MR4525296

Reviewed

Article Cite Review PDF

Jafari, Amir (IR-SHAR); Najafi Amin, Amin (IR-SHAR)

A new basis for the U-invariants of binary forms. (English summary)

Comm. Algebra **51** (2023), no. 1, 248–253.

Classifications

15A72 - Vector and tensor algebra, theory of invariants

16W22 - Actions of groups and semigroups; invariant theory (associative rings and algebras)

Citations

Review

Let V_n be the K-vector space of all binary homogeneous forms of degree n. There is a natural action of the group SL_2 on this space. The subgroup U of upper triangular unipotent matrices in SL_2 can be identified with the additive group of the field K through $\left[\begin{smallmatrix} 1 & \lambda \\ 0 & 1 \end{smallmatrix}\right] \mapsto \lambda \in K$. The action of U extends to the polynomial algebra $K[a_0, a_1, \ldots, a_n] = \mathcal{O}(V_n)$. The homogeneous polynomials $p(a) = p(a_0, \ldots, a_n)$ that satisfy the equation $\lambda \cdot p(a) = p(a)$ are called U-invariants. In 1868, Gordan proved that the algebra $K[a_0, a_1, \ldots, a_n]^U$ of U-invariants is finitely generated. Computing an explicit minimal set of generators is very hard. It has been done only for $n \leq 10$. In the paper under review the authors construct an explicit set of generators for $K[a_0, a_0^{-1}, a_1, \ldots, a_n]^U$ of degrees one, two and three. They are algebraically independent polynomials $p_1 = a_0, p_2, p_3, p_4, \ldots$ such that $K[a_0, a_0^{-1}, a_1, \ldots, a_n]^U = K[a_0, a_0^{-1}, p_1, \ldots, p_n]$. This means that any U-invariant $p \in K[a_0, a_1, \ldots, a_n]$, after being multiplied by an appropriate power of a_0, is expressed as a polynomial in $p_1, p_2, \ldots, p_n$, where $p_i \in K[a_0, a_1, \ldots, a_i]$. Further, the authors give a basis for the U-invariants of the reducible SL_2-representation $V_{n_1} \oplus \cdots \oplus V_{n_m}$ with $0 < n_1 \leq n_2 \leq \cdots \leq n_m$.

Reviewer: Ciampella, Adriana

References

Hide references Search References

This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

1. Brouwer, A. E., Popoviciu, M. (2010). The invariants of the binary decimic. *J. Symbolic Comput*. 45(8): 837–843. DOI: 10.1016/j.jsc.2010.03.002. **MR**2657667
2. Freudenburg, G. (2017). *Algebraic Theory of Locally Nilpotent Derivations*. Encyclopedia of Mathematical Sciences, Vol. 136, 2nd ed. Berlin: Springer-Verlag. **MR**3700208
3. Gordan, P. (1868). Beweis, dass jede Covariante und Invariante einer binären Form eine ganze Function mit numerischen Coefficienten einer endlichen Anzahl solcher Formen ist. *J. Reine Angew. Math*. 69: 323–354. **MR**1579424
4. Grace, J. H., Young, A. (2010). *The Algebra of Invariants*. Cambridge, England: Cambridge University Press. Reprint of 1903 Original. **MR**2850282
5. Hilbert, D. (1993). *Theory of Algebraic Invariants*. Cambridge, England: Cambridge University Press. Translated from German and with preface by Laubenbacher, R. C., edited and with an introduction by Strumfels, B. **MR**1266168
6. Kraft, H., Procesi, C. (2021). Perpetuants: a lost treasure. *IMRN*. 2021(5):3597–3632. DOI: 10.1093/imrn/rnaa032. **MR**4227580
7. Schur, I. (1968). *Vorlesungen über Invariantentheorie. Die Grundlehren der Mathematischen Wissenschaften*, Vol. 143. Berlin: Springer-Verlag. **MR**0229674

Previous Release