/instrument combination was chosen because the full redshift range of the calibration sample can be optimally probed with these instruments, and their performance in obtaining redshifts for  $i \sim 24.5$  galaxies has been demonstrated in numerous publications (e.g., Steidel et al. 2004; Newman et al. 2013; Kriek et al. 2014). For LRIS we follow Steidel et al. (2004) and assume the 300 groove  $\text{mm}^{-1}$  grism blazed at 5000 Å on the blue side and the 600 groove  $\text{mm}^{-1}$  grating blazed at 10,000 Å on the red side with the D560 dichroic. With DEIMOS the 600 groove  $\text{mm}^{-1}$  grating tilted to 7000 Å was assumed. MOSFIRE was assumed to be in its default configuration. Sensitivities were estimated using the official exposure time calculators (ETCs) provided by Keck by scaling from a 24th magnitude flat spectrum object. We assume  $1''$  seeing, a  $1''$  wide slit, an airmass of 1.3, and we include appropriate slit losses. For all instruments we scaled the SNR to a binning of  $R \sim 1500$ , the minimum required resolution for calibration redshifts. The assumed SNRs in a one hour exposure at 24th magnitude (AB) are given in Table 2.

We assume that the galaxies in the cells needing spectroscopy have the redshifts, galaxy spectral types, and reddening derived from template fitting with *Le Phare*. The modeled galaxy spectral types, redshifts, and observed magnitudes were used to determine the required SNR and the instrument such

that a  $>99\%$  reliable redshift can be obtained. For star-forming galaxies at  $z < 2.7$  we require  $\text{SNR} = 2$  on the continuum because bright rest-frame optical emission lines will be used to determine the redshift. For star-forming galaxies at  $z \geq 2.7$  we require  $\text{SNR} = 3$  on the continuum to clearly detect the Lyman break and the rest frame ultraviolet (UV) absorption features with LRIS or DEIMOS (e.g., Steidel et al. 2003). For galaxies classified as passive, we require  $\text{SNR} = 5$  on the continuum (e.g., Kriek et al. 2009; Onodera et al. 2012), while objects intermediate between passive and star-forming were allowed to linearly scale between an SNR of 5 and 2 with an increasingly star-forming spectral template, because the spectral feature strength increases with star formation rate.

The magnitude measured in the band closest to the most prominent spectral feature was assumed for the SNR calculation, and the instrument with the highest sensitivity at that feature was assumed. For passive galaxies it was assumed that the 4000 Å break must be targeted at  $z < 2.3$  and the 1216 Å Lyman forest break at higher redshifts, with DEIMOS used at  $z < 1.3$ , LRIS at  $1.3 < z < 1.4$ , MOSFIRE at  $1.4 < z < 2.3$ , LRIS at  $2.3 < z < 3.5$ , and DEIMOS at  $z > 3.5$ . For other galaxies, the strongest of  $\text{H}\alpha$ ,  $\text{H}\beta$ ,  $\text{O}[\text{III}]$ , and  $\text{O}[\text{II}]$  was targeted at  $z < 2.7$ , with DEIMOS at  $z < 1.5$  and MOSFIRE at  $1.5 < z < 2.7$ . The 1216 Å Lyman forest break was targeted at higher redshifts, with LRIS at  $2.7 < z < 3.5$  and DEIMOS at  $z > 3.5$ .

Objects were then grouped into masks by instrument and exposure time, assuming a multiplexing of 70 for DEIMOS and 20 for LRIS and MOSFIRE, making the assumption that deep observations could be obtained for rare faint objects by observing them in multiple masks. Assuming nights are 10 h long, overheads are 10%, 20% of the objects need to be observed by more than one instrument to confirm the redshift, and 30% are losses due to weather, we obtain the estimate of required observing time given in Table 2.

An exploratory program in early 2015 used samples from poorly sampled regions of color space as fillers on 2–4 hr Keck DEIMOS slit masks, finding that  $>98\%$  of sources were readily identified from strong  $\text{O}[\text{II}]$ ,  $\text{O}[\text{III}]$ , and/or  $\text{H}\alpha$  emission, while the non-detected sources had photometric redshifts for which no line detection was expected by DEIMOS.

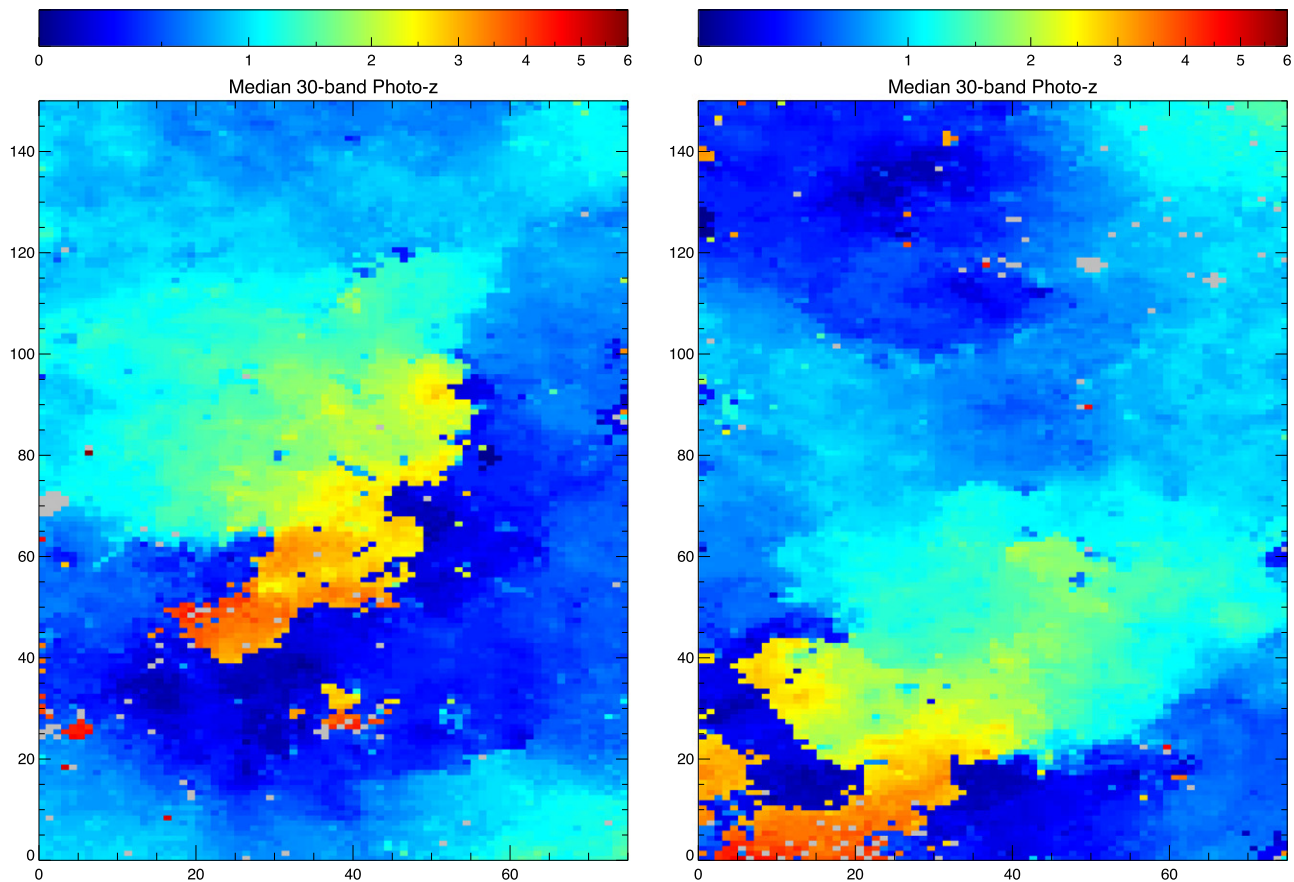
We note that an additional  $\sim 12$  nights would be required to get to 99.8% completeness in color cells, and  $\sim 49$  (for a total of  $\sim 100$ ) more nights to reach 99.9% completeness. This confirms the difficulty in obtaining truly complete samples noted by previous work, as well as the importance of systematically rejecting sources (Newman et al. 2015).

## APPENDIX B

### ALTERNATE SOM EXAMPLES

Figure 10 shows two alternate maps generated with the same COSMOS data, but with different starting conditions and training orders. Note that the overall topological features are the same. The representativeness of these maps (in the sense described in Section 4.3) is essentially identical and the map shown throughout the paper. However, the positions and orientations of different photometric clusters are random.





**Figure 10.** Two alternate versions of the SOM created with different initial conditions/training order, colored by the cell 30-band median photo- $z$  (compare to Figure 4, right panel).

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